Mathematics

JEE Advanced Revision Booklet

A Comprehensive Revision Program

Content

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JEE Advanced Revision Booklet	Quadratic Equations

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. If $x^2+xy=12$ and $2xy+3y^2+5=0$ then $x+4y$ can be:

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

2. If 'a' and 'b' are distinct zeroes of the polynomial x^3-2x+c and $a^2\left(2a^2+4ab+3b^2\right)=3$ then $b^2\left(3a^2+4ab+2b^2\right)$ is equal to :

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6

3. Let α and β be the real roots of the equation $x^2-x(k-2)+\left(k^2+3k+5\right)=0.$ The maximum value of $\alpha^2+\beta^2$ is :

(A) 18 **(B)** 19 **(C)** 50/9 **(D)** 50/19

4. For $x\in R,$ the maximum value of $\sqrt{x^4-3x^2-6x+13}-\sqrt{x^4-x^2+1}$ is :

(A) 3 **(B)** $\sqrt{10}$ **(C)** $\sqrt{13}-\sqrt{3}$ **(D)** $2\sqrt{3}$

5. Suppose $A=\{x\,;5x-a\le 0\}, B=\{x\,;6x-b>0\},$ $a,b\in N$ and $A\cap B\cap N=\{2,3,4\}$. The number of such pairs (a, b) is:

(A) 20 **(B)** 25 **(C)** 30 **(D)** 35

6. The number of real solutions to the equation $\sqrt{3x^2-18x+52}+\sqrt{2x^2-12x+162} = \sqrt{-x^2+6x+280}$ is(are) :

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

7. a, b, c, d are distinct integers such that $(x-a)(x-b)(x-c)(x-d)=4$ has an integral root r. Then $a+b+c+d$ is equal to :

(A) r **(B)** $2r$ **(C)** $3r$ **(D)** $4r$

8. Let the n real roots of the equation $x^n-2nx^{n-1}+2n(n-1)x^{n-2}+ax^{n-3}+bx^{n-4}+........+c=0$ be $\alpha_1,\alpha_2,\alpha_3......\alpha_n$ then $\sum_{k=1}^{n}(-1)^{k-1}\alpha_k$ is :

(A) Zero **(B)** One **(C)** Two **(D)** Three

9. The number of monic quadratic polynomials of the form x^2+ax+b with integer roots, where 1, a, b are in AP is(are) :

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4

10. Let $A=[-2,4)$, $B=\{x\,;\,x^2-ax-4\le 0\}$. If $B\subseteq A$, then the range of real a is :

(A) $[-1,2)$ **(B)** $[-1,2]$ **(C)** [0, 3] **(D)** [0, 3)

11. The sum of all real x such that, $\dfrac{4x^2+15x+17}{x^2+4x+12}=\dfrac{5x^2+16x+18}{2x^2+5x+13}$ is :

(A) 0 **(B)** $\dfrac{-11}{3}$ **(C)** $-\dfrac{20}{3}$ **(D)** $\dfrac{23}{3}$

12. The number of solutions to the equation $2\sqrt{1+x\sqrt{1+(x+1)\sqrt{1+(x+2)\sqrt{1+(x+3)(x+5)}}}}=x$ is :

(A) 0 **(B)** 2 **(C)** 4 **(D)** 16

PARAGRAPH FOR QUESTIONS 13 - 15

Given that $a>0$, $\left|ax^2+bx+c\right|\le 1$, if $-1\le x\le 1$, $a, b, c\in R$ and $ax+b$ has its maximum value 2, when $-1\le x\le 1$. Then :

13. $a=$

(A) 3 **(B)** 1 **(C)** 2 **(D)** 4

14. $b=$

(A) -1 **(B)** 2 **(C)** 1 **(D)** 0

15. $c=$

(A) -1 **(B)** 0 **(C)** 1 **(D)** 2

PARAGRAPH FOR QUESTIONS 16 - 18

Consider the equation $x^4-(k-1)x^2+(2-k)=0$. The complete set of possible values of real k for which the equation has:

16. Four distinct real roots is :

(A) $(-\infty, 2)$ **(B)** $(2\sqrt{2}-1,\ 2)$

(C) $(\sqrt{2}-1,\ 2\sqrt{2}-1)$ **(D)** $(2, \infty)$

17. 3 distinct real roots is :

(A) $\{2\}$ **(B)** $\{\sqrt{2}-1,\ 2\}$ **(C)** $\{\sqrt{5}-1\}$ **(D)** $\{2\sqrt{2},\ \sqrt{3}-\sqrt{2}\}$

18. 2 distinct real roots is :

(A) (0, 2) **(B)** $(-\infty,\ 2\sqrt{2}-1)$ **(C)** $(2, \infty)$ **(D)** $\{2\sqrt{2}-1\}\cup(2, \infty)$

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

19. The function $f(x) = ax^2 - c$ satisfies $-4 \le f(1) \le -1$ and $-1 \le f(2) \le 5$. Which of the following statements is true?

(A) $-1 \le f(3) \le 20$ **(B)** $2 \le f(3) \le 18$ **(C)** $-\frac{1}{2} \le f(3) \le 20$ **(D)** $0 \le f(3) \le 20$

20. Let $a \ne 0, b, c$ be integers and $\sin\theta, \cos\theta$ be the rational roots of the equation $ax^2 + bx + c = 0$. Then:

(A) a is a perfect square **(B)** $a + 2c$ is a perfect square
(C) $a - 2c$ is a perfect square **(D)** b is a perfect square

21. If all roots of the polynomials $6x^2 - 24x - 4a$ and $x^3 + ax^2 + bx - 8$ are non-negative real numbers, then:

(A) $a = -6$ **(B)** $a = 2$ **(C)** $b = 10$ **(D)** $b = 12$

22. Let $P(x) = x^4 + ax^3 + bx^2 + cx + 1$ and $Q(x) = x^4 + cx^3 + bx^2 + ax + 1$ with $a, b, c \in R$ and $a \ne c$. If $P(x) = 0$ and $Q(x) = 0$ have two common roots then :

(A) $b = -2$ **(B)** $b = 2$ **(C)** $a + c = 0$ **(D)** $a - 2c = 0$

23. All the roots of $x^3 + ax^2 + bx + c$ are positive integers greater than 2 and the coefficient satisfy $a + b + c = -46$:

(A) $a = -14$ **(B)** $a = 14$
(C) Number of distinct roots of the equation=3 **(D)** Number of distinct roots of the equation=2

24. If the equations $ax^3 + (-a + b)x^2 + (-b + c)x - c = 0$ and $2x^3 + x^2 + 2x - 5 = 0$ have a common root $(a \ne 0, a, b, c \in R)$ then $a + b + c$ is equal to :

(A) 0 **(B)** $5a$ **(C)** $3b$ **(D)** $2c$

25. Let $f(x) = ax^2 + bx + c,\ a, b, c \in R.$ Suppose $|f(x)| \le 1,\ \forall\ x \in [0, 1]$ then :

(A) $|a| \le 8$ **(B)** $|a + 2b + 4c| \le 4$ **(C)** $|a| + |b| + |c| \le 17$ **(D)** $|3a + 2b| \le 8$

26. Consider the equation $\sqrt{x + \sqrt{2x - 1}} + \sqrt{x - \sqrt{2x - 1}} = A$

(A) For A = $\sqrt{2}$, $x \in \left[\frac{1}{2}, 1\right]$ **(B)** For A = $\sqrt{2}$, $x \in \left[0, \frac{1}{2}\right]$

(C) For A = 1, $x \in \phi$ **(D)** For A = 2, $x = \frac{3}{2}$

27. Suppose $f(x) = -x^2 + bx + 1$ and $g(x) = x^2 + 2x + c,\ b, c \in R,$ are such that maximum $f(x) \le$ minimum $g(x)$ as x varies over R. Then possible values that c can take is(are) :

(A) 1 **(B)** $\sqrt{2}$ **(C)** 2 **(D)** $\sqrt{5}$

28. The greatest value of the function $f(x) = \frac{1}{2bx^2 - x^4 - 3b^2}$ on the interval $[-2, 1]$ depending on the parameter b is(are) :

(A) $-\frac{1}{3b^2}$ if $b \in [0, 2]$ **(B)** $\frac{1}{4b - 4 - 3b^2}$ if $b \in [0, 4]$

(C) $\frac{1}{8b - 16 - 3b^2}$ if $b \le 2$ **(D)** $-\frac{1}{3b^2}$ if $b \ge 2$

29. Given that a, b, c are positive distinct real numbers such that quadratic expressions $ax^2 + bx + c,\ bx^2 + cx + a$ and $cx^2 + ax + b$ are always non-negative. Then the expression $\dfrac{a^2 + b^2 + c^2}{ab + bc + ca}$ can never lie in :

(A) $(-\infty, 2]$ **(B)** $(-\infty, 1]$ **(C)** $(2, 4)$ **(D)** $[4, \infty)$

30. The equation $8x^4 - 16x^3 + 16x^2 - 8x + a = 0,\ a \in R$ has :

(A) Atleast two real roots $\forall\ a \in R$

(B) Atleast two imaginary roots $\forall\ a \in R$

(C) The sum of all non-real roots equal to 2, if $a > \dfrac{3}{2}$

(D) The sum of all non-real roots equal to 1, if $a \le \dfrac{3}{2}$

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column I are labelled as (A), (B), (C) & (D) whereas statements in Column 2 are labeled as p, q, r, s & t. More than one choice from Column 2 can be matched with Column 1.

31. **MATCH THE COLUMN :**

	Column 1		Column 2
(A)	If a, b, c are length of sides of a triangle, then the roots of the equation $a^2x^2 + \left(b^2 + a^2 - c^2\right)x + b^2 = 0$ are	**(p)**	of opposite signs
(B)	If a, b, c are unequal positive numbers and b is A.M. of a and c, then the roots of the equation $ax^2 + 2bx + c = 0$ are	**(q)**	both positive
(C)	If $a \in R$, then roots of the equation $x^2 - (a+1)x - a^2 - 4 = 0$ are	**(r)**	both negative
(D)	If a, b, c are unequal positive numbers and b is H.M. of a and c, then the roots of the equation $ax^2 + 2bx + c = 0$ are	**(s)**	real and distinct
		(t)	imaginary

32. Let α, β, γ be three numbers such that $\alpha + \beta + \gamma = 2,\ \alpha^2 + \beta^2 + \gamma^2 = 6$ and $\alpha^3 + \beta^3 + \gamma^3 = 11$, then :

	Column 1		Column 2
(A)	$\alpha^4 + \beta^4 + \gamma^4$ is equal to	**(p)**	13
(B)	$\alpha^5 + \beta^5 + \gamma^5$ is equal to	**(q)**	26
(C)	$\left(\alpha^2 - 4\right)\left(\beta^2 - 4\right)\left(\gamma^2 - 4\right)$ is equal to	**(r)**	57
(D)	$\alpha^6 + \beta^6 + \gamma^6$ is equal to	**(s)**	119
		(t)	129

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

33. The solution of the equation $\frac{8}{\{x\}}=\frac{9}{x}+\frac{10}{[x]}$ is of the form $\frac{k+1}{k}$, $k \in N$ then $k =$ ___________1. ([x] denotes largest integer less than or equal to x, and {x} denotes fractional part of x)

34. The value of 'a' so that the equation $x^3-6x^2+11x+a-6=0$ has exactly three integer solutions is _______.

35. Remainder when $P\left(x^5\right)$ is divided by $P(x)=x^4+x^3+x^2+x+1$ is ________.

36. If $a,b,c \in I, a>10$ and $(x-a)(x-12)+2=(x+b)(x+c)$ for all $x \in R$ then $|b-c|=$_________.

37. For real $a,b,c,$ $a+b+c=2,$ $a^2+b^2+c^2=6$ and $a^3+b^3+c^3=8$ then $(1-a)(1-b)(1-c)=$_______.

38. Let $f(x)=x^2+bx+c,$ $b,c \in R.$ If $f(x)$ is a factor of both x^4+6x^2+25 and $3x^4+4x^2+28x+5,$ then the minimum value of $f(x)$ is ________.

39. Given that m is a real number not less than –1, such that equation $x^2+2(m-2)x+m^2-3m+3=0$ has two distinct real roots x_1 and x_2. Find the maximum value of $\frac{1}{2}\left(\frac{mx_1^2}{1-x_1}+\frac{mx_2^2}{1-x_2}\right)$.

40. Let p be an integer such that both roots of the equation $5x^2-5px+(66p-1)=0$ are positive integers. Then the value of $\left[\frac{p}{10}\right]$ is equal to ([.] denotes greatest integer function)

JEE Advanced Revision Booklet	Trigonometry

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. Two rays are drawn through a point A at an angle of $30°$. A point B is taken on one of them at a distance a from the point A. A perpendicular is drawn from the point B to the other ray and another perpendicular is drawn from its foot to AB to meet AB at another point from where the similar process is repeated indefinitely. The length of the resulting infinite polygon line is :

(A) $a\left(2-\sqrt{3}\right)$ **(B)** $a\left(2+\sqrt{3}\right)$ **(C)** a **(D)** None of these

2. The least value of $sin^2\frac{A}{2}+sin^2\frac{B}{2}+sin^2\frac{C}{2}$ is : (Where A, B, C are interior angles of a triangle)

(A) 3/2 **(B)** 3/4 **(C)** 1 **(D)** None of these

3. If A, B, C, D are the smallest positive angles in ascending order of magnitude which have their sines equal to the positive quantity k, then the value of $4\,sin\frac{A}{2}+3\,sin\frac{B}{2}+2\,sin\frac{C}{2}+sin\frac{D}{2}$ is equal to :

(A) $2\sqrt{1-k}$ **(B)** $\sqrt{1+k}$ **(C)** $2\sqrt{k}$ **(D)** None of these

4. If $x\,sin\theta = y\,sin\left(\theta+\frac{2\pi}{3}\right) = z\,sin\left(\theta+\frac{4\pi}{3}\right)$, then $\sum xy =$

(A) 1/2 **(B)** $-1/2$ **(C)** 0 **(D)** None of these

5. If $u=\sqrt{a^2\,cos^2\theta+b^2\,sin^2\theta}+\sqrt{a^2\,sin^2\theta+b^2\,cos^2\theta}$, then the difference between maximum and minimum values of u^2 is given by :

(A) $(a-b)^2$ **(B)** $(a+b)^2$ **(C)** a^2+b^2 **(D)** None of these

6. The number of pairs $(x,\,y)$ satisfying the equations $sin\,x+sin\,y=sin(x+y)$ and $|x|+|y|=1$ is :

(A) 0 **(B)** 1 **(C)** 2 **(D)** None of these

7. The value of $cot^{-1}\left(2^2+\frac{1}{2}\right)+cot^{-1}\left(2^3+\frac{1}{2^2}\right)+cot^{-1}\left(2^4+\frac{1}{2^3}\right)+\ldots\infty$ is :

(A) $tan^{-1}\frac{1}{3}$ **(B)** $tan^{-1}\frac{1}{2}$ **(C)** 1 **(D)** None of these

8. In a ΔABC, $\angle B=\frac{\pi}{3}$ and $\angle C=\frac{\pi}{4}$, also D divides BC internally in the ratio 1 : 3, then $\frac{sin\angle BAD}{sin\angle CAD}$ is equal to :

(A) $\frac{1}{\sqrt{6}}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{\sqrt{3}}$ **(D)** None of these

9. The two adjacent sides of a cyclic quadrilateral are 2, 5 and the angle between them is $60°$. If the area of the quadrilateral is $4\sqrt{3}$, then the remaining two sides are :

(A) 1, 2 **(B)** 2, 2 **(C)** 2, 3 **(D)** None of these

10. In a $\Delta ABC,\ \angle A > \angle B$. Let $\angle A,\ \angle B$ satisfy the equation $3\,sin\,x - 4\,sin^3\,x - k = 0$, where $0 < k < 1$, then $\angle C$ is equal to :

(A) $\frac{\pi}{3}$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{2\pi}{3}$ **(D)** None of these

11. Points D, E are taken on the side BC of ΔABC, such that $BD = DE = EC$ and let $\angle BAD = x,\ \angle DAE = y,\ \angle EAC = z$; then $\frac{sin(x+y)\,sin(y+z)}{sin\,x\,sin\,z} =$

(A) 4 **(B)** 6 **(C)** 8 **(D)** None of these

12. If P be any interior point of the equilateral ΔABC of side length 2 units and also $x_a,\ x_b,\ x_c$ be the distances of P from the sides BC, CA, AB respectively, then $x_a + x_b + x_c =$

(A) $\sqrt{3}$ **(B)** $3\sqrt{2}$ **(C)** 4 **(D)** None of these

13. If $(x-a)\cos\theta + y\sin\theta = (x-a)\cos\phi + y\sin\phi = a,\ tan\frac{\theta}{2} - tan\frac{\phi}{2} = 2e$ and $\theta,\ \phi$ are unequal angles less than $360°$, then y^2 is equal to :

(A) $2ax - (1+e^2)x^2$ **(B)** $2ax - (1-e^2)x^2$ **(C)** $2ax + (1-e^2)x^2$ **(D)** $2ax + (1+e^2)x^2$

14. The number of solutions of the equation : $x^2 + (x+1)\,sin\frac{\pi x}{6} = \frac{3+x}{2}$; $-2 \le x \le 0$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

15. The radius of the circle passing through the incentre $\Delta\,ABC$ and through the end points of BC is given by:

(A) $\frac{a}{2}$ **(B)** $\frac{a}{2}sec\frac{A}{2}$ **(C)** $\frac{a}{2}sin\,A$ **(D)** $a\,sec\frac{A}{2}$

16. The median AD of a triangle ABC is bisected at E and BE meets AC in F ; then $AF : AC =$

(A) 3/4 **(B)** 1/3 **(C)** 1/2 **(D)** 1/4

17. If in a $\Delta ABC,\ \sum sin\,3A = 0$, then at least one angle of ΔABC is :

(A) $60°$ **(B)** $30°$ **(C)** $90°$ **(D)** $45°$

18. If $\left[sin^{-1}\,cos^{-1}\,sin^{-1}\,tan^{-1}\,x\right] = 1$ whose [.] denotes the greatest integer function, then x belongs to :

(A) $[tan\ sin\ cos\,1,\ tan\ sin\ cos\ sin\,1]$ **(B)** $[tan\ cos\ sin\,1,\ tan\ cos\ sin\ cos\,1]$

(C) $[-1, 1]$ **(D)** $[sin\ cos\ tan\,1,\ sin\ cos\ sin\ tan\,1]$

19. If $sin\,x + cosec\,x + tan\,y + cot\,y = 4$, where $x,\ y \in \left[0, \frac{\pi}{2}\right]$, then $tan\left(\frac{y}{2}\right)$ is a root of the equation :

(A) $\alpha^2 + 2\alpha + 1 = 0$ **(B)** $\alpha^2 + 2\alpha - 1 = 0$ **(C)** $2\alpha^2 - 2\alpha - 1 = 0$ **(D)** None of these

20. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ and $t_1 = (tan\,\theta)^{tan\,\theta}$, $t_2 = (tan\,\theta)^{cot\,\theta}$, $t_3 = (cot\,\theta)^{tan\,\theta}$ and $t_4 = (cot\,\theta)^{cot\,\theta}$, then :

(A) $t_3 > t_4 > t_1 > t_2$ **(B)** $t_4 > t_3 > t_1 > t_2$ **(C)** $t_4 > t_3 > t_2 > t_1$ **(D)** None of these

21. The period of the function $f(x) = e^{sin^2 x + sin^2\left(x+\frac{\pi}{3}\right) + cos\,x\,cos\left(x+\frac{\pi}{3}\right)}$ is :

(A) 1 **(B)** $\pi/2$ **(C)** π **(D)** Cannot be determined

22. If $t = x + y + z$, then $sin\,x + sin\,y + sin\,z - sin\,t$ equals :

(A) $4\,tan\left(\frac{y+z}{2}\right)tan\left(\frac{z+x}{2}\right)tan\left(\frac{x+y}{2}\right)$ **(B)** $4\,cot\left(\frac{y+z}{2}\right)cot\left(\frac{z+x}{2}\right)cot\left(\frac{x+y}{2}\right)$

(C) $4\,sin\left(\frac{y+z}{2}\right)sin\left(\frac{z+x}{2}\right)sin\left(\frac{x+y}{2}\right)$ **(D)** $4\,cos\left(\frac{y+z}{2}\right)cos\left(\frac{z+x}{2}\right)cos\left(\frac{x+y}{2}\right)$

23. The value of $cos^{-1}x + cos^{-1}\left(\frac{x}{2} + \frac{1}{2}\sqrt{3-3x^2}\right)$ is equal to : $\left(\frac{1}{2} \le x \le 1\right)$

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{3}$ **(C)** π **(D)** 0

24. A quadrilateral *ABCD* in which *AB* = *a*, *BC* = *b*, *CD* = *c* and *DA* = *d* is such that one circle can be inscribed in it and another circle can be circumscribed about it. *cos A* =

(A) $\frac{ad+bc}{ad-bc}$ **(B)** $\frac{ad-bc}{ad+bc}$ **(C)** $\frac{ac+bd}{ac-bd}$ **(D)** $\frac{ac-bd}{ac+bd}$

25. If the equation $a_1 + a_2\,cos\,2x + a_3\,sin^2 x = 1$ is satisfied by every real value of *x*, then the number of possible values of the triplet (a_1, a_2, a_3) is :

(A) 0 **(B)** 1 **(C)** 3 **(D)** Infinite

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

26. Let sides of a ΔABC are in A.P. and $a < min.\{b, c\}$, then *cos A* is equal to :

(A) $\frac{1}{2c}(4b-3c)$ **(B)** $\frac{1}{2c}(4c-3b)$ **(C)** $\frac{1}{2b}(4b-3c)$ **(D)** $\frac{1}{2b}(4c-3b)$

27. If points *D*, *E* and *F* divide sides *BC*, *CA* and *AB* respectively in ratio $\lambda : 1$ (in order) and or $ar(\Delta DEF) = 0.4$ $ar(\Delta ABC)$, then λ is equal to :

(A) $\frac{2-\sqrt{3}}{2}$ **(B)** $\frac{3-\sqrt{5}}{2}$ **(C)** $\frac{2+\sqrt{3}}{2}$ **(D)** $\frac{3+\sqrt{5}}{2}$

28. If in a right angled triangle the greatest side is *a*, then $tan\left(\frac{C}{2}\right) =$

(A) $\frac{a-b}{c}$ **(B)** $\frac{a+b}{c}$ **(C)** $\frac{a-c}{b}$ **(D)** $\frac{a+b}{b}$

29. Which of the following is true ?

(A) $tan\left|tan^{-1}x\right|=\left|x\right|$ **(B)** $cot\left|cot^{-1}x\right|=x$

(C) $tan^{-1}\left|tan\,x\right|=\left|x\right|$ **(D)** $sin\left|sin^{-1}x\right|=\left|x\right|$

30. If $f(x)=\left(sin^{-1}x\right)^3+\left(cos^{-1}x\right)^3$, then :

(A) Minimum value of $f(x)=-\dfrac{\pi^3}{8}$ **(B)** Minimum value of $f(x)=\dfrac{\pi^3}{32}$

(C) Maximum value of $f(x)=-\dfrac{\pi^3}{8}$ **(D)** Maximum value of $f(x)=\dfrac{7\pi^3}{8}$

31. If $f(x)=sec^{-1}\left[1+cos^2x\right]$, where [.] denotes the greatest integer function, then :

(A) The domain of *f* is R **(B)** The domain of *f* is [1, 2]

(C) The range of *f* is [1. 2] **(D)** The range of *f* is $\left\{sec^{-1}1,\ sec^{-1}2\right\}$

32. If $(sin\,\alpha)x^2-2x+b\ge 2$ for all real values of $x\le 1$ and $\alpha\in\left(0,\dfrac{\pi}{2}\right)\cup\left(\dfrac{\pi}{2},\pi\right)$, then *b* can be equal to :

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5

33. If $sin\,\alpha+sin\,\beta=\dfrac{3\sqrt{2}}{5}$ and $cos\,\alpha+cos\,\beta=\dfrac{4\sqrt{2}}{5}$, then :

(A) $sin(\alpha+\beta)=\dfrac{12}{13}$ **(B)** $sin(\alpha+\beta)=\dfrac{24}{25}$ **(C)** $cos(\alpha+\beta)=\dfrac{5}{13}$ **(D)** $cos(\alpha+\beta)=\dfrac{7}{25}$

34. If in a triangle *ABC*, atleast one of the following points, orthocenter, centroid, incentre and circumcentre lie outside the triangle, then :

(A) Triangle is obtuse angled
(B) Exactly 2 of these centres will lie outside the triangle
(C) Incentre may be collinear with other three centres
(D) Atleast one of the ex-radii is smaller than the inradius of the triangle

35. The value of θ lying between $\theta=0$ and $\theta=\dfrac{\pi}{2}$ and satisfying the equation :

$$\begin{vmatrix}1+cos^2\theta & sin^2\theta & 4\,sin\,4\theta\\ cos^2\theta & 1+sin^2\theta & 4\,sin\,4\theta\\ cos^2\theta & sin^2\theta & 1+4\,sin\,4\theta\end{vmatrix}=0 \text{ is :}$$

(A) $\dfrac{11\pi}{24}$ **(B)** $\dfrac{7\pi}{24}$ **(C)** $\dfrac{5\pi}{24}$ **(D)** $\dfrac{\pi}{24}$

36. If $x=sin\left(2\,tan^{-1}2\right)$ and $y=sin\left(\dfrac{1}{2}tan^{-1}\dfrac{4}{3}\right)$, then which of the following options is(are) correct ?

(A) $x=y^2$ **(B)** $y^2=1-x$ **(C)** $x^2=\dfrac{y}{2}$ **(D)** $x>y$

37. The equation $(1-tan\theta)(1+tan\theta)sec^2\theta+2^{tan^2\theta}=0$ has :

(A) No solution in the interval $\left(-\frac{\pi}{2},0\right)$ **(B)** Two solutions in the interval $\left(-\frac{\pi}{2},0\right)$

(C) No solution in the interval $\left(0,\frac{\pi}{2}\right)$ **(D)** Two solutions in the interval $\left(0,\frac{\pi}{2}\right)$

38. Which of the following is not a possible value of $f(x)=tan3x\,cot\,x$?

(A) 1 **(B)** 2 **(C)** 4 **(D)** 5

39. In the ΔABC, $b:c=2:1$ and $sin(B-C)=\frac{3}{5}$. Then :

(A) ΔABC is right-angled **(B)** ΔABC is obtuse angled

(C) $a:c=3:1$ **(D)** $a:c=\sqrt{5}:1$

40. Which of the following is/are true?

(A) $\tan 1>\tan^{-1}1$ **(B)** $\sin 1>\cos 1$ **(C)** $\tan 1<\sin 1$ **(D)** $\cos(\cos 1)>\frac{1}{\sqrt{2}}$

41. Which of the following is/are positive?

(A) $\log_{\sin 1}\tan 1$ **(B)** $\log_{\cos 1}(1+\tan 3)$

(C) $\log_{\log_{10}5}(\cos\theta+\sec\theta)$ **(D)** $\log_{\tan 15°}(2\sin 18°)$

42. If $2(\cos(x-y)+\cos(y-z)+\cos(z-x))=-3$, then:

(A) $\cos x\cos y\cos z=1$ **(B)** $\cos x+\cos y+\cos z=0$

(C) $\sin x+\sin y+\sin z=1$ **(D)** $\cos 3x+\cos 3y+\cos 3z=12\cos x\cos y\cos z$

43. If $2a=2\tan 10°+\tan 50°;\ 2b=\tan 20°+\tan 50°$

$2c=2\tan 10°+\tan 70°;\ 2d=\tan 20°+\tan 70°$

Then which of the following is / are correct ?

(A) $a+d=b+c$ **(B)** $a+b=c$ **(C)** $a>b<c>d$ **(D)** $a<b<c<d$

44. The value of $\frac{\sin x-\cos x}{\sin^3 x}$ is equal to :

(A) $\text{cosec}^2x(1-\cot x)$ **(B)** $1-\cot x+\cot^2 x-\cot^3 x$

(C) $\text{cosec}^2x-\cot x-\cot^3 x$ **(D)** $\frac{1-\cot x}{\sin^2 x}$

45. The inequality $4\sin 3x+5\geq 4\cos 2x+5\sin x$ is true for $x\in$:

(A) $\left[-\pi,\frac{3\pi}{2}\right]$ **(B)** $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ **(C)** $\left[\frac{5\pi}{8},\frac{13\pi}{8}\right]$ **(D)** $\left[\frac{23\pi}{14},\frac{41\pi}{14}\right]$

46. The equation $\cos x\cos 6x=-1$:

(A) has 50 solutions in $[0,100\pi]$ **(B)** has 3 solutions in $[0,3\pi]$

(C) has even number of solutions in $(3\pi,13\pi)$ **(D)** has one solution in $\left[\frac{\pi}{2},\pi\right]$

47. Identify the correct options:

(A) $\frac{\sin 3\alpha}{\cos 2\alpha} > 0$ for $\alpha \in \left(\frac{3\pi}{8}, \frac{23\pi}{48}\right)$ **(B)** $\frac{\sin 3\alpha}{\cos 2\alpha} < 0$ for $\alpha \in \left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$

(C) $\frac{\sin 2\alpha}{\cos \alpha} < 0$ for $\alpha \in \left(-\frac{\pi}{2}, 0\right)$ **(D)** $\frac{\sin 2\alpha}{\cos \alpha} > 0$ for $\alpha \in \left(\frac{13\pi}{48}, \frac{14\pi}{48}\right)$

48. The equation $\sin^4 x + \cos^4 x + \sin 2x + k = 0$ must have real solutions if:

(A) $k = 0$ **(B)** $|k| \le \frac{1}{2}$ **(C)** $-\frac{3}{2} \le k \le \frac{1}{2}$ **(D)** $-\frac{1}{2} \le k \le \frac{3}{2}$

49. Let $f(\theta) = \left(\cos\theta - \cos\frac{\pi}{8}\right)\left(\cos\theta - \cos\frac{3\pi}{8}\right)\left(\cos\theta - \cos\frac{5\pi}{8}\right)\left(\cos\theta - \cos\frac{7\pi}{8}\right)$ then:

(A) maximum value of $f(\theta) \forall \theta \in R$ is $\frac{1}{4}$ **(B)** maximum value of $f(\theta) \forall \theta \in R$ is $\frac{1}{8}$

(C) $f(0) = \frac{1}{8}$ **(D)** Number of principle solutions of $f(\theta) = 0$ is 8

50. If r_1, r_2, r_3 are radii of the escribed circles of a triangle ABC and r is the radius of its incircle, then the root(s) of the equation $x^2 - r(r_1r_2 + r_2r_3 + r_3r_1)x + (r_1r_2r_3 - 1) = 0$ is/are:

(A) r_1 **(B)** $r_2 + r_3$ **(C)** 1 **(D)** $r_1r_2r_3 - 1$

51. Let A, B, C be angles of a triangle ABC and let $D = \frac{5\pi + A}{32}$, $E = \frac{5\pi + B}{32}$, $F = \frac{5\pi + C}{32}$, then: (where $D, E, F \ne \frac{n\pi}{2}, n \in I, I$ denote set of integers)

(A) $\cot D \cot E + \cot E \cot F + \cot D \cot F = 1$ **(B)** $\cot D + \cot E + \cot F = \cot D \cot E \cot F$

(C) $\tan D \tan E + \tan E \tan F + \tan F \tan D = 1$ **(D)** $\tan D + \tan E + \tan F = \tan D \tan E \tan F$

52. In a ΔABC if $\frac{r}{r_1} = \frac{r_2}{r_3}$, then which of the following is/are true? (where symbols used have usual meanings)

(A) $a^2 + b^2 + c^2 = 8R^2$ **(B)** $\sin^2 A + \sin^2 B + \sin^2 C = 2$

(C) $a^2 + b^2 = c^2$ **(D)** $\Delta = s(s + c)$

53. ABC is a triangle whose circumcentre, incentre and orthocentre are O, I and H respectively which lie inside the triangle, then :

(A) $\angle BOC = A$ **(B)** $\angle BIC = \frac{\pi}{2} + \frac{A}{2}$ **(C)** $\angle BHC = \pi - A$ **(D)** $\angle BHC = \pi - \frac{A}{2}$

54. In a triangle ABC, $\tan A$ and $\tan B$ satisfy the inequality $\sqrt{3}x^2 - 4x + \sqrt{3} < 0$, then which of the following must be correct?
(where symbols used have usual meanings)

(A) $a^2 + b^2 - ab < c^2$ **(B)** $a^2 + b^2 > c^2$

(C) $a^2 + b^2 + ab > c^2$ **(D)** $a^2 + b^2 < c^2$

55. $f(x) = \sin^{-1}(\sin x), g(x) = \cos^{-1}(\cos x)$, then:

(A) $f(x) = g(x)$ if $x \in \left(0, \frac{\pi}{4}\right)$ **(B)** $f(x) < g(x)$ if $x \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

(C) $f(x) < g(x)$ if $x \in \left(\pi, \frac{5\pi}{4}\right)$ **(D)** $f(x) > g(x)$ if $x \in \left(\pi, \frac{5\pi}{4}\right)$

56. The solution(s) of the equation $\cos^{-1} x = \tan^{-1} x$ satisfy

(A) $x^2 = \frac{\sqrt{5}-1}{2}$ **(B)** $x^2 = \frac{\sqrt{5}+1}{2}$

(C) $\sin(\cos^{-1} x) = \frac{\sqrt{5}-1}{2}$ **(D)** $\tan(\cos^{-1} x) = \frac{\sqrt{5}-1}{2}$

57. A solution of the equation $\cot^{-1} 2 = \cot^{-1} x + \cot^{-1}(10-x)$ where $1 < x < 9$ is:

(A) 7 **(B)** 3 **(C)** 2 **(D)** 5

58. Consider the equation $\sin^{-1}\left(x^2 - 6x + \frac{17}{2}\right) + \cos^{-1} k = \frac{\pi}{2}$, then:

(A) the largest value of k for which equation has 2 distinct solution is 1

(B) the equation must have real root if $k \in \left(-\frac{1}{2}, 1\right)$

(C) the equation must have real root if $k \in \left(-1, \frac{1}{2}\right)$

(D) the equation has unique solution if $k = -\frac{1}{2}$

59. The value of x satisfying the equation $(\sin^{-1} x)^3 - (\cos^{-1} x)^3 + (\sin^{-1} x)(\cos^{-1} x)(\sin^{-1} x - \cos^{-1} x) = \frac{\pi^3}{16}$ cannot be equal to:

(A) $\cos\frac{\pi}{5}$ **(B)** $\cos\frac{\pi}{4}$ **(C)** $\cos\frac{\pi}{8}$ **(D)** $\cos\frac{\pi}{12}$

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

60. If $a\sin\theta - b\cos\theta = -\sin 4\theta$ and $a\cos\theta + b\sin\theta = \frac{5}{2} - \frac{3}{2}\cos 4\theta$, then $(a+b)^{2/5} + (a-b)^{2/5}$ is _______.

61. Let incircle of radius 4 units of a triangle *ABC* touches the side *BC* at *D*. If $BD = 6,\ DC = 8$ and Δ be the area of triangle, then $\sqrt{\sqrt{\Delta - 3}} =$ __________.

62. The total number of solutions of $\tan\{x\} = \cot\{x\}$; where $\{x\}$ denotes the fractional part of x in $[0, 2\pi]$ is ________.

63. Consider the equation $\tan^{-1} x + \cos^{-1}\left(\frac{y}{\sqrt{1+y^2}}\right) = \sin^{-1}\left(\frac{3}{\sqrt{10}}\right)$. Let α = sum of positive integral solutions of *x* and β = sum of positive integral solutions of *y*. Then $\beta - \alpha =$ __________.

64. If $\sin x + \sin^2 x + \sin^3 x = 1$, then $\cos^6 x - 4\cos^4 x + 8\cos^2 x =$ __________.

65. The number of solutions of $sin^{-1}\left(\frac{1+x^2}{2x}\right)=\frac{\pi}{2}sec(x-1)$ is ________.

66. If the square of the diameter of a circle circumscribing a ΔABC is equal to half the sum of the squares of its sides then $\sum \sin^2 A$ is ________.

67. If $tan\left(142\frac{1}{2}°\right)=2+\sqrt{2}-\sqrt{\mu}-\sqrt{\lambda}$, then $\mu+\lambda$ = __________.

68. If $tan\left(\frac{2\pi}{3}-x\right)=\frac{sin\frac{2\pi}{3}-sin\,x}{cos\frac{2\pi}{3}-cos\,x}$ where $0<x<\frac{3\pi}{2}$, and the values of x are x_1 and x_2, then the value of $\frac{12}{\pi}|x_2-x_1|$ is ___________.

69. If in a $\Delta ABC,\ a=5,\ b=4$ and $cos(A-B)=\frac{31}{32}$, then the third side c is equal to __________.

70. If $10\sin^4\alpha+15\cos^4\alpha=6$ and the value of $9\text{cosec}^4\alpha+8\sec^4\alpha$ is S, then find the value of $\frac{S}{25}$.

71. Given that for $a, b, c, d \in R,$ if $a\sec(200°)-c\tan(200°)=d$ and $b\sec(200°)+d\tan(200°)=c,$ then find the value of $\left(\frac{a^2+b^2+c^2+d^2}{bd-ac}\right)\sin 20°.$

72. If $\sum_{r=1}^{n}\left(\frac{\tan 2^{r-1}}{\cos 2^r}\right)=\tan p^n-\tan q,$ then find the value of $(p+q)$.

73. If $x=\alpha$ satisfy the equation $3^{\sin 2x+2\cos^2 x}+3^{1-\sin 2x+2\sin^2 x}=28,$ then $(\sin 2\alpha-\cos 2\alpha)^2+8\sin 4\alpha$ is equal to:

74. If the value of $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}+\cos\frac{7\pi}{7}=-\frac{l}{2}$. Find the value of l.

75. If $x+\sin y=2014$ and $x+2014\cos y=2013, 0\le y\le\frac{\pi}{2},$ then find the value of $[x+y]-2005$ (where [.] denotes greatest integer function)

76. The complete set of values of x satisfying $\frac{2\sin 6x}{\sin x-1}<0$ and $\sec^2 x-2\sqrt{2}\tan x\le 0$ in $\left(0, \frac{\pi}{2}\right)$ is $[a, b)\cup(c, d],$ then find the value of $\left(\frac{cd}{ab}\right)$.

77. The range of value's of k for which the equation $2\cos^4 x-\sin^4 x+k=0$ has atleast one solution is $[\lambda, \mu]$. Find the value of $(9\mu+\lambda)$.

78. The number of solutions of the system of equations:

$$2\sin^2 x+\sin^2 2x=2$$
$$\sin 2x+\cos 2x=\tan x$$

in $[0, 4\pi]$ satisfying $2\cos^2 x+\sin x\le 2 is$:

79. If the sum of all values of $\theta, 0 \le \theta \le 2\pi$ satisfying the equation $(8\cos 4\theta - 3)(\cot\theta + \tan\theta - 2)(\cot\theta + \tan\theta + 2) = 12$ is $k\pi,$ then k is equal to:

80. In a ΔABC; inscribed circle with centre I touches sides AB, AC and BC at D, E, F respectively. Let area of quadrilateral $ADIE$ is 5 square units and area of quadrilateral $BFID$ is 10 square units. Find the value of $\dfrac{\cos\left(\frac{C}{2}\right)}{\sin\left(\frac{A-B}{2}\right)}.$

81. If Δ be area of incircle of a triangle ABC and $\Delta_1, \Delta_2, \Delta_3$ be the area of excircles then find the least value of $\dfrac{\Delta_1\Delta_2\Delta_3}{729\Delta^3}.$

82. In an acute angled triangle $ABC, \angle A = 20°,$ let DEF be the feet of altitudes through A, B, C respectively and H is the orthocentre of $\Delta ABC.$ Find $\dfrac{AH}{AD} + \dfrac{BH}{BE} + \dfrac{CH}{CF}.$

83. Let ΔABC be inscribed in a circle having radius unity. The three internal bisectors of the angles A, B and C are extended to intersect the circumcircle of ΔABC at A_1, B_1 and C_1 respectively. find

$$\frac{AA_1\cos\frac{A}{2} + BB_1\cos\frac{B}{2} + CC_1\cos\frac{C}{2}}{\sin A + \sin B + \sin C}$$

84. In $\Delta ABC,$ if circumradius `$R'$ and inradius `r' are connected by relation $R^2 - 4Rr + 8r^2 - 12r + 9 = 0,$ then the greatest integer which is less than the semiperimeter of ΔABC is:

85. The complete set of values of x satisfying the inequality $\sin^{-1}(\sin 5) > x^2 - 4x$ is $(2 - \sqrt{\lambda - 2\pi}, 2 + \sqrt{\lambda - 2\pi}),$ then $\lambda =$

86. In ΔABC; if $(II_1)^2 + (I_2I_3)^2 = \lambda R^2,$ where I denotes incentre; I_1, I_2 and I_3 denote centres of the circles escribed to the sides BC, CA and AB respectively and R be the radius of the circum circle of $\Delta ABC.$ Find $\lambda.$

87. If $2\tan^{-1}\dfrac{1}{5} - \sin^{-1}\dfrac{3}{5} = -\cos^{-1}\dfrac{63}{\lambda},$ then $\lambda =$

88. If $\displaystyle\sum_{n=0}^{\infty} 2\cot^{-1}\left(\frac{n^2+n+4}{2}\right) = k\pi,$ then find the value of $k.$

89. Find number of solutions of the equation $\sin^{-1}(|\log_6^2(\cos x) - 1|) + \cos^{-1}(|3\log_6^2(\cos x) - 7|) = \dfrac{\pi}{2},$ if $x \in [0, 4\pi].$

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. if p^{th}, q^{th} and r^{th} terms of an H.P. be respectively x, y, z, then $(p-q)xy+(q-r)yz+(r-p)xz=$

(A) $xyz+pqr$ **(B)** pqr **(C)** xyz **(D)** 0

2. If the $(m+1)^{th}, (n+1)^{th}$ and $(r+1)^{th}$ terms of an A.P. are n G.P. and m, n, r are in H.P., then the ratio of the common difference to the first term in the A.P. is equal to :

(A) $\frac{2}{n}$ **(B)** $\frac{1}{n}$ **(C)** $-\frac{1}{n}$ **(D)** $-\frac{2}{n}$

3. $\sum_{r=1}^{99} r!\left(r^2+r+1\right)$ is equal to:

(A) $102!-100!$ **(B)** $100(100!)-1$ **(C)** $99(100!)-1$ **(D)** $100(99!)-1$

4. The sum $\sum_{k=1}^{n} \frac{k^2-\frac{1}{2}}{k^4+\frac{1}{4}}$ is equal to:

(A) $\frac{2n^2-2n+1}{2n^2+2n+1}$ **(B)** $\frac{2n^2-n}{2n^2+2n+1}$ **(C)** $\frac{n^2}{2n^2+2n+1}$ **(D)** $\frac{2n^2}{2n^2+2n+1}$

5. The value of $\frac{\left(1^4+\frac{1}{4}\right)\left(3^4+\frac{1}{4}\right)....\left((2n-1)^4+\frac{1}{4}\right)}{\left(2^4+\frac{1}{4}\right)\left(4^4+\frac{1}{4}\right)....\left((2n)^4+\frac{1}{4}\right)}$ is equal to:

(A) $\frac{1}{4n^2+2n+1}$ **(B)** $\frac{1}{8n^2+4n+1}$ **(C)** $\frac{1}{4\left(2n^2+n+1\right)}$ **(D)** $\frac{n}{8n^2-4n+1}$

6. The sum $\sum_{k=1}^{n} \frac{1}{(k+1)\sqrt{k}+k\sqrt{k+1}}$ is equal to:

(A) $\frac{\sqrt{n+1}-2}{\sqrt{n+1}}$ **(B)** $\frac{\sqrt{n+1}-1}{\sqrt{n+1}}$ **(C)** $\frac{\sqrt{n+1}+1}{\sqrt{n+1}}$ **(D)** $\frac{n+1}{n\sqrt{n+1}}$

7. Let S_n, S_{2n}, S_{3n} are respectively the sums of first n, 2n, 3n terms of an arithmetic progression, then $S_{3n}=$

(A) $2(S_{2n}-S_n)$ **(B)** $\frac{3}{2}(S_{2n}-S_n)$ **(C)** $3(S_{2n}-S_n)$ **(D)** $6(S_{2n}-S_n)$

8. The sum $\frac{19}{1\cdot2\cdot3}\frac{1}{4}+\frac{28}{2\cdot3\cdot4}\frac{1}{8}+\frac{39}{3\cdot4\cdot5}\frac{1}{16}+\frac{52}{4\cdot5\cdot6}\frac{1}{32}+...+$ upto infinite terms is equal to:

(A) $\frac{1}{2}$ **(B)** 1 **(C)** 2 **(D)** 4

9. The sum of infinite series $1^2 - \frac{2^2}{5} + \frac{3^2}{5^2} - \frac{4^2}{5^3} + \frac{5^2}{5^4} - \frac{6^2}{5^2} + ...\infty$ is equal to:

(A) $\frac{5}{27}$ **(B)** $\frac{25}{27}$ **(C)** $\frac{25}{108}$ **(D)** $\frac{25}{54}$

10. $\frac{n}{1\cdot2\cdot3} + \frac{n-1}{2\cdot3\cdot4} + \frac{n-2}{3\cdot4\cdot5} + ...$ upto n terms is equal to:

(A) $\frac{1}{2(n+2)} + \frac{n+1}{4} - \frac{1}{2}$ **(B)** $\frac{1}{2(n+2)} + \frac{n+1}{4} + \frac{1}{2}$

(C) $\frac{1}{n+2} + \frac{n+1}{4} - \frac{1}{2}$ **(D)** $\frac{1}{2(n+2)} + \frac{n+1}{2} + \frac{1}{2}$

<u>For Questions 11 - 13</u>

If $\phi(r) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... + \frac{1}{r}$ and $\sum_{r=1}^{n} (2r+1)\phi(r) = P(n)\phi(n+1) - Q(n)$, where $P(n)$ and $Q(n)$ are polynomial function of 'n', then

11. $\sum_{r=0}^{10} P(r)$ is equal to:

(A) 235 **(B)** 506 **(C)** 285 **(D)** 385

12. $\sum_{r=0}^{\infty} \frac{1}{Q(r)}$ is equal to:

(A) 1 **(B)** 2 **(D)** 4 **(D)** 8

13. $P(13) - Q(13)$ is equal to:

(A) 81 **(B)** 78 **(C)** 91 **(D)** 65

<u>For Questions 14 - 16</u>

Let $a_m \left(m = 1, 2, 3...., p\right)$ be the possible integral values of a for which the graphs of $f(x) = ax^2 + 2bx + b$ and $g(x) = 5x^2 - 3bx - a$ meets at some point for all real values of b. let $t_r = \prod_{m=1}^{p} (r - a_m)$ and $S_n = \prod_{r=1}^{n} t_r, n \in N$.

14. The minimum possible value of a is:

(A) $\frac{1}{5}$ **(B)** $\frac{5}{26}$ **(C)** $\frac{3}{38}$ **(D)** $\frac{2}{43}$

15. The sum of values of n for which S_n vanishes is:

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11

16. The value of $\sum_{r=5}^{\infty} \frac{1}{t_r}$ is equal to:

(A) $\frac{1}{3}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{15}$ **(D)** $\frac{1}{18}$

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

17. Let $a_1, a_2, a_3, ..., a_n$ be the first 'n' terms of an A.P. having common difference 'd' $(d \neq 0)$, then the greatest value of product of two terms equidistant from the extreme terms is:

(A) $a_1 a_n + \frac{d^2(n-1)^2}{4}$ if n is odd **(B)** $a_1 a_n + \frac{d^2(n+1)^2}{4}$ if n is odd

(C) $a_1 a_n + \frac{d^2 n(n+2)}{4}$ if n is even **(D)** $a_1 a_n + \frac{d^2}{4} n(n-2)$ is n is even

18. For all permissible value of x, consider $y = \frac{\sin 3x(\cos 6x + \cos 4x)}{\sin x(\cos 8x + \cos 2x)}$ and range of y is $(-\infty, a) \cup (b, \infty)$. If 2b is the first terms of G.P. and 'a' is its common ratio, then: (S_∞ denotes the sum of infinite terms of G.P.)

(A) $b - a = \frac{10}{3}$ **(B)** $3a + b = 4$ **(C)** $S_\infty = 9$ **(D)** $S_\infty = \frac{27}{10}(a+b)$

19. Let $\{a_n\}$ consists of positive numbers and for any positive integer n, $\frac{a_n + 2}{2} = \sqrt{2s_n}$, where $s_n = \sum_{i=1}^{n} a_1$. Then :

(A) $a_{21} = 82$ **(B)** $a_{12} = 48$ **(C)** $a_{13} = 50$ **(D)** $a_{14} = 54$

20. If x, y, z are three distinct positive real numbers and are in H.P., then $\frac{3x+2y}{2x-y} + \frac{3z+2y}{2z-y}$ is greater then:

(A) 9 **(B)** 10 **(C)** 12 **(D)** 15

21. The sequence $\{a_n\}, n \in N$ satisfies $a_1 = 1$ and $5^{a_{n+1} - a_n} = 1 + \frac{1}{n + \frac{2}{3}}$. Then: (where $[\cdot]$ denotes greatest integer function)

(A) $[a_{501}] = 3$ **(B)** $[a_{207}] = 3$ **(C)** $[a_{223}] = 4$ **(D)** $[a_{625}] = 4$

22. If a, b, c are three positive real numbers, then $\frac{a^2+1}{b+c} + \frac{b^2+1}{c+a} + \frac{c^2+1}{a+b}$ can be never be equal to:

(A) 1 **(B)** 2 **(C)** $\frac{8}{3}$ **(D)** 3

23. Let 'p' be the first of 'n' arithmetic means between two positive numbers and 'q' be first of 'n' harmonic means between same two numbers. The $\frac{q}{p}$ can lie in interval(s):

(A) $(-\infty, 1]$ **(B)** $\left(1, \left(\frac{n+1}{n-1}\right)^2\right)$

(C) $\left(\left(\frac{n-1}{n+1}\right)^2, \left(\frac{n+1}{n-1}\right)^2\right)$ **(D)** $\left[\left(\frac{n+1}{n-1}\right)^2, \infty\right)$

24. Let x, y, z are distinct positive integers and $m = \left(\frac{x^2+y^2+z^2}{x+y+z}\right)^{(x+y+z)}$, $n = x^x y^y z^z$, $P = \left(\frac{x+y+z}{3}\right)^{(x+y+z)}$, then:

(A) $m > n$ **(B)** $n > p$ **(C)** $m < n$ **(D)** $n < p$

25. Let $f(x) = \lim_{n\to\infty}\left(\frac{1}{x+1}+\frac{2}{x^2+1}+\frac{4}{x^4+1}+...+\frac{2^n}{x^{2^n}+1}\right)$ for $x > 1$, then:

(A) $\sum_{r=2}^{6}\frac{1}{f(r)} = 20$ **(B)** $f(5) = \frac{1}{6}$ **(C)** $f(5) = \frac{1}{4}$ **(D)** $\sum_{r=2}^{6}\frac{1}{f(r)} = 15$

26. For a positive integer 'n', let $S(n) = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{2^n-1}$. Then:

(A) $S(200) > 100$ **(B)** $S(200) < 100$ **(C)** $S(100) < 100$ **(D)** $S(100) > 100$

27. Let $a_1, a_2, a_3, ...a_n$ be first 'n' terms of a G.P. with first term 'a' and common ratio 'r'. Then:

(A) $\frac{1}{a_1^2-a_2^2}+\frac{1}{a_2^2-a_3^2}+...+\frac{1}{a_{n-1}^2-a_n^2} = \frac{\left(1-r^{2n}\right)}{a^2r^{2n-4}\left(1-r^2\right)^2}$

(B) $\frac{1}{a_1^2-a_2^2}+\frac{1}{a_2^2-a_3^2}+...+\frac{1}{a_{n-1}^2-a_n^2} = \frac{\left(1-r^{2n-2}\right)}{a^2r^{2n-4}\left(1-r^2\right)^2}$

(C) $\frac{1}{a_1^m+a_2^m}+\frac{1}{a_2^m+a_3^m}+..+\frac{1}{a_{n-1}^m+a_n^m} = \frac{\left(r^{mn-m}-1\right)}{a^m\left(1+r^m\right)\left(r^{mn-m}-r^{mn-2m}\right)}$

(D) $\frac{1}{a_1^m+a_2^m}+\frac{1}{a_2^m+a_3^m}+...+\frac{1}{a_{n-1}^m+a_n^m} = \frac{\left(r^{mn-m}-1\right)}{a^m\left(1-r^m\right)\left(r^{mn-m}-r^{mn-2m}\right)}$

28. Let the equation $x^3+px^2+qx-q=0$, where $p, q \in R, q \neq 0$ has 3 real roots α, β, γ in H.P., then:

(A) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2} \geq \frac{1}{3}$ **(B)** $9p+2q+27 = 0$

(C) Maximum value of $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}$ is $\frac{1}{3}$ **(D)** $\frac{p}{q} \geq -\frac{1}{3}$

29. The sum $\sum_{n=1}^{\infty}\sum_{k=1}^{n-1}\frac{k}{2^{n+k}}$ is equal to

(A) $\sum_{k=1}^{\infty}\frac{k}{2^k}$ **(B)** $\sum_{k=1}^{\infty}\frac{k}{4^k}$

(C) $\sum_{m=1}^{\infty}\left(\frac{m}{2^m}\sum_{n=m+1}^{\infty}\frac{1}{2^n}\right)$ **(D)** $\frac{4}{27}$

30. For any natural number $n > 1$, consider the sum S =

$$\frac{1}{n^2+1}+\frac{2}{n^2+2}+\frac{3}{n^2+3}+...+\frac{n}{n^2+n}$$, then

(A) $S < \frac{1}{2}+\frac{1}{2n}$ **(B)** $S > \frac{1}{2}+\frac{1}{2n}$ **(C)** $S > \frac{1}{2}$ **(D)** $S < 1$

31. If $n \in N, n > 5$ then which of the following holds true ?

(A) $n^n > 1 \cdot 3 \cdot 5 ... (2n-1)$ **(B)** $2^n > 1 + n\sqrt{2^{n-1}}$

(C) $\frac{1}{n+1}+\frac{1}{n+1}+\frac{1}{n+3}+...\frac{1}{2n} > \frac{1}{2}$ **(D)** $2^n < 1 + n\sqrt{2^{n-1}}$

32. Let $\{a_n\}$ be a sequence of real numbers such that $a_1 = 2, a_{n+1} = a_n^2 - a_n + 1$ for $n = 1, 2, 3, ...$ Let

$S = \frac{1}{a_1}+\frac{1}{a_2}+...+\frac{1}{a_{2018}}$, then

(A) $S < 1 - \frac{1}{(2018)^{2018}}$ **(B)** $S > 1 - \frac{1}{(2018)^{2018}}$

(C) $S < 1$ **(D)** $S > 1 - \frac{1}{(2017)^{2017}}$

33. Let $a_k = \frac{k}{(k-1)^{4/3}+k^{4/3}+(k+1)^{4/3}}$ and $S_n = \sum_{k=1}^{n} a_k$, then

(A) $S_{26} > \frac{17}{4}$ **(B)** $S_{26} < \frac{17}{4}$ **(C)** $S_{999} < 50$ **(D)** $S_{999} > 50$

34. Let $S = \frac{1}{\sqrt{1}+\sqrt{3}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\frac{1}{\sqrt{9}+\sqrt{11}}+\frac{1}{\sqrt{9997}+\sqrt{9999}}$, then

(A) $S < 24$ **(B)** $S > 24$ **(C)** $S > 18$ **(D)** $S < 18$

35. Define $f_n(x) = (1+x)(1+2x)(1+4x)...(1+2^n x) = a_{n,0} + a_{n,1}^{x} + a_{n,2}^{x^2} + ... + a_{n,n}x^2$, where n is a positive integer, then

(A) $a_{100,2} = \frac{(2^{100}-1)(2^{102}-4)}{3}$ **(B)** $a_{100,2} = \frac{(2^{101}-1)(2^{101}-2)}{3}$

(C) $a_{100,2} - a_{99,2} = 2^{201} - 2^{101}$ **(D)** $a_{100,2} - a_{99,2} = 2^{200} - 2^{100}$

36. Let a, b, c be positive integers such that $a + b + c = n$, then

(A) $\left(a^a b^b c^c\right)^{1/n} \le \frac{a^2+b^2+c^2}{n}$ **(B)** $\left(a^b b^c c^a\right)^{1/n} \le \frac{ab+bc+ca}{n}$

(C) $\left(a^b b^c c^a\right)^{1/n} \le \frac{a^2+b^2+c^2}{n}$ **(D)** $\left(a^a b^b c^c\right)^{1/n} + \left(a^b b^c c^a\right)^{1/n} + \left(a^c b^a c^b\right)^{1/n} \le n$

37. Let $S = 2016^2 + 2015^2 + 2014^2 - 2013^2 - 2012^2 - 2011^2 + 2010^2 + 2009^2 + 2008^2 - 2007^2 + -2006^2 -$ $2005^2 + ...6^2 + 5^2 + 4^2 - 3^2 - 2^2 - 1^2$, then S is divisible by

(A) 8 **(B)** 27 **(C)** 112 **(D)** 2017

38. Let $f(x)=\frac{1}{x+1}+\frac{2x}{(x+1)(x+2)}+\frac{3x^2}{(x+1)(x+2)(x+3)}+...+\frac{nx^{n-1}}{(x+1)(x+2)(x+3)...(x+n)}$ then:

(A) $f(x)=\frac{x}{1+x}-\frac{x^n}{(x+1)(x+2)...(x+n)}$ **(B)** $1-\frac{x^n}{(x+1)(x+2)...(x+n)}$

(C) $f'(x)=\left(-\frac{x^n}{(x+1)(x+2)...(x+n)}\right)\left(\sum_{r=1}^{n}\frac{r}{x+r}\right)$

(D) $f'(x)=\left(-\frac{x^{n-1}}{(x+1)(x+2)..(x+n)}\right)\left(\sum_{r=1}^{n}\frac{r}{x+r}\right)$

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column I are labelled as (A), (B), (C) & (D) whereas statements in Column II are labeled as P, Q, R, S & T. More than one choice from Column II can be matched with Column I.

39.

	Column –I	Column –II	
(A)	If $A=\sum_{r=1}^{n}r^2, B=\sum_{m=1}^{n}\sum_{r=1}^{m}r-\frac{1}{2}\sum_{r=1}^{n}r$, then $\frac{A}{B}$ is equal to	(P)	1
(B)	For positive numbers a, b, c the minimum value of $\frac{a\left(b^2+c^2\right)+b\left(c^2+a^2\right)+c\left(a^2+b^2\right)}{abc}$ is equal to	(Q)	2
(C)	If $x+y+z=1, x,y,z>0$, then the minimum value of $\frac{2x^2}{y+z}+\frac{2y^2}{z+x}+\frac{2z^2}{x+y}$ is equal to	(R)	3
(D)	If $\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}=3$ where $x,y,z\in N$, then $x+y+z$ is equal to	(S)	4
		(T)	6

40.

	Column –I	Column –II	
(A)	Let *a*, *b*, *c* are positive real numbers such that $a^3b^2c=12$, then the minimum value of $49a+3b+c$ is equal to	(P)	1
(B)	The minimum value of $\left\|2x^3-\frac{3}{x^2}\right\|$ for $x<0$ is equal to	(Q)	5
(C)	The maximum value of $\frac{x^5\left(8-x^3\right)}{\sqrt[3]{25}}$ for $0<x<2$ is equal to	(R)	7
(D)	If $x^7y^5=a$ and $7x+5y\geq 12\forall x,y>0$, then the minimum value of 'a' is equal to	(S)	15
		(T)	42

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

41. If $\sum_{r=1}^{n} a_r = \sum_{k=1}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j} 2$ and $\lambda = \lim_{n\to\infty}\left(\sum_{r=1}^{n}\frac{1}{a_r}\right)^n$ then $\left[\frac{1}{\lambda}\right]$ is equal to (where $[\cdot]$ denotes greatest integer function).

42. Let $a_i + b_i = 1 \forall i = 1,2,....,6$ and $a = \frac{1}{6}(a_1 + a_2 + ... + a_6), b = \frac{1}{6}(b_1 + b_2 + ... + b_6)$. Then

$a_1b_1 + a_2b_2 + ... + a_6b_6 = nab - (a_1 - a_2)^2 - (a_2 - a)^2 - ... - (a_6 - a)^2$ where n is equal to

43. The value of expression $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + ...[\sqrt{100^2}]$, (where[.] denotes greatest integer function) is equal to___.

44. If the value of $\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{3^i 3^j 3^k}(i \neq j \neq k)$ is equal to $\frac{m}{n}$, where m, n are coprime natural numbers, then $m+n$ is equal to_____.

45. Integers 1, 2, 3,…, n where $n > 2$ are written on a board. Two numbers m, k such that $1 < m < n, 1 < k < n$ are removed and the average of the remaining numbers is found to be 17. What is the maximum sum of the two removed numbers ?

46. Let the equation $x^4 - 16x^3 + px^2 - 256x + q = 0$ has four positive real roots in G.P., then $p+q$ is equal to

47. Let $x_1, x_2, x_3, ..., x_{2018}$ be real numbers different from 1, such that $x_1 + x_2 +,,, + x_{2018} = 1$ and $\frac{x_1}{1-x_1} + \frac{x_2}{1-x_2} + ... + \frac{x_{2018}}{1-x_{2018}} = 1$ then the value of $\frac{x_1^2}{1-x_1} + \frac{x_2^2}{1-x_2} + ... + \frac{x_{2018}^2}{1-x_{2018}}$ is equal to_____.

48. Let $x_1, x_2, ..., x_{2018}$ be positive real numbers such that $x_1 + x_2 + ... + x_{2018} = 1$. Determine the smallest constant k such that $k\sum_{i=1}^{2018}\frac{x_i^2}{1-x_i} \geq 1$

49. Let x, y, z are positive real numbers satisfy $2x - 2y + \frac{1}{z} = \frac{1}{2018}, 2y - 2z + \frac{1}{x} = \frac{1}{2018}$, $2z - 2x + \frac{1}{y} = \frac{1}{2018}$ then $x + y - z$ is equal to_____.

50. Let a, b, c be positive real number such that $a + b + c \geq 4$, then find the minimum value of

$$\frac{a^3}{(a-b)(a-c)} + \frac{b^3}{(b-c)(b-a)} + \frac{c^3}{(c-a)(c-b)}.$$

JEE Advanced Revision Booklet	Complex Numbers

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. If z_1, z_2, z_3 be three complex numbers such that $|z_1+1| \le 1, |z_2+2| \le 2$ and $|z_3+4| \le 4$, then the maximum value of $|z_1|+|z_2|+|z_3|$ is :

(A) 7 **(B)** 10 **(C)** 12 **(D)** 14

2. The value of $i\log(x-i)+i^2\pi+i^3\log(x+i)+i^4(2\tan^{-1}x)$, (where, $x > 0$ and $i=\sqrt{-1}$) , is :

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

3. The complex number satisfying arg $(z + i) = \frac{\pi}{4}$ and $\arg(2z+3-2i)=\frac{3\pi}{4}$ simultaneously, is :

(A) $\frac{1}{4}-\frac{3}{4}i$ **(B)** $\frac{1}{4}+\frac{3}{4}i$ **(C)** $-\frac{1}{4}-\frac{3}{4}i$ **(D)** None of these

4. Equation of tangent drawn to the circle $|z| = r$ at the point A (z_0), is :

(A) $\text{Re}\left(\frac{z}{z_0}\right)=1$ **(B)** $\text{Re}\left(\frac{z_0}{z}\right)=1$ **(C)** $\text{Im}\left(\frac{z}{z_0}\right)=1$ **(D)** $\text{Im}\left(\frac{z_0}{z}\right)=1$

5. Consider a square *OABC* in the Argand plane, where '*O*' is origin and $A \equiv A(z_0)$. Then the equation of the circle that can be inscribed in this square is : (vertices of square are given in anti-clockwise order)

(A) $|z-z_0(1+i)|=|z_0|$ **(B)** $2\left|z-\frac{z_0(1+i)}{2}\right|=|z_0|$

(C) $\left|z-\frac{z_0(1+i)}{2}\right|=|z_0|$ **(D)** None of the above

6. If $\left|\frac{z-z_1}{z-z_2}\right|=3$, where z_1 and z_2 are fixed complex numbers and *z* is a variable complex number, then *z* lies on a :

(A) Circle with z_1 as its interior point **(B)** Circle with z_2 as its interior point

(C) Circle with z_1 and z_2 as its interior points **(D)** Circle with z_1 and z_2 as its exterior points

7. Let z_1 and z_2 be the non-real roots of the equation $3z^2 + 3z + b = 0$. If the origin together with the points represented by z_1 and z_2 form an equilateral triangle, then the value of *b* is :

(A) 1 **(B)** 2 **(C)** 3 **(D)** None of the above

8. The equation $(1+a)x^2+2a^2x+a^2+b^2-1=0$ has roots of opposite sign, if $a + ib$ lies, $(a > -1)$:

(A) On straight line $x + y = 1$

(B) Inside a circle of centre (0, 0) and radius '1'

(C) On a parabola of vertex (0, 0) and focal length '1'

(D) None of the above

9. The roots of $z^n = (z+1)^n$

(A) Lie on the vertices of a regular n-polygon **(B)** Lie on a circle

(C) Are collinear **(D)** None of the above

10. If $|z-4+3i| \le 1$ and α and β be the least and greatest values of $|z|$ and K be the least value of $\frac{x^4+x^2+4}{x}$ on the interval $(0,\infty)$, then K is equal to :

(A) α **(B)** β **(C)** $\alpha+\beta$ **(D)** None of the above

11. The complex numbers $\sin x + i\cos 2x$ and $\cos x - i\sin 2x$ are conjugate to each other for :

(A) $x = n\pi$ **(B)** $x = \left(n+\frac{1}{2}\right)\pi$ **(C)** $x = 0$ **(D)** No value of x

12. If $\left(\frac{3-z_1}{2-z_1}\right)\left(\frac{2-z_2}{3-z_2}\right) = K$, then points $A(z_1), B(z_2), C(3,0)$ and $D(2,0)$ (taken clockwise) will :

(A) lie on a circle only for $K > 0$ **(B)** lie on a circle only for $K < 0$

(C) lie on a circle $\forall K \in R$ **(D)** be the vertices of a square $\forall K \in (0,1)$

13. Let 'z' be a complex number and 'a' be a real parameter such that $z^2 + az + a^2 = 0$, then:

(A) locus of z is a pair of straight lines **(B)** locus of z is a circle

(C) $\arg(z) = \pm\frac{5\pi}{3}$ **(D)** $|z| = 2|a|$

14. The locus represented by the equation $|z-1|+|z+1| = 2$ is :

(A) an ellipse with foci (1, 0) and (–1, 0)

(B) one of the family of circles passing through the points of intersection of the circles $|z-1| = 1$ and $|z+1| = 1$.

(C) the radical axis of the circles $|z-1| = 1$ and $|z+1| = 1$.

(D) the portion of the real axis between the points (1, 0) (-1, 0) including both.

15. If $|z-2| = \min\{|z-1|, |z-5|\}$, where z is a complex number, then :

(A) $\text{Re}(z) = \frac{3}{2}$ **(B)** $\text{Re}(z) = \frac{7}{2}$ **(C)** $\text{Re}(z) \in \left\{\frac{3}{2}, \frac{7}{2}\right\}$ **(D)** None of these

16. If $z_1, z_2, z_3, \ldots, z_{n-1}$ are the roots of the equation $z^{n-1}+z^{n-2}+z^{n-3}+\ldots+z+1 = 0$, where $n \in N$, $n > 2$ and ω is the cube root of unity, then :

(A) ω^n, ω^{2n} are also the roots of the given equation

(B) $\omega^{1/n}, \omega^{2/n}$ are also the roots of the given equation

(C) $z_1, z_2, z_3, \ldots, z_{n-1}$ form a geometric progression

(D) $a^{\frac{z_r+1}{z_r}}$ is variable for $a > 0$ and $r = 1, 2, \ldots, n-2$

17. If $A(z_1)$, $B(z_2)$, $C(z_3)$ are the vertices of a triangle ABC inscribed in the circle $|z| = 2$. Internal angle bisector of the angle A, meet the circumcircle again at $D(z_4)$, then :

(A) $z_4^2 = z_2 z_3$ **(B)** $z_4 = \frac{z_2 z_3}{z_1}$ **(C)** $z_4 = \frac{z_1 z_2}{z_3}$ **(D)** $z_4 = \frac{z_1 z_3}{z_2}$

18. If $|z-2-i| = |z|\sin\left(\frac{\pi}{4} - \arg z\right)$, then locus of z is/an :

(A) ellipse **(B)** circle **(C)** parabola **(D)** pair of straight lines

19. If $|z| = 2$ and $\frac{z_1 - z_3}{z_2 - z_3} = \frac{z-2}{z+2}$, then z_1, z_2, z_3 will be vertices of a/an :

(A) equilatéral triangle **(B)** acute angled triangle
(C) right angled triangle **(D)** None of the above

20. For the complex number z, the minimum value of $|z| + |z - \cos\alpha - i\sin\alpha|$ is :

(A) 0 **(B)** 1 **(C)** 2 **(D)** None of the above

Paragraph for Questions 21 - 23

Let $f(x) = x^4 - 6x^3 + 26x^2 - 46x + 65$. All the roots of $f(x) = 0$ are of the form $a_k + ib_k$ for $k = 1, 2, 3, 4$, where $i = \sqrt{-1}$. Further a_k and b_k are all integers. Also $\lambda = |b_1| + |b_2| + |b_3| + |b_4|$ and $\mu = a_1 + a_2 + a_3 + a_4$. If set S is formed whose elements are all $a_i{}'s$ and $b_i{}'s$, then :

21. The value of $\lambda + \mu$ is equal to :

(A) 16 **(B)** 4 **(C)** 10 **(D)** 6.

22. Roots of the equation are :

(A) $-1 \pm 2i, -2 \pm 3i$ **(B)** $2 \pm 2i, 1 \pm 3i$ **(C)** $-2 \pm 2i, -1 \pm 3i$ **(D)** $1 \pm 2i, 2 \pm 3i$.

23. Number of functions from set $S \to C$, where C has 8 distinct element is :

(A) 8^4 **(B)** 8^8 **(C)** 8^5 **(D)** 8^6.

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

24. Let $\alpha = e^{i2\pi/11}, \lambda = \alpha^6, \mu = \alpha^7, \beta = \alpha^2$. Then :

(A) $\text{Re}\left(\lambda + \lambda^2 + \lambda^3 + \lambda^4 + \lambda^5\right) = -\frac{1}{2}$ **(B)** $(\mu-\beta)(\mu-\beta^2)(\mu-\beta^3).....(\mu-\beta^9)(\mu-\beta^{10}) = 0$

(C) $(i-\beta)(i-\beta^2)(i-\beta^3).....(i-\beta^{10}) = i$ **(D)** None of these

25. $\alpha_1, \alpha_2, \alpha_3, ...\alpha_{100}$ are all the 100th roots of unity. The numerical value of $\sum\sum_{1 \le i < j \le 100} (\alpha_i \alpha_j)^5$, is :

(A) 20 **(B)** 0 **(C)** $(20)^{1/20}$ **(D)** $\sum_{i=1}^{100} \alpha_i$

26. If A and B respresent complex numbers z_1 and z_2. $P(z)$ is any complex number satisfying $\left|z - \frac{z_1 + z_2}{2}\right| = k, (k > 0)$, then:

(A) Maximum area of $(\Delta PAB) = \frac{1}{2}k\,|z_1 - z_2|$

(B) There are two possible positions of P on argand plane when area of ΔPAB is maximum

(C) Area of ΔPAB = constant $\left(< \frac{1}{2}k\,|z_1 - z_2|\right)$, for 4 possible positions of P.

(D) ΔPAB is equilateral triangle of maximum area if $4k^2 = 3|z_1 - z_2|^2$

27. Locus of z, if $\arg[z-(1+i)] = \begin{cases} \dfrac{3\pi}{4}, \text{ when} |z| \le |z-2| \\ \dfrac{-\pi}{4}, \text{ when} |z| > |z-2| \end{cases}$, is :

(A) a pair of straight lines passing through (2, 0)
(B) a pair of straight lines passing through (2, 0), (1, 1)
(C) a line segment **(D)** a set of two rays

28. Let $z \in C$ and if $A = \left\{z : \arg(z) = \dfrac{\pi}{4}\right\}$ and $B = \left\{z : \arg(z-3-3i) = \dfrac{2\pi}{3}\right\}$. Then $n(A \cap B)$ is equal to :

(A) 1 **(B)** $\sum_{r=0}^{99} i^r$ **(C)** 3 **(D)** 0

29. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = 1$ then $|z_1 - z_2|^2 + |z_2 - z_3|^2 + |z_3 - z_1|^2$ cannot exceed :
(A) 6 **(B)** 9 **(C)** 12 **(D)** 5

30. Let $f(z)$ be a polynomial function of a complex number z. On division by $z-i, z+i$ and z^2+1 we obtain remainder as α, β and $g(z)$, $(\alpha, \beta \in C)$. Then :

(A) $\alpha = i$ and $\beta = 1+i \Rightarrow g(z) = i\left(\dfrac{z}{2}+1\right)+\dfrac{1}{2}$ **(B)** $\alpha = i$ and $\beta = 1-i \Rightarrow g(z) = \left(z-\dfrac{1}{2}\right)-\dfrac{iz}{2}$

(C) $\alpha = i$ and $\beta = 1+i \Rightarrow g(z) = i\left(z+\dfrac{1}{2}\right)+\dfrac{1}{2}$ **(D)** $\alpha = i$ and $\beta = 1-i \Rightarrow g(z) = \left(z+\dfrac{1}{2}\right)+\dfrac{iz}{2}$

31. If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are complex numbers such that $|z_1| = 1, |z_2| = 2$ and $\text{Re}(z_1 z_2) = 0$, then the pair of complex numbers $\omega_1 = a_1 + \dfrac{ia_2}{2}$ and $\omega_2 = 2b_1 + ib_2$ satisfy :

(A) $|\omega_1| = 1$ **(B)** $|\omega_2| = 2$ **(C)** $\text{Re}(\omega_1\omega_2) = 0$ **(D)** $\text{Im}(\omega_1\omega_2) = 0$

32. If from a point P representing the complex number z_1 on the curve $|z| = 2$, pair of tangents are drawn to the curve $|z| = 1$, meeting at point $Q(z_2)$ and $R(z_3)$, then :

(A) complex number $\dfrac{z_1+z_2+z_3}{3}$ will lie on the curve $|z| = 1$

(B) $\left(\dfrac{4}{z_1}+\dfrac{1}{z_2}+\dfrac{1}{z_3}\right)\left(\dfrac{4}{\bar{z}_1}+\dfrac{1}{\bar{z}_2}+\dfrac{1}{\bar{z}_3}\right) = 9$

(C) $\arg\left(\dfrac{z_2}{z_3}\right) = \dfrac{2\pi}{3}$

(D) orthocentre and circumcentre of ΔPQR will coincide

33. The complex number satisfying $|z+\bar{z}| + |z-\bar{z}| = 2$ and $|z+i| + |z-i| = 2$ is / are :
(A) i **(B)** $-i$ **(C)** $1+i$ **(D)** $1-i$

34. If z is a complex number such that $\arg\left(\frac{z-3\sqrt{3}}{z+3\sqrt{3}}\right)=\frac{\pi}{3}$, then the locus of z is :

(A) $|z-3i|=6$ **(B)** $|z-3i|=6$, $Im\ z>0$
(C) $|z-3i|=6$, $Im z<0$ **(D)** Major arc of a circle

35. Value of $\left(\sin\left(\log i^{i}\right)\right)^{3}+\left(\cos\left(\log i^{i}\right)\right)^{3}$ is :

(A) 1 **(B)** -1 **(C)** $\sum_{k=1}^{8} e^{i\frac{2\pi k}{9}}$ **(D)** 2i

36. If $x^6=(4-3i)^5$, then the product of all of its roots is : (where $\theta=-\tan^{-1}(3/4)$)

(A) $5^5(\cos(\pi+5\theta)+\sin(\pi+5\theta))$ **(B)** $-5^5(\cos 5\theta+i\sin 5\theta)$
(C) $5^5(\cos 5\theta-i\sin 5\theta)$ **(D)** $-5^5(\cos 5\theta-i\sin 5\theta)$

37. If $|z-1|+|z+3|\le 8$, then the possible values of $|z-4|$ belongs to :

(A) $(-9, 0)$ **(B)** $[-9, -1]$ **(C)** $[1, 9]$ **(D)** $[5, 9]$

38. The possible values of parameter α for which $|z-(\alpha^2-7\alpha+11+i)|=1$ and $\arg z\ge\frac{\pi}{2}$ is satisfied for at least one z are :

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6

39. If $\alpha\ne 1,\ \alpha^5=1,$ then $\log_{\sqrt{3}}\left|1+\alpha+\alpha^2+\alpha^3-\frac{2}{\alpha}\right|$ is equal to :

(A) 2 **(B)** 3 **(C)** 4 **(D)** $\min\left(x+\frac{1}{x}\right),\ (x>0)$

40. If $\alpha=\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}$ and $\left|\sum_{r=0}^{3n-1}\alpha^{2^r}\right|^2=32$ then n is :

(A) 8 **(B)** 4 **(C)** $\log_e e^8$ **(D)** $-4\sum_{k=1}^{6} e^{i\frac{2\pi k}{7}}$

41. The number of roots of the equation $z^{15}=1$ satisfying $|\arg z|<\pi/2$ are :

(A) 6 **(B)** 7 **(C)** 8 **(D)** $\frac{7}{i}\sum_{r=1}^{97} i^r$

42. If z is a complex number satisfying $|z|^2-|z|-2<0$, then the possible value(s) of $\left|z^2+z\sin\theta\right|$. for all values of θ, is(are) :

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7

43. If $z_n=\cos\frac{\pi}{n(n+1)(n+2)}+i\sin\frac{\pi}{n(n+1)(n+2)}$ for n = 1, 2, 3, …,k, then the value of $\left|\lim_{k\to\infty}(z_1z_2....z_k)\right|$ is :

(A) 1 **(B)** 2 **(C)** $-\sum_{k=1}^{98} e^{i\frac{2\pi k}{99}}$ **(D)** $\sum_{k=0}^{98} e^{i\frac{2\pi k}{99}}$

44. Complex number whose real and imaginary parts x and y are integers and satisfy the equation $3x^2 - |xy| - 2y^2 + 7 = 0$

(A) do not exist **(B)** exist and have equal modulus
(C) form two conjugate pairs **(D)** do not form conjugate pairs

45. If all the three roots of $az^3 + bz^2 + cz + d = 0$ have negative real parts, $(a, b, c \in R)$ then

(A) $ab > 0$ **(B)** $bc > 0$ **(C)** $ad > 0$ **(D)** $bc - ad > 0$

46. If z_1, z_2 and z_3 are three complex numbers such that $|z_1| = |z_2| = |z_3| = 1$, then $|z_1 - z_2|^2 + |z_2 - z_3|^2 + |z_3 - z_1|^2$ is strictly less than

(A) 6 **(B)** 9 **(C)** 12 **(D)** 18

47. If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are complex numbers such that $|z_1| = 1, |z_2| = 2$ and Re $(z_1 z_2) = 0$ then the pair of complex numbers $w_1 = a_1 + \frac{ia_2}{2}$ and $w_2 = 2b_1 + ib_2$ satisfy

(A) $|w_1| = 1$ **(B)** $|w_2| = 2$ **(C)** Re $(w_1 w_2) = 0$ **(D)** Im $(w_1 w_2) = 2$

48. For complex number z and w, if $|z|^2 w - |w|^2 z = z - w$ then

(A) $z = w$ **(B)** $z\overline{w} = 1$ **(C)** $z = -w$ **(D)** $\overline{z}w = 1$

49. If $z^3 + (3 + 2i)z + (-1 + ia) = 0$ has one real root, then the value of a lies in the interval $(a \in R)$

(A) (–2, 1) **(B)** (–1, 0) **(C)** (0, 1) **(D)** (–2, 3)

50. If $p(x)$, $q(x)$, $r(x)$ and $s(x)$ are polynomials such that $p(x^3) + xq(x^3) + x^2 r(x^3) = (1 + x + x^2)s(x)$ then

(A) $p(1) = s(1)$ **(B)** $p(1) = r(1)$ **(C)** $p(1) = 3s(1)$ **(D)** $p(1) = 2r(1)$

51. If the roots of the equation $z^4 + \lambda z^3 + (-36 + 15i)z^2 + mz = 0$ are the vertices of a square then $(\lambda + m)$ can be equal to

(A) $35 + 45i$ **(B)** $-35 - 45i$ **(C)** $35 - 45i$ **(D)** $-35 + 45i$

52. Complex numbers z_1, z_2, z_3 and z_4 correspond to the points A, B, C and D, respectively, on a circle $|z| = 1$. If $z_1 + z_2 + z_3 + z_4 = 0$, Then $ABCD$ is necessarily

(A) a triangle **(B)** a square **(C)** a rhombus **(D)** a parallelogram

53. Two triangles having vertices as z_1, z_2, z_3 and a, b, c are similar. Then

(A) $az_1 + bz_2 + cz_3 = 0$ **(B)** $\frac{z_1}{a} + \frac{z_2}{b} + \frac{z_3}{c} = 0$
(C) $z_1(b - c) + z_2(c - a) + z_3(a - b) = 0$ **(D)** $a(z_2 - z_3) + b(z_3 - z_1) + c(z_1 - z_2) = 0$

54. If $k \in R - \{0\}, z$ is a complex number and $k + |k + z^2| = |z^2|$ then the value (s) of arg z is/are

(A) $-\pi$ **(B)** $-\frac{\pi}{2}$ **(C)** $\frac{\pi}{2}$ **(D)** π

55. Let z_1 and z_2 be two complex numbers represented by points on the circle $|z_1| = 1$ and $|z_2| = 2$, respectively. Then

(A) max $|2z_1 + z_2| = 4$ **(B)** max $|z_1 - z_2| = 1$
(C) $\left|z_2 + \frac{1}{z_1}\right| \le 3$ **(D)** None of these

56. Equation of line through a and ib such that $a, b \in R$ and $a, b \neq 0$ is

(A) $z\left(\frac{1}{2a}+\frac{1}{2ib}\right)+\bar{z}\left(\frac{1}{2a}-\frac{1}{2ib}\right)=1$ **(B)** $z\left(\frac{1}{2a}+\frac{i}{2b}\right)+\bar{z}\left(\frac{1}{2a}+\frac{i}{2b}\right)=1$

(C) $1\,z\left(\frac{1}{2a}-\frac{i}{2b}\right)+\bar{z}\left(\frac{1}{2a}-\frac{i}{2b}\right)=1$ **(D)** $z\left(\frac{1}{2a}-\frac{i}{2b}\right)+\bar{z}\left(\frac{1}{2a}+\frac{i}{2b}\right)=1$

57. w_1, w_2 be roots of $(a+\bar{c})z^2+(b+\bar{b})z+(\bar{a}+c)=0$ If $|z_1|<|z_2|<1$, then

(A) $|w_1|<1$ **(B)** $|w_1|=1$ **(C)** $|w_2|<1$ **(D)** $|w_2|=1$

58. A complex number z satisfies the equation $|Z^2-9|+|Z^2|=41$, then the true statements among the following are

(A) $|Z+3|+|Z-3|=10$ **(B)** $|Z+3|+|Z-3|=8$

(C) Maximum value of $|Z|$ is 5 **(D)** Maximum value of $|Z|$ is 6

59. Let a, b, c be distinct complex numbers with $|a| = |b| = |c| = 1$ and z_1, z_2 be the roots of the equation $az^2+bz+c=0$ with $|z_1|=1$. Let P and Q represent the complex numbers z_1 and z_2 in the Argand plane with $\angle POQ = \theta$, $0° < \theta < 180°$ (where O being the origin). Then

(A) $b^2=ac;\ \theta=\frac{2\pi}{3}$ **(B)** $\theta=\frac{2\pi}{3};\ PQ=\sqrt{3}$

(C) $PQ=2\sqrt{3};\ b^2=ac$ **(D)** $\theta=\frac{\pi}{3};\ b^2=ac$

60. Let $Z_1=x_1+iy_1,\ Z_2=x_2+iy_2$ be complex numbers in fourth quadrant of argand plane and $|Z_1|=|Z_2|=1$, $\text{Re}(Z_1Z_2)=0$. The complex number

$Z_3=x_1+ix_2,\ Z_4=y_1+iy_2,\ Z_5=x_1+iy_2,\ Z_6=x_2+iy_1$, will always satisfy

(A) $|Z_4|=1$ **(B)** $\arg(Z_1Z_4)=-\frac{\pi}{2}$

(C) $\frac{Z_5}{\cos(\arg Z_1)}+\frac{Z_6}{\sin(\arg Z_1)}$ is purely real **(D)** $Z_3^2+(\bar{Z}_6)^2$ is purely imaginary

61. If the imaginary part of $\frac{z-3}{e^{i\theta}}+\frac{e^{i\theta}}{z-3}$ is zero, then z can lie on

(A) a circle with unit radius **(B)** a circle with radius 3 units

(C) a straight line through the point (3, 0) **(D)** a parabola with the vertex (3, 0)

62. If z_1, z_2, z_3 are any three roots of the equation $z^6=(z+1)^6$ then arg $\left(\frac{z_1-z_3}{z_2-z_3}\right)$ can be equal to

(A) 0 **(B)** π **(C)** $\frac{\pi}{4}$ **(D)** $-\frac{\pi}{4}$

63. Let z_1, z_2, z_3 are the vertices of ΔABC, respectively, such that $\frac{z_3-z_2}{z_1-z_2}$ is purely imaginary number. A square on side AC is drawn outwardly. $P(z_4)$ is the centre of square, then

(A) $|z_1-z_2|=|z_2-z_4|$ **(B)** $\arg\left(\frac{z_1-z_2}{z_4-z_2}\right)+\arg\left(\frac{z_3-z_2}{z_4-z_2}\right)=\frac{\pi}{2}$

(C) $\arg\left(\frac{z_1-z_2}{z_4-z_2}\right)+\arg\left(\frac{z_3-z_2}{z_4-z_2}\right)=0$ **(D)** z_1, z_2, z_3 and z_4 lie on a circle.

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column 1 are labelled as (A), (B), (C) & (D) whereas statements in Column 2 are labeled as p, q, r, s & t. More than one choice from Column 2 can be matched with Column 1.

64. If $|z-1|+|z+1|=k$ then locus of z is :

	Column 1		Column 2
(A)	If $k = 2$	**(p)**	Ellipse of eccentricity 2/3
(B)	If $k = 5$	**(q)**	No locus
(C)	if $0 < k < 2$	**(r)**	Line segment
(D)	If $k = 3$	**(s)**	Ellipse of eccentricity 2/5

65. If $\arg\left(\frac{z-(1+i)}{z-(3+4i)}\right)=\theta$ Then locus of z is :

	Column 1		Column 2
(A)	Line segment	**(p)**	If $\theta=\frac{2\pi}{3}$
(B)	Line ray	**(q)**	If $\theta=\pi$
(C)	Major arc of a circle	**(r)**	If $\theta=\frac{\pi}{3}$
(D)	Minor arc of a circle	**(s)**	If $\theta=0$

66. If an equilateral triangle *ABC* with vertices at z_1, z_2 and z_3 be inscribed in the circle $|z| = 2$ and again a circle is inscribed in the triangle *ABC* touching the sides *AB*, *BC* and *CA* at $D(z_4)$, $E(z_5)$ and $F(z_6)$ respectively :

	Column 1		Column 2
(A)	Then value of $\mathrm{Re}(z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1)$ is equal to	**(p)**	2
(B)	If $\frac{4z_1}{z_3}$ is equal to $a(-1+i\sqrt{3})$, then a is	**(q)**	– 6
(C)	The value of $\|z_1+z_2\|^2+\|z_2+z_3\|^2+\|z_3+z_1\|^2$ is	**(r)**	12
(D)	If *P* is any point on incircle the value of $DP^2+EP^2+FP^2$ is	**(s)**	6
		(t)	– 2

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

67. The number of solution(s) of the equation $z^2 - z - |z|^2 - \frac{64}{|z|^5} = 0$ is ____________.

68. If arg(z) < 0, then $-\frac{10}{\pi}$ arg $\left(\frac{z-\bar{z}}{2}\right)$ is equal to ____________.

69. Let w, $\overline{w}$ is complex cube root of unity and P(z) is point on a circle $|z| = 4$ such that $|z - 1|$ is maximum and centroid of triangle formed by $z, -w, -\overline{w}$ is α then -7 Re(α) is____________.

70. If z is a complex number and the minimum value of $|z|+|z-1|+|2z-3|$ is λ and if $y = 2[x]+3 = 3[x-\lambda]$ then find the value of $\frac{1}{5}([x+y])$. (where [.] denotes the greatest integer function).

71. The area of the region bounded by curves.

(i) $|z-z_1| = |z-z_3|$

(ii) $|\text{Re}(z)-\text{Re}(z_1)| = |\text{Re}(z)-\text{Re}(z_3)|$

(iii) $|z-z_2| - |z-z_1| = |z_1 - z_2|$

(where $z_1 = 1+i,\ z_2 = 2+i,\ z_3 = -3+3i$) is $\frac{p}{q}$, (p, q are co-prime) then find $p+q$.

72. Let $\alpha = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$ and let $A_k = x + y\alpha^k + p\alpha^{2k} + w\alpha^{3k} + f\alpha^{4k}$ where x, y, p, w, f are points on the circle $|z| = 1$, then $\frac{|A_0|^2 + |A_1|^2 + |A_2|^2 + |A_3|^2 + |A_4|^2}{5}$ is equal to _______.

73. Let A, B, C be the set of complex numbers defined as $A = \{z: ||z+2| - |z-2|| = 2\}$, $B = \left\{z : \arg\left(\frac{z-1}{z}\right) = \frac{\pi}{2}\right\}$ and $C = \{z: \arg(z-1) = \pi\}$, then $n(A \cap B \cap C)$ is ______.

74. Let $z = (\cos 12° + i\sin 12° + \cos 48° + i\sin 48°)^6$, then $Im(z)$ is equal to ____________.

75. If $8iz^3 + 12z^2 - 18z + 27i = 0$, then $2|z|$ is equal to ____________.

76. Given a, b, c are cube roots of $q(q < 0)$, then for any value of x, y, z given by
$\left|\frac{a^2x^2+b^2y^2+c^2z^2}{b^2x^2+c^2y^2+a^2z^2}\right| + (x_1^2 - 2y_1^2)\omega + \left([x^2]+[y^2]+[z^2]\right)\omega^2 = 0$, (where [.] denotes the greatest integer function, ω is cube root of unity, x_1, y_1 are positive integers and y_1, is a prime number) then value of $[x^2 + x_1] + [y^2 + y_1] + [z^2 + x_1^2 + y_1^2] - 10$ is ____________.

77. If $\omega \neq 1$ is a cube root of unity and z is a complex number such that $|z| = 1$ then $\left|\dfrac{2+3\omega+4z\omega^2}{4\omega+3\omega^2 z+2z}\right|$ = _____.

78. If z is a complex number such that $\left|z+\dfrac{1}{z}\right|=2$ then minimum value of $|z|$ is____

79. If $|z| = 1$ and $z^{2n}+1 \neq 0$ then $\dfrac{z^n}{z^{2n}+1}-\dfrac{(\bar{z})^n}{(\bar{z})^{2n}+1}$ is equal to _____.

80. Let $A(z)$ and $B(z_1)$ be two variable points such that $zz_1 = |z|^2$. If the area enclosed by $|z-\bar{z}|+|z_1+\bar{z}_1|=10$ is A then the value of $A/8$ is ____.

81. Sum of all the solutions of $z=|z|+z^2$ is ____.

82. Let $z = x+iy$ and $\arg\left(e^{z^2}\right)=\arg\left(e^{(z+i)}\right)$. If $y = f(x)$ is a function, then f(3) is equal to _____.

83. Let z_i, i = 1, 2, …, 6 be the roots of $z^6+z^4=2$ then $\sum_{i=1}^{6}|z_i|^4$ is equal to____.

84. Difference between the square of the least and the square of the greatest values of $|z|$, where $z=e^{i2\phi}\sin\phi+\cos\phi\,(\phi\in R)$, is____.

85. The value of $4\alpha\left(\beta^4-\alpha^4\right)$, if $\alpha+i\beta,\ \beta\neq 0$ is a root of $z^5=1$, is____.

86. If $z\bar{z}=1$, then the value of $\left[\left|2+\dfrac{1}{z}\right|+|2-z|^2\right]$ is ____.

87. If $1+2|z^2|=|z^2+1|+2|z+1|^2$, then the value of $\dfrac{|z(z+1)|}{2}$ is ____.

88. Sum of all the solutions of $z^2+|z|=(\bar{z})^2$ is ____.

89. If the roots Z_1, Z_2, Z_3 of the equation $Z^3-Z^2+mZ-1=0$ lie on $|Z|=1$ and $|(Z_1+3)(Z_2+3)(Z_3+3)|=10\lambda$ then λ = ____.

90. Given $|3z_1-2z_2-4|^2=|3z_1-1|^2+|2z_2+3|^2\left(z_2\neq-\dfrac{3}{2}\right)$ If cube roots of $w=\dfrac{3z_1-1}{2z_2+3}$ are w_1, w_2, w_3; where ($\arg w_1 < \arg w_2 < \arg w_3$), then the value of $\dfrac{w_2^2}{w_1 w_3}$ is ____.

91. Number of complex number z satisfying $z^3=\bar{z}$ is ____.

92. Let α and β be two complex numbers satisfying $|\alpha+1+i|=1$ and $|\beta-2-3i|=6$. Then the value of $6|\alpha|_{\max}-|\beta|_{\max}$ is ____.

93. Let z be a complex number such that $\left|2z+\dfrac{1}{z}\right|=1$ and $\arg(z)=\theta$. Then minimum value of $\sin^2\theta$ is____.

94. Let AB and CD be parallel chords of the circle $|z| = r$. If z_1, z_2, z_3 and z_4 represent A, B, C and D, respectively, and $z_1z_2=kz_3z_4$ then $\dfrac{75k}{4}$ equals ____.

95. The number of complex numbers which are conjugate of their own cube, is____.

96. If $A(z_1); B(z_2); C(z_3)$ are the vertices of triangle such that $z_3=\dfrac{z_2-iz_1}{1-i}$ & $|z_1|=3; |z_2|=4$ & $|z_2+iz_1|=|z_1|+|z_2|$ then area of ΔABC is ___.

JEE Advanced Revision Booklet	Permutations & Combinations

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. Number of ways in which four different toys and five indistinguishable marbles can be distributed between Amar, Akbar and Anthony, if each child receives atleast one toy and one marble, is :
(A) 42 **(B)** 100 **(C)** 150 **(D)** 216

2. You are given an unlimited supply of each of the digits 1, 2, 3 or 4. Using only these four digits, you construct n digit numbers. Such n digit numbers will be called LEGITIMATE if it contains the digit 1 either an even number times or not at all. Number of n digit legitimate numbers are :
(A) $2^n + 1$ **(B)** $2^{n+1} + 2$ **(C)** $2^{n+2} + 4$ **(D)** $2^{n-1}(2^n + 1)$

3. It 5 letters are put in the 5 envelopes. Find the no. of ways so that atleast 2 letters are in wrong envelope:
(A) 120 **(B)** 119 **(C)** 118 **(D)** 117

4. Number of positive integral solutions satisfying the equation $(x_1 + x_2 + x_3)(y_1 + y_2) = 77$, is :
(A) 150 **(B)** 270 **(C)** 420 **(D)** 1024

5. There are counters available in 3 different colours (atleast four of each colour). Counters are all alike except for the colour. If 'm' denotes the number of arrangements of four counters if no arrangement consists of counters of same colour and 'n' denotes the corresponding figure when every arrangement consists of counters of each colour, then :
(A) $m = 2n$ **(B)** $6m = 13n$ **(C)** $3m = 5n$ **(D)** $5m = 3n$

6. An ice cream parlour has ice creams in eight different varieties. Number of ways of choosing 3 ice creams taking atleast two ice creams of the same variety, is :
(Assume that ice creams of the same variety to be identical & available in unlimited supply)
(A) 56 **(B)** 64 **(C)** 100 **(D)** None of these

7. Three digit numbers in which the middle one is a perfect square are formed using the digits 1 to 9. Their sum is :
(A) 134055 **(B)** 270540 **(C)** 170055 **(D)** None of these

8. A guardian with 6 wards wishes everyone of them to study either Law or Medicine or Engineering, Number of ways in which he can make up his mind with regard to the education of his wards if every one of them be fit for any of the branches to study, and atleast one child is to be sent in each discipline is:
(A) 120 **(B)** 216 **(C)** 729 **(D)** 540

9. There are $(p + q)$ different books on different topics in Mathematics. $(p \neq q)$

If L = The number of ways in which these books are distributed between two students X and Y such that X get p books and Y gets q books.

M = The number of ways in which these books are distributed between two student X and Y such that one of them gets p books and another gets q books.

N = The number of ways in which these books are divided into groups of p books and q books then :
(A) $L = M = N$ **(B)** $L = 2M = 2N$ **(C)** $2L = M = 2N$ **(D)** $L = M = 2N$

10. The number 916238457 is an example of nine digit number which contains each of the digit 1 to 9 exactly once. It also has the property that the digits 1 to 5 occur in their natural order, while the digits 1 to 6 do not. Number of such numbers are :

(A) 2268 **(B)** 2520 **(C)** 2975 **(D)** 1560

11. Number of functions defined from $f:\{1,2,3,4,5,6\}\to\{7,8,9,10\}$ such that the sum $f(1)+f(2)+f(3)+f(4)+f(5)+f(6)$ is odd, is :

(A) 2^{10} **(B)** 2^{11} **(C)** 2^{12} **(D)** $2^{12}-1$

12 The number of non-negative integral solutions of $x+y+z\le n$ where $n\in N$ is :

(A) ${}^{n+4}C_4$ **(B)** ${}^{n+5}C_5$ **(C)** ${}^{n+3}C_3$ **(D)** None of these

13. From 4 men and 6 ladies a committee of 5 is to be selected. The number of ways in which the committee can be formed so that men are in majority, is :

(A) 66 **(B)** 156 **(C)** 60 **(D)** None of these

14. Six persons *A*, *B*, *C*, *D*, *E* and *F* are to be seated at a circular table. The number of ways this can be done if *A* must have either *B* or *C* on his right and *B* must have either *C* or *D* on his right is :

(A) 36 **(B)** 12 **(C)** 24 **(D)** 18

15. The total number of integral solution for x, y, z such that $xyz = 24$ is :

(A) 30 **(B)** 60 **(C)** 90 **(D)** 120

16. How many five digit numbers can be formed from 1, 2, 3, 4, 5 (without repetition), when the digit at the unit place must be greater than that in the tenth place ?

(A) 54 **(B)** 60 **(C)** 5!/3 **(D)** $2 \times 4!$

17. Number of ways in which 7 green bottles and 8 blue tube bottles can be arranged in a row if exactly 1 pair of green bottles is side by side, is (Assume all bottles to be alike except for the colour).

(A) 84 **(B)** 360 **(C)** 504 **(D)** None of these

18. Seven different coins are to be divided amongst three persons. If no two of the persons receive the same number of coins but each receives atleast one coin & none is left over, then the number of ways it which the division may be made is

(A) 420 **(B)** 630 **(C)** 710 **(D)** None of these

19. The number of ways in which 8 distinguishable apples can be distributed among 3 boys such that every boy should get atleast 1 apple and atmost 4 apples is $K.\ {}^7P_3$ where *K* has the value equal to :

(A) 14 **(B)** 66 **(C)** 44 **(D)** 22

20. A rack has 5 different pairs of shoes. The number of ways in which 4 shoes can be chosen from it, so that there will be no complete pair is :

(A) 1920 **(B)** 200 **(C)** 110 **(D)** 80

21. There are 10 seats in a double decker bus, 6 in the lower deck and 4 on the upper deck. Ten passenger board the bus, of them 3 refuse to go to the upper deck and 2 insist on going up. The number of way in which the passengers can be accommodated is : (Assume all seats to be duly numbered)

(A) 172800 **(B)** 162800 **(C)** 152800 **(D)** 182800

22. An old man while dialing a 7 digit telephone number remembers that the first four digits consists of one 1'*s*, one 2'*s* and two 3'*s*. He also remembers that the fifth digit is either a 4 or 5 while has no memory of the sixth digit, he remembers that the seventh digit is 9 minus the sixth digit. Maximum number of distinct trials he has to try to make sure that he dials the correct telephone number, is

(A) 360 **(B)** 240 **(C)** 216 **(D)** None of these

23. The total number of different ways a grandfather along with two of his grandsons and four grand daughters can be seated in a line for a photograph so that he is always in the middle and the two grandsons are never adjacent to each other :

(A) 728 **(B)** 600 **(C)** 528 **(D)** 328

24. Number of ways in which 9 different toys be distributed among 4 children belonging to different age groups in such a way that distribution among the 3 elder children is even and the youngest one is to receive one toy more, is:

(A) $\frac{(5!)}{8}$ **(B)** $\frac{9!}{2}$ **(C)** $\frac{9!}{3!(2!)^3}$ **(D)** None of these

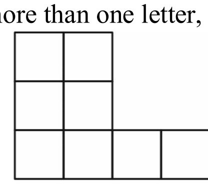
25. In an election three districts are to be canvassed by 2, 3 and 5 men respectively. If 10 men volunteer, the number of ways they can be allotted to the different districts is :

(A) $\frac{10!}{2!3!5!}$ **(B)** $\frac{10!}{2!5!}$ **(C)** $\frac{10!}{(2!)^2 5!}$ **(D)** $\frac{10!}{(2!)^2 3!\,5!}$

26. The number of ordered pairs (*m*, *n*), *m*, $n \in \{1, 2, ..., 50\}$ such that $6^n + 9^m$ is a multiple of 5:

(A) 2500 **(B)** 1250 **(C)** 625 **(D)** 500

27. Number of ways in which the letters of the word "NATION" can be filled in the given figure such that no row remains empty and each box contains not more than one letter, are:



(A) $11\underline{|6}$ **(B)** $12\underline{|6}$ **(C)** $13\underline{|6}$ **(D)** $14\underline{|6}$

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

28. The combinatorial *C*(*n*, *r*) is equal to :

(A) number of possible subsets of *r* members from a set of *n* distinct members

(B) number of possible binary messages of length *n* with exactly *r* 1's

(C) number of non decreasing 2-*D* paths from the lattice point (0,0) to (*r*, *n*)

(D) number of ways of selecting *r* things out of *n* different things when a particular thing is always included plus the number of ways of selecting '*r*' things out of *n*, when a particular thing is always excluded

29. Identify the correct statement(s).

(A) Number of naughts standing at the end of $\underline{|125}$ is 30

(B) A telegraph has 10 arms an each arm is capable of 9 distinct positions excluding the position of rest. The number of signals that can be transmitted is $10^{10} - 1$

(C) Number greater than 4 lacs which can be formed by using only the digits 0, 2, 2, 4, 4 and 5 is 90

(D) In a table tennis tournament, each player plays with every other player. If the number of games played is 5050 then the number of players in the tournament is 100

30. There are 10 questions, each question is either True or False. Number of different sequences of not all correct answers is also equal to :

(A) Number of ways in which a normal coin tossed 10 times would fall in a definite order if both Heads and Tails are present.

(B) Number of ways in which a multiple choice question containing 10 alternatives with one or more than one correct alternatives, can be attempted

(C) Number of ways in which it is possible to draw coins from 10 coins of different denominations taken some or all at a time.

(D) Number of different selections of 10 indistinguishable things taken some or all at a time.

31. The continued product, 2. 6. 10. 14….. to n factors is equal to :

(A) ${}^{2n}C_n$ **(B)** ${}^{2n}P_n$

(C) $(n + 1)\,(n + 2)\,(n + 3)\ldots.(n + n)$ **(D)** None of these

32. The number of ways in which five different books to be distributed among 3 persons so that each person gets at least one book, is equal to the number of ways in which:

(A) 5 persons are allotted 3 different residential flats so that each person is allotted at most one flat and no two persons are allotted the same flat

(B) number of parallelograms (some of which may be overlapping) formed by one set of 6 parallel lines and other set of 5 parallel lines that goes in other direction

(C) 5 different toys are to be distributed among 3 children, so that each child gets at least one toy

(D) 3 mathematics professors are assigned five different lectures to be delivered, so that each professor gets at least one lecture.

33. The maximum number of permutations of $2n$ letters in which there are only a's and b's, taken all at a time is given by :

(A) ${}^{2n}C_n$ **(B)** $\frac{2}{1}.\frac{6}{2}.\frac{10}{3}.......\frac{4n-6}{n-1}.\frac{4n-2}{n}$

(C) $\frac{n+1}{1}.\frac{n+2}{2}.\frac{n+3}{3}.\frac{n+4}{4}.......\frac{2n-1}{n-1}.\frac{2n}{n}$ **(D)** $\frac{2^n.[1.3.5.........(2n-3)(2n-1)]}{n!}$

34. Number of ways in which 3 different numbers in $A.P.$ can be selected from 1, 2, 3,…...n is :

(A) $\left(\frac{n-1}{2}\right)^2$ if n is even **(B)** $\frac{n(n-2)}{4}$ if n is odd

(C) $\frac{(n-1)^2}{4}$ if n is odd **(D)** $\frac{n(n-2)}{4}$ if n is even

35. The combinatorial coefficient ${}^{n-1}C_p$ denotes :

(A) The number of ways in which n things of which p are alike and rest different can be arranged in a circle

(B) The number of ways in which p different things can be selected out of n different thing if a particular thing is always excluded

(C) Number of ways in which n alike balls can be distributed in p different boxes so that no box remains empty and each box can hold any number of balls

(D) The number of ways in which $(n - 2)$ white balls and p black balls can be arranged in a line if no two black balls are together, balls are all alike except for the colour

36. Triplet (x, y, z) is chosen from the set $\{1, 2, 3, \ldots, n\}$, such that $x \le y < z$. The number of such triplets is :

(A) n^3 **(B)** ${}^{n+1}C_3$ **(C)** ${}^{n}C_2$ **(D)** ${}^{n}C_2 + {}^{n}C_3$

37. Which of the following statements are correct?

(A) Number of 6 letter words that can be formed using letters of the word "CENTRIFUGAL" if each word must contain all the vowels is 3.7 !

(B) There are 15 balls of which some are white and the rest black. If the number of ways in which the balls can be arranged in row, is maximum then the number of white balls must be equal to 7 or 8. Assume balls of the same colour to be alike

(C) There are 12 things, 4 alike of one kind, 5 alike and of another kind and the rest are all different. The total number of combinations is 240

(D) Number of selections that can be made of 6 letters from the word "COMMITIEE" is 35

38. In a plane, there are two families of lines $y = x + r$, $y = -x + r$, where $r \in \{0,1,2,3,4\}$. The number of squares of diagonals of length 2 formed by the lines is :

(A) $\left(\frac{2}{3}\right)(4!)$ **(B)** $\left(\frac{3}{2}\right)(3!)$ **(C)** 16 **(D)** 9

39. Number of ways in which the letters of the word 'B U L B U L' can be arranged in a line in any order is also equal to the

(A) number of ways in which 2 alike Apples and 4 alike Mangoes can be distributed in 3 children so that each child receives any number of fruits.

(B) Number of ways in which 6 different books can be tied up into 3 bundles, if each bundle is to have equal number of books.

(C) coefficient of $x^2y^2z^2$ in the expansion of $(x+y+z)^6$

(D) number of ways in which 6 different prizes can be distributed equally in three children.

40. Let $p = 2520$, $x =$ number of divisors of p which are multiples of 6, y = number of divisors of p which are multiples of 9, then :

(A) $x = 24$ **(B)** $x = 12$ **(C)** $y = 16$ **(D)** $y = 12$

41. A person wants to invite one or more of his friends for a dinner party. In how many ways can he do so if he has eight friends?

(A) 2^8 **(B)** $2^8 – 1$ **(C)** 8^2 **(D)** ${}^8C_1 + {}^8C_2 + {}^8C_3 + \ldots + {}^8C_8$

42. In an examination, a candidate is required to pass in all four subjects he is studying. The number of ways in which he can fail is :

(A) ${}^4P_1 + {}^4P_2 + {}^4P_3 + {}^4P_4$ **(B)** $4^4 – 1$

(C) $2^4 – 1$ **(D)** ${}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$

43. The number of ways to select 2 ordered numbers from {0, 1, 2, 3, 4} such that the sum of the squares of the selected numbers is divisible by 5 are (repetition of digits is allowed)

(A) ${}^9C_1$ **(B)** ${}^9C_8$ **(C)** 9 **(D)** 7

44. If n objects are arranged in a row, then the number of ways of selecting three of these objects so that no two of them are next to each other is :

(A) $\frac{(n-2)(n-3)(n-4)}{6}$ **(B)** ${}^{n-2}C_3$

(C) ${}^{n-3}C_3 + {}^{n-3}C_2$ **(D)** None of these

45. m points on one straight line are joined to n points on another straight line. The number of points of intersection of the line segments thus formed (not lying on given two lines) is :

(A) ${}^mC_2.{}^nC_2$ **(B)** $\dfrac{mn(m-1)(n-1)}{4}$ **(C)** $\dfrac{{}^mC_2.{}^nC_2}{2}$ **(D)** ${}^mC_2+{}^nC_2$

46. Consider seven digit number $x_1, x_2, \ldots, x_7$, where $x_1, x_2, \ldots, x_7 \neq 0$ having the property that x_4 is the greatest digit and digits towards the left and right of x_4 are in decreasing order (from left to right). Then total number of such numbers in which all digits are distinct is :

(A) ${}^9C_7.{}^6C_3$ **(B)** ${}^9C_6.{}^5C_3$ **(C)** ${}^{10}C_7.{}^6C_3$ **(D)** ${}^9C_2.{}^6C_3$

47. A contest consists of ranking 10 songs of which 6 are Indian classic and 4 are western songs. Number of ways of ranking so that. (mention correct statements)

(A) There are exactly 3 Indian classic songs in top 5 is $(5!)^3$
(B) Top rank goes to Indian classic song is 6(9!)
(C) The rank of all western songs are consecutive is 4!7!
(D) The 6 Indian classic songs are in a specified order is ${}^{10}P_4$

48. $P = n(n^2-1)(n^2-4)(n^2-9)....(n^2-100)$ is always divisible by; ($n \in I$)

(A) 2! 3! 4! 5! 6! **(B)** $(5!)^4$ **(C)** $(10!)^2$ **(D)** 10! 11!

49. A fair coin is tossed n times. Let a_n denote the number of cases in which no two heads occur consecutively. Then which of the following is true?

(A) $a_1 = 2$ **(B)** $a_2 = 3$ **(C)** $a_5 = 14$ **(D)** $a_8 = 55$

50. All the five-digit numbers in which each successive digit exceeds its predecessor are arranged in the increasing order. The 105th number does not contain the digit:

(A) 1 **(B)** 3 **(C)** 4 **(D)** 5

51. The number of ways of selecting two 1×1 squares from a chess board such that they:

(A) Have a common vertex is 98
(B) Have a common side is 112
(C) Neither have a common vertex nor have a common side is 1806
(D) None of these

52. The kindergarten teacher has 25 kids in her class. She takes 5 of them at a time to zoological garden as often as she can without taking the same 5 kids more than once. Then the number of visits, the teacher makes to the garden exceeds that of a kid by:

(A) ${}^{25}C_5 - {}^{24}C_4$ **(B)** ${}^{24}C_5$ **(C)** ${}^{25}C_5 - {}^{24}C_5$ **(D)** ${}^{24}C_4$

53. Suppose a lot contains 2n objects of which n are identical. The number of ways to select n objects out of these $2n$ objects must be:

(A) 2^n
(B) $({}^{2n+1}C_0 + {}^{2n+1}C_1 + \ldots + {}^{2n+1}C_n)^{1/2}$
(C) The number of possible subsets of the set $\{a_1, a_2 \ldots, a_n\}$
(D) None of these

54. The number of selections of 4 letters taken from the word "COLLEGE" must be:

(A) 18
(B) 22
(C) Coefficient of x^4 in the expansion of $(1+x)^3\left(1+x+x^2\right)^2$
(D) Coefficient of x^4 in the expansion of $(1+x)^2\left(1+x+x^2\right)^3$

55. The integers from 1 to 1000 are written in order around a circle. Starting at 1, every fifteenth numbers is marked (i.e., 1, 16, 31 etc.). This process is continued until a number is reached which has already been marked, then unmarked numbers are:

(A) 200 **(B)** 400 **(C)** 600 **(D)** 800

56. The number of ways in which 10 students can be divided into three teams, one containing 4 and others 3 each, is:

(A) $\frac{10!}{4!\,3!\,3!}$ **(B)** 2100 **(C)** ${}^{10}C_4 \times {}^5C_2$ **(D)** $\frac{10!}{6!\,3!\,3!}.\frac{1}{2}$

57. The number of isosceles triangles with integer sides if no side exceeding 2008 is:

(A) $(1004)^2$ if equal sides do not exceed 1004
(B) $2(1004)^2$ if equal sides exceed 1004
(C) $3(1004)^2$ if equal sides have any length $\le$ 2008
(D) $(2008)^2$ if equal sides have any length $\le$ 2008

58. The number of ways of distributing 10 different books among 4 students (S_1, S_2, S_3 and S_4) such that S_1 and S_2 get 2 books each and S_3 and S_4 gets 3 books each is:

(A) 12600 **(B)** 25200 **(C)** ${}^{10}C_4$ **(D)** $\frac{10!}{2!\,2!\,3!\,3!}$

59. Given that the divisors of $n = 3^p.\,5^q.\,7^r$ are of the form $4\lambda+1,\ \lambda \ge 0$. Then

(A) $p+r$ is even **(B)** $p+q+r$ is even or odd
(C) q can be any integer **(D)** if p is odd, then r is odd

60. For the equation $x+y+z+w=19$, the number of positive integral solutions is equal to:

(A) The number of ways in which 15 identical things can be distributed among 4 persons
(B) The number of ways in which 19 identical things can be distributed among 4 persons
(C) Coefficient of x^{19} in $(x^0+x^1+...+x^{19})^4$
(D) Coefficient of x^{19} in $(x+x^2+x^3+...+x^{19})^4$

61. Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and let $\vec{r}$ be a variable vector such that $\vec{r}.\hat{i},\ \vec{r}.\hat{j}$ and $\vec{r}.\hat{k}$ are positive integers. If $\vec{r}.\vec{a} \le 12$ then the number of values of $\vec{r}$ is:

(A) ${}^{12}C_9 - 1$ **(B)** ${}^{12}C_3$ **(C)** ${}^{12}C_9$ **(D)** ${}^{12}C_3 - 1$

62. The number of 5 letter words formed with the letters of the word Calculus is divisible by:

(A) 2 **(B)** 3 **(C)** 5 **(D)** 7

63. The coefficient of x^{50} in the expansion of $\sum_{k=0}^{100} {}^{100}C_k (x-2)^{100-k} 3^k$ is also equal to:

(A) Number of ways in which 50 identical books can be distributed in 100 students, if each student can get almost one book

(B) Number of ways in which 100 different white balls and 50 identical red balls can be arranged in a circle, if no two red balls are together

(C) Number of dissimilar terms in $(x_1 + x_2 + x_3 + ... + x_{50})^{51}$

(D) $\dfrac{2.6.10.14.....98}{51!}$

64. Let a, b, c, d be non-zero distinct digits. The number of 4-digit numbers $abcd$ such that $ab + cd$ is even is divisible by:

(A) 3 **(B)** 4 **(C)** 7 **(D)** 11

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column 1 are labelled as (A), (B), (C) & (D) whereas statements in Column 2 are labeled as p, q, r, s & t. More than one choice from Column 2 can be matched with Column 1.

65. In how many ways 7 digit numbers can be formed by using the digits 1, 2, 3, 4, 5 such that

	Column 1		**Column 2**
(A)	Repetition is allowed	**(p)**	78, 120
(B)	Exactly 3 digits will appear	**(q)**	76, 860
(C)	Atleast 2 digit will appear	**(r)**	78, 125
(D)	Atleast 3 digit will appear	**(s)**	18060

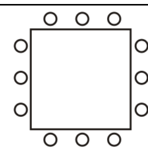
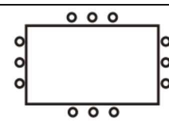
66. In how many ways 7 distinct hats can be arranged among 7 persons such that :

	Column 1		**Column 2**
(A)	No person will get its own hat	**(p)**	1331
(B)	Exactly 3 person will get their own hats	**(q)**	407
(C)	Atleast 2 person will get their own hat	**(r)**	1854
(D)	Atleast 3 person will get their own hat	**(s)**	315

67. A dice is thrown 7 times. Find the number of possible outcomes if :

	Column 1		**Column 2**
(A)	All digits will appear	**(p)**	279930
(B)	Exactly 3 digits will appear	**(q)**	15120
(C)	Atleast 2 digits will appear	**(r)**	264816
(D)	Atmost 5 digits will appear	**(s)**	36, 120

68. In how many ways 12 persons can be seated :

	Column 1		Column 2
(A)	Linearly	**(p)**	$3\times\lvert\underline{11}$
(B)	Circularly	**(q)**	$6\times\lvert\underline{11}$
(C)	Around a square table 	**(r)**	$\frac{\lvert\underline{12}}{12}$
(D)	Around a rectangular table 	**(s)**	$\lvert\underline{12}$

69. If p is the total number of ways of arranging 36 girls around a table and q is the exponent of 2 in p then the value of q, if :

	Column 1		Column 2
(A)	The table is circular is	**(p)**	Total number of zeros in $(10^{33}+10^{10})$
(B)	The table is square and 9 girls each side is	**(q)**	Total number of trailing zeros in 138!
(C)	The table is rectangular and 10 girls along length and 8 around along breadth is	**(r)**	The exponent of 5 in 132!
(D)	The table is hexagonal and 6 girls each side is	**(s)**	Exponent of 3 in 70!
		(t)	Total number of trailing zeros in $10^{100}+10^{33}$

SUBJECTIVE INTEGER TYPE

Each of the following question has an integer answer between 0 and 9.

70. A staircase has 10 steps. A person can go up the steps one at a time, two at a time, or any combination of 1's and 2's. If the number of ways in which the person can go up the stairs is p, then find $\frac{p}{89}$.

71. $A_1A_2,.......A_{2n}$ is regular $2n$ sided polygon $(n \ge 3)$. Find the ratio of number of obtuse angled triangles to the number of acute angled triangles formed by joining the vertices of the polygon.

72. There are n persons sitting around a circular table. They start singing a 2 minute song in pairs such that no two persons sitting together will sing together. This process is continued for 28 minutes. Find n.

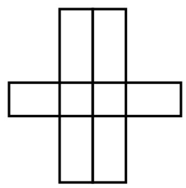
73. If the total number of strictly increasing functions defined from $f:\{1,2,3,4,5,6\}\rightarrow\{1,2,3,....9\}$ is k then $\frac{k}{12}$ is equal to ________.

74. If the total number of non-decreasing functions defined from $f:\{1,2,3,4,5\}\to\{1,2,3,4,5,6,7,8,9\}$ is m then $\frac{m}{143}$ is equal to _______.

75. If total number of ways of distributing 5 distinct objects and 4 identical objects among 3 persons is (blank group is allowed) $3^p \times q$ then $q-2p$ is equal to _______. (HCF of p and q is 5)

76. If the number of ways in which 8 people can be arranged in a line if A and B must be next to each other and C must be somewhere behind D is equal to 'm' then sum of all the digits of m is equal to _______.

77. Six X's have to be placed in the squares of the figure given below, such that each row contains at least one X. If the total no. of different ways this can be done is 'm' then $\frac{m}{13}$ is equal to __________.



78. A conference attended by 200 delegates is held in a hall. The hall has 7 doors, marked A, B, …….., G. At each door, an entry book is kept and the delegates entering through that door sign it in the order in which they enter. If each delegate is free to enter any time and through any door he likes, if the total no. of different sets of seven lists would arise in all is equal to ${}^{n}P_{r}$ then 'n-r' is equal to (Assume that every person signs only at his first entry).

79. If u_r denoted the number of one-one functions from $(x_1, x_2, ..., x_r)$ to $(y_1, y_2, ..., y_r)$ such that $f(x_i) \neq y_i$, for $i = 1,2,3,..........,r$ then $u_4 =$

80. Three friends went to a shopping mall with \$ 6, 7 and 8 with them in how many ways they can pay a bill of \$ 10 if they have note of only one denomination.

81. In how many rotationally distinct ways can the vertices of a cube be coloured with black or white colour?

82. Fifteen coupons are numbered 1, 2, 3, … 15. Seven coupons are selected such that the largest number appearing on the selected coupon is 9, if total number of ways is ${}^{n}C_8$, then n is______.

83. Ramesh has $2n$ number of fruits out of which n of them are identical and remaining n are distinct, If the total number of ways he can distribute these fruits to his two children Bhavesh and Sanjesh such that both of them will receive equal number of fruits is 16 then n is equal to______.

84. In an Ice cream parlor at South City Mall Kolkata, 4 different varieties of ice creams namely Vanila, Strawberry, Chocolate and Butter Scotch were available. On a particular day it was noticed that each customer bought at least one ice cream and at max 10 ice creams, on further investigation it was noticed that no two customer bought same set of ice creams then if the number of customers visited the ice cream shop on that particular day is k then $\frac{k}{100}$ is______.

85. Mr. Anshuman has thrown a dice 6 times in k ways we can get a sum greater than 17 then $\frac{k}{10000}$ is______.

86. If k is the number of positive integral solutions of the in equality $a+b+3c\leq 30$? Then $\frac{k}{5}$ is______.

87. Each set has 'm' parallel lines. If the total number of parallelograms thus formed is 225 then m is equal to______.

88. If λ be the number of 3-digit numbers are of the form xyz with $x<y$, $z<y$ and $x\neq 0$, the value of λ is______.

89. In k ways can you place 2 rooks on a chessboard such that they are not in attacking positions, if rooks can attack only in a same row or in a same column? Then $\frac{k}{100}$ is______.

90. Consider a polygon of k sides. If the number of triangles that can be drawn taking vertices of these polygons as vertices of triangles and no sides of triangles is common with any sides of the polygon is 50 then k is______.

91. In a class of 10 students if two prizes (1^{st} and 2^{nd}) has to be given in three subjects Physics, Chemistry & Mathematics and this can be done in k ways, then $\frac{k}{1000}$ is______.

92. Consider 5 points in a plane are situated so that no two of the straight lines joining them are parallel, perpendicular, or coincident. From each point perpendiculars are drawn to all the lines joining the other four points. Determine the maximum number of intersections that these perpendiculars can have?

93. Consider a 6×6 square which is dissected into 9 rectangles by lines parallel to its sides such that all the rectangles have integral sides. What is the minimum number of congruent rectangles?

94. Consider a set $X=\{1, 2, 3, \ldots, 9, 10\}$. If the number of pairs $\{A, B\}$ such that $A\subseteq X$ and $B\subseteq X$ also $A\neq B$ and $A\cap B=\{2, 3, 5, 7\}$ is $3\lambda-4$ then $\lambda-\mu$ is______.

95. At IITD, roll number of N students are given from 1 to N. Three students are selected from these N students such that their roll numbers are not consecutive, the total number of ways this selection can be done is 10, then N is equal to______.

96. Consider a set $S=\{1, 2, \ldots, 100\}$ two elements p and q are selected from this set S such that 7^p+7^q is divisible by 5, How many ways this selection can be made?

97. In a particular batch of VIDYAMANDIR CLASSES Boston, there are 4 boys and certain number of girls. In every mock test only 5 students including at least 3 boys can appear. If different group of students write the Mock exam every time and number of times test conducted is 66 then find the total number of students in the class.

98. The total number of ways in which 10 Men and 10 Women can form 10 mixed complex (a mixed couple contain a Man and a Woman) is N, then $\frac{N}{9!}$ is equal to______.

99. Find the minimum value of k such that (k!!) is completely divisible by all two-digit prime numbers.

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. Let k and n be the positive integers and $S_k = 1^k + 2^k + 3^k + ... + n^k$. Then ${}^{m+1}C_1 S_1 + {}^{m+1}C_2 S_2 + {}^{m+1}C_3 S_3 + ... {}^{m+1}C_m S_m$ is equal to :

(A) $(n+1)^{m+1}$ **(B)** $(n+1)^m - n$ **(C)** $(n+1)^{m+1} - 1$ **(D)** $(n+1)^{m+1} - n - 1$

2. The sum of the series ${}^nC_1^2 + \frac{1+2}{2}\,{}^nC_2^2 + \frac{1+2+3}{3}\,{}^nC_3^2 ..... + \frac{1+2+3+....n}{n}\,{}^nC_n^2$ is equal to :

(A) $\frac{1}{2}(n\,{}^{2n-1}C_n + {}^{2n}C_n)$ **(B)** $\frac{1}{2}(n\,{}^{2n-1}C_n + {}^{2n}C_n - 1)$

(C) $\frac{1}{2}((n+1)\,{}^{2n-1}C_n - 1)$ **(D)** $\frac{1}{2}(n\,{}^{2n-1}C_n + {}^{2n}C_n - 2)$

3. $\frac{{}^nC_0}{2} - \frac{{}^nC_1}{6} + \frac{{}^nC_2}{10} - \frac{{}^nC_3}{14} + ... + \frac{(-1)^n\,{}^nC_n}{(4n+2)}$, $n \in N$ is equal to :

(A) $\frac{2^{n-1} n!}{3\cdot5\cdot7.....(2n-1)(2n+1)}$ **(B)** $\frac{2^n n!}{1\cdot3\cdot5\cdot7.....(2n-3)(2n-1)}$

(C) $\frac{2^n n!}{3\cdot5.....(2n-1)(2n+1)}$ **(D)** $\frac{2^n (n-1)!}{1\cdot3\cdot5.....(2n-3)(2n-1)}$

4. ${}^nC_1 - \left(1+\frac{1}{2}\right){}^nC_2 + \left(1+\frac{1}{2}+\frac{1}{3}\right){}^nC_3 - \left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right){}^nC_4 + ... + (-1)^{n-1}\left(1+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{n}\right){}^nC_n =$

(A) $\frac{n-1}{n}$ **(B)** $\frac{1}{n}$ **(C)** $\frac{1}{n+1}$ **(D)** $\frac{2^n}{n+1}$

5. $\sum_{r=1}^{n} \frac{(-1)^{r-1}\,{}^nC_r (1-x)^r}{r} =$

(A) $x + \frac{x^2}{2} + \frac{x^3}{3} + ... + \frac{x^n}{n}$ **(B)** $\frac{1-x}{1} + \frac{1-x^2}{2} + \frac{1-x^3}{3} + ... + \frac{1-x^n}{n}$

(C) $(x-1) + \frac{x^2-1}{2} + \frac{x^3-1}{3} + ... + \frac{x^n-1}{n}$ **(D)** $n - \frac{x}{1} - \frac{x^2}{2} - \frac{x^3}{3} ... \frac{x^n}{n}$

6. ${}^nC_0\, x^{2n} + \frac{{}^nC_1}{2} x^{2n-2}(2-x^2) + \frac{{}^nC_2}{3} x^{2n-4}(2-x^2)^2 + ... + \frac{{}^nC_n (2-x^2)^n}{(n+1)} =$

(A) $\frac{2^n - x^{2n+2}}{(n+1)(2-x^2)}$ **(B)** $\frac{2^n - x^{2n}}{(n+1)(2-x^2)}$ **(C)** $\frac{2^{n+1} - x^{2n+2}}{(n+1)(2-x^2)}$ **(D)** $\frac{2^{n+1} - x^{2n}}{(n+1)(2-x^2)}$

7. If $p + q = 1$, then $\sum_{r=0}^{n} r^3\,{}^nC_r\, p^r q^{n-r} =$

(A) $np(n^2 p + 3(n-1)p + 1)$ **(B)** $np((n^2 - n)p^2 + 2(n-1)p + 1)$

(C) $np((n^2 - 3n + 2)p^2 + 2(n-1)p + 1)$ **(D)** $np((n^2 - 3n + 2)p^2 + 3(n-1)p + 1)$

For Questions 8 - 10

If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r^2}{{}^nC_r} = P(n)\,a_{n+2} + Q(n)\,a_{n+1} + a_n + R(n)$, where $P(n), Q(n), R(n)$ are the polynomial functions of n, then :

8. $P(5) =$

(A) 56 **(B)** 42 **(C)** 36 **(D)** 30

9. $Q(5) =$

(A) -5 **(B)** -10 **(C)** -15 **(D)** -18

10. $R(5) =$

(A) -24 **(B)** -20 **(C)** -30 **(D)** -42

For Questions 11 - 13

Let $f_1(x) = (x-2)^2,\ f_2(x) = ((x-2)^2-2)^2,\ f_3(x) = \left((x-2)^2-2)^2-2\right)^2$,..... and so on ; so that

$$f_k(x) = \underbrace{\left(.....\left((x-2)^2-2\right)^2.....-2\right)^2}_{k \text{ times}} = A_k + B_k x + C_k x^2 + D_k x^3 + ...$$

11. B_5 is equal to :

(A) -2048 **(B)** -32 **(C)** -1024 **(D)** -512

12. C_3 is equal to :

(A) 256 **(B)** 352 **(C)** 320 **(D)** 336

13. C_k is equal to :

(A) $\frac{4^{2k-1}-4^{k-1}}{3}$ **(B)** 4^{2k-2} **(C)** $\frac{4^{2k-1}+4^{k-1}}{5}$ **(D)** 4^{k+1}

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

14. $\sum_{r=1}^{n} r(n-r)\,{}^nC_r^2$ is equal to :

(A) $n^2\,{}^{2n-2}C_{n-2}$ **(B)** $(n-1)^2\,{}^{2n-2}C_{n-1}$

(C) $n(n-1)\,{}^{2n-2}C_{n-2}$ **(D)** $n(n-1)\,{}^{2n-2}C_{n-1}$

15. The value of the sum ${}^nC_1^2 - 2\cdot{}^nC_2^2 + 3\cdot{}^nC_3^2 - 4\cdot{}^nC_4^2 + ... + (-1)^n n\cdot{}^nC_n^2$ where $n \in N,\ n > 3$ will be equal to :

(A) $-n\,{}^{n-1}C_{\frac{n-2}{2}}$ if $n = 4k,\ k \in I$ **(B)** $n\,{}^{n-1}C_{\frac{n-1}{2}}$ if $n = 4k+1,\ k \in I$

(C) $n\,{}^{n-1}C_{\frac{n-2}{2}}$ if $n = 4k+2,\ k \in I$ **(D)** $-n\,{}^{n-1}C_{\frac{n-1}{2}}$ if $n = 4k+3,\ k \in I$

16. If $n \in N$ and $(1+x+x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then $\sum_{r=0}^{n} (-1)^r a_r \, {}^nC_r$ is equal to :

(A) 0 if $n = 57$ **(B)** 0 if $n = 77$ **(C)** ${}^{24}C_8$ if $n = 24$ **(D)** ${}^{39}C_{13}$ if $n = 39$

17. In the expansion of $(x+a)^n$, $n \in N$, if the sum of odd numbered terms be α and the sum of even numbered terms be β, then :

(A) $4\alpha\beta = (x+a)^{2n} - (x-a)^{2n}$ **(B)** $2(\alpha^2+\beta^2) = (x+a)^{2n} + (x-a)^{2n}$

(C) $\alpha^2 - \beta^2 = (x^2-a^2)^n$ **(D)** $\alpha^2+\beta^2 = (x+a)^{2n} + (x-a)^{2n}$

18. If the middle term of the expression $(1+x)^{24}$, $x > 0$, is the only greatest term of the expansion, then :

(A) $x < 1$ **(B)** $x < \frac{13}{12}$ **(C)** $x > \frac{12}{13}$ **(D)** $x > 1$

19. ${}^nC_m + 3\,{}^{n-1}C_m + 5\,{}^{n-2}C_m + 7\,{}^{n-3}C_m + ... + (2(n-m)+1)\,{}^mC_m$ is equal to :

(A) ${}^{n+2}C_{m+3} + {}^{n+3}C_{m+3}$ **(B)** ${}^{n+2}C_{m+2} + 2\,{}^{n+2}C_{m+3}$

(C) ${}^{n+1}C_{m+2} + {}^{n+2}C_{m+2}$ **(D)** ${}^{n+1}C_{m+1} + 2\,{}^{n+1}C_{m+2}$

20. Let n be a positive integer and $(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n-1}x^{2n-1} + a_{2n}x^{2n}$, then :

(A) $\sum_{r=0}^{n-1} a_r = \frac{1}{2}(3^n - a_n)$ **(B)** $\sum_{r=0}^{2n} (-1)^r a_r^2 = a_n$

(C) $a_0^2 - a_1^2 + a_2^2 - a_3^2 + ... + (-1)^{n-1} a_{n-1}^2 = \frac{a_n}{2}(1-(-1)^n a_n)$

(D) $(r+1)a_{r+1} = (n-r)a_r + (2n-r+1)a_{r-1}$, $1 \le r \le 2n-1$

21. Let $n \in N$, $n \ge 4$ and $P = \prod_{r=0}^{n} {}^nC_r$, then :

(A) $P > \left(\frac{2^n}{n+1}\right)^{n+1}$ **(B)** $P < \left(\frac{2^n}{n+1}\right)^{n+1}$ **(C)** $P < \left(\frac{2^n-2}{n-1}\right)^{n-1}$ **(D)** $P < \left(\frac{2^n-2}{n-1}\right)^{n}$

22. Let $S_1 = \sum_{r=0}^{n} ({}^{2n+1}C_{2r})^2$ and $S_2 = \sum_{r=0}^{n} ({}^{2n+1}C_{2r+1})^2$, then :

(A) $S_1 = \frac{1}{2}({}^{4n+2}C_{2n} + (-1)^n\,{}^{2n+1}C_n)$ **(B)** $S_2 = \frac{1}{2}({}^{4n+2}C_{2n} - (-1)^n\,{}^{2n+1}C_n)$

(C) $S_1 = \frac{1}{2}({}^{4n+2}C_{2n+1})$ **(D)** $S_2 = \frac{1}{2}({}^{4n+1}C_{2n+1})$

23. Let the coefficient of x^{20} in the expressions $(1+x^2-x^3)^{1000}$, $(1-x^2+x^3)^{1000}$, $(1-x^2-x^3)^{1000}$ and $(1+x^2+x^3)^{1000}$ be respectively a, b, c and d, then :

(A) $a = d$ **(B)** $a > b$ **(C)** $a > c$ **(D)** $b < c$

24. Let $\sum_{r=0}^{200} \alpha_r (1+x)^r = \sum_{r=0}^{200} \beta_r x^r$, where $\alpha_r = 1 \; \forall \; r \geq 98$, then the greatest coefficient in the expansion of $(1+x)^{201}$ is :

(A) $^{201}C_{100}$ **(B)** β_{98} **(C)** β_{99} **(D)** β_{100}

25. Let $x = (5\sqrt{3}+8)^{2n+1}$, $n \in N$, then :

(A) $[x]$ is even **(B)** $[x]$ is odd **(C)** $x\{x\} = (11)^{2n+1}$ **(D)** $x\{x\} = (13)^{2n+1}$

where $[\cdot]$ denotes greatest integer and $\{\cdot\}$ denote fraction part function

26. If $(1+x+x^2)^n = a_0 + a_1 x + a_2 x^2 + ... + a_{2n} x^{2n}$, then the value of $a_1 - a_3 + a_5 - a_7 + ...$ is equal to :

(A) 1 if $n = 4k$ **(B)** -1 if $n = 4k+1$ **(C)** 0 if $n = 4k+2$ **(D)** -1 if $n = 4k+3$

27. If the unit digit of $13^n + 7^n - 3^n$, $n \in N$, is 3 then possible value(s) of n is/are :

(A) 27 **(B)** 103 **(C)** 11 **(D)** 101

28. If the coefficient of x^t and x^{t+1} in $\sum_{r=0}^{n} (1+x)^r$ where $t < n-2$ are equal, then :

(A) n is odd **(B)** n is even

(C) The sum of coefficients of x^t and $x^{t+1} = {}^{n+1}C_{t+2}$

(D) The sum of coefficients of x^t and $x^{t+1} = {}^{n+2}C_{t+2}$

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column I are labelled as (A), (B), (C) & (D) whereas statements in Column II are labeled as P, Q, R, S & T. More than one choice from Column II can be matched with Column I.

29. Match the column :

	Column-I		Column-II
(A)	If the fourth term in the expansion of $\left(\frac{x}{a}+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, then a^2 is divisible by	**(P)**	2
(B)	$\sum_{p=1}^{4} \sum_{r=p}^{4} {}^4C_r \, {}^rC_p$ is divisible by	**(Q)**	4
(C)	The coefficient of x^{13} in $(1-x)^5(1+x+x^2+x^3)$ is divisible by	**(R)**	5
(D)	$\sum_{r=0}^{4} {}^4C_r (r-2)^2$ is divisible by	**(S)**	8
		(T)	13

30. Match the column:

	Column-I		Column-II
(A)	The coefficient of x^4 in $(2-x+3x^2)^6$ is	**(P)**	1024
(B)	${}^{11}C_0\ {}^{22}C_{11} - {}^{11}C_1\ {}^{20}C_{11} + {}^{11}C_2\ {}^{18}C_{11} - {}^{11}C_3\ {}^{16}C_{11} + ... =$	**(Q)**	2048
(C)	${}^5C_1\ {}^5C_5 - {}^5C_2\ {}^{10}C_5 + {}^5C_3\ {}^{15}C_5 - {}^5C_4\ {}^{20}C_5 + {}^5C_5\ {}^{25}C_5 =$	**(R)**	990
(D)	The coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^{11}$ is	**(S)**	3125
		(T)	3660

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

31. The value of ${}^{50}C_6 - {}^5C_1\ {}^{40}C_6 + {}^5C_2\ {}^{30}C_6 - {}^5C_3\ {}^{20}C_6 + {}^5C_4\ {}^{10}C_6$ is equal to_____.

32. The remainder obtained when $6^{2007} + 8^{2007}$ is divided by 49 is equal to_____.

33. ${}^{10}C_0\ {}^{20}C_{10} - {}^{10}C_1\ {}^{18}C_{10} + {}^{10}C_2\ {}^{16}C_{10} - {}^{10}C_3\ {}^{14}C_{10} + {}^{10}C_4\ {}^{12}C_{10} - {}^{10}C_5\ {}^{10}C_{10}$ is equal to_____.

34. The coefficient of x^5y^5 in the expression of $((1+x+y+xy)(x+y))^5$ is equal to_____.

35. The coefficient of $x^{\frac{n^2+n-14}{2}}$ in $(x-1)(x^2-2)(x^3-3)(x^4-4)....(x^n-n),\ n \ge 30$ is equal to_____.

36. ${}^nC_0\ {}^{2n}C_n - {}^nC_1\ {}^{2n-1}C_n + {}^nC_2\ {}^{2n-2}C_n - {}^nC_3\ {}^{2n-3}C_n + ...+...+(-1)^n\ {}^nC_n\ {}^nC_n =$

37. The value of $\dfrac{(18^3+7^3+3\cdot 18\cdot 7\cdot 25)}{(3^6+6\cdot 243\cdot 2+15\cdot 81\cdot 4+20\cdot 27\cdot 8+15\cdot 9\cdot 16+6\cdot 3\cdot 32+64)}$ is equal to_____.

38. For what x is the 4th term in the expansion of $\left[\left(5^{1/3}\right)^{-1/2\log_{10}(6-\sqrt{8x})} + \left(\dfrac{5^{\log_{10}(x-1)}}{25^{\log_{10}5}}\right)^{1/6}\right]^m$ is equal to $\dfrac{84}{5}$, if it is known that $\dfrac{14}{9}$ of binomial coefficient of 3rd term, binomial coefficient of 4th term and binomial coefficient of 5th term in the expansion constitute a G.P.

39. Let $x = (5+2\sqrt{6})^n,\ n \in N,$ then find the value of $x - x^2 + x[x],$ where $[\cdot]$ denotes greatest integer function.

40. If the coefficient of x^2 + coefficient of x in the expansion of $(1+x)^m(1-x)^n,\ (m \ne n)$ is equal to $-m,$ then the value of $n-m$ is equal to_____.

JEE Advanced Revision Booklet	Straight Line

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1 If $L \equiv \left(\frac{1}{x_l}, l\right); M = \left(\frac{1}{x_m}, m\right); N = \left(\frac{1}{x_n}, n\right)$ where $x_k \neq 0$, denotes the k^{th} terms of a *H.P.* for $k \in N$, then :

(A) $ar(\Delta LMN) = \frac{l^2m^2n}{2}\sqrt{(l-m)^2 + (m-n)^2 + (n-l)^2}$ **(B)** ΔLMN is a right angled triangle

(C) The points *L*, *M*, *N* are collinear **(D)** ΔLMN is equilateral

2. Two points P_1 and P_2 are at distances r_1 and r_2 respectively from the origin *O* and OP_1 and OP_2 makes angle θ_1 and θ_2 respectively with the *x*-axis. Let there be a point *P* on P_1P_2 such that *OP* makes an angle $\frac{\theta_2 + \theta_1}{2}$ with the *x*-axis. Then *OP* is :

(A) $\frac{2r_1r_2}{r_1 + r_2}\cos\frac{\theta_2 - \theta_1}{2}$ **(B)** $\frac{2r_1r_2}{r_1 + r_2}\sin\frac{\theta_2 - \theta_1}{2}$

(C) $\frac{r_1r_2}{r_1 + r_2}\cos\frac{\theta_2 + \theta_1}{2}$ **(D)** $\frac{r_1r_2}{r_1 + r_2}\sin\frac{\theta_2 + \theta_1}{2}$

3. If *A* (3, 0) and *B*(6, 0) are two fixed points and $U(\alpha, \beta)$ is a variable point in the plane. *AU* and *BU* meet the *y*-axis at *C* and *D* respectively and *AD* meet *OU* at *V*. Then the coordinate of the point through which *CV* always passes is :

(A) (3, 4) **(B)** (4, 0) **(C)** (0, 2) **(D)** (2, 0)

4. If $\frac{a}{\sqrt{bc}} - 2 = \sqrt{\frac{b}{c}} + \sqrt{\frac{c}{b}}$ where *a*, *b*, *c* > 0, then family of lines $\sqrt{a}x + \sqrt{b}y + \sqrt{c} = 0$ passes through the point:

(A) (1, 1) **(B)** (1, –2) **(C)** (–1, 2) **(D)** (–1, 1)

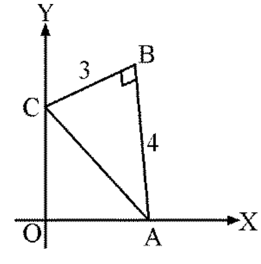
5. *I*(1,0) is the centre of incircle of triangle *ABC*, the equation of *BI* is $x - 1 = 0$ and equation of C*I* is $x - y - 1 = 0$, then angle *BAC* is:

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{3}$ **(C)** $\frac{\pi}{2}$ **(D)** $\frac{2\pi}{3}$

6. If the points where the lines $3x - 2y - 12 = 0$ and $x + ky + 3 = 0$ intersect both the coordinate axes are concyclic, then the number of possible real values of *k* is:

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

7. In the adjacent figure ΔABC is right angled at *B*. If *AB* = 4 and *BC* = 3 and side *AC* slides along the coordinate axes in such a way that '*B*' always remains in the first quadrant, then *B* always lie on straight line :



(A) $y = x$ **(B)** $3y = 4x$

(C) $3x = 4y$ **(D)** $3y + 4x = 0$

8. Consider the triangle OAB where $O \equiv (0,0), B(3,4)$. If orthocenter of triangle is $H(1, 4)$, then coordinates of 'A' is :

(A) $\left(0,\frac{15}{4}\right)$ **(B)** $\left(0,\frac{17}{4}\right)$ **(C)** $\left(0,\frac{21}{4}\right)$ **(D)** $\left(0,\frac{19}{4}\right)$

9. Two vertices of a triangle are (5, –1) and (–2, 3). If orthocenter of the triangle is origin, then the co-ordinates of third vertex is :

(A) (4, 7) **(B)** (3, 7) **(C)** (–4, –7) **(D)** (4, –7)

10. It is desired to construct a right angled triangle $ABC(\angle C = \pi/2)$ in xy-plane so that its sides are parallel to co-ordinates axes and the medians through A and B lie on the lines $y = 3x+1$ and $y = mx+2$ respectively. The values of 'm' for which such a triangle is possible is/are :

(A) –12 **(B)** 12 **(C)** 4/3 **(D)** 1/12

11. m, n are integer with $0 < n < m$. A is the point (m, n) on the Cartesian plane. B is the reflection of A in the line $y = x$. C is the reflection of B in the y-axis, D is the reflection of C in the x-axis and E is the reflection of D in the y-axis. The area of the pentagon $ABCDE$ is :

(A) $2m(m+n)$ **(B)** $m(m+3n)$ **(C)** $m(2m+3n)$ **(D)** $2m(m+3n)$

12. The ends of the base of an isosceles triangle are at (2, 0) and (0, 1) and the equation of one side is $x = 2$ then the orthocenter of the triangle is :

(A) $\left(\frac{3}{4},\frac{3}{2}\right)$ **(B)** $\left(\frac{5}{4},1\right)$ **(C)** $\left(\frac{3}{4},1\right)$ **(D)** $\left(\frac{4}{3},\frac{7}{12}\right)$

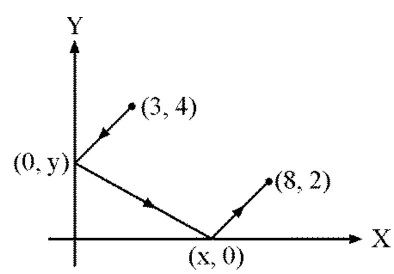
13. Given $A \equiv (1,1)$ and AB is any line through it cutting the x-axis in B. If AC is perpendicular to AB and meets the y-axis in C, then the equation of locus of mid-point P of BC is :

(A) $x+y=1$ **(B)** $x+y=2$ **(C)** $x+y=2xy$ **(D)** $2x+2y=1$

14. A piece of cheese is located at (12, 10) in a coordinate plane. A mouse is at (4, –2) and is running up the line $y=-5x+18$. At the point (a, b), the mouse starts getting farther from the cheese rather than closer to it. The value of $(a + b)$ is :

(A) 6 **(B)** 10 **(C)** 18 **(D)** 14

15. Suppose that a ray of light leaves the point (3, 4), reflects off the y-axis towards the x-axis, reflects off the x-axis, and finally arrives at the point (8, 2). The value of x is :



(A) $x = 4\frac{1}{2}$ **(B)** $x = 4\frac{1}{3}$

(C) $x = 4\frac{2}{3}$ **(D)** $x = 5\frac{1}{3}$

16. In a triangle ABC, if A (2, –1) and $7x-10y+1=0$ and $3x-2y+5=0$ are equations of an altitude and an angle bisector respectively drawn from B, then equation of BC is :

(A) $x+y+1=0$ **(B)** $5x+y+17=0$ **(C)** $4x+9y+30=0$ **(D)** $x-5y-7=0$

17. The distance between the two parallel lines is 1 unit. A point '*A*' is chosen to lie between the lines at a distance '*d*' from one of them. Triangle *ABC* is equilateral with *B* on one line and *C* on the other parallel line. The length of the side of the equilateral triangle is :

(A) $\frac{2}{3}\sqrt{d^2+d+1}$ **(B)** $2\sqrt{\frac{d^2-d+1}{3}}$ **(C)** $2\sqrt{d^2-d+1}$ **(D)** $\sqrt{d^2-d+1}$

Paragraph for Questions 18 - 20

Consider 3 non-collinear points $A(9, 3)$, $B(7, -1)$ and $C(1, -1)$. Let $P(a, b)$ be the centre and R is the radius of circle 'S' passing through points *A*, *B*, *C*. Also $H(\bar{x}, \bar{y})$ are the coordinates of the orthocenter of triangle *ABC* whose area be denoted by Δ .

18. If *D*, *E* and F are the middle points of *BC*, *CA* and *AB* respectively then the area of the triangle *DEF* is :

(A) 12 **(B)** 6 **(C)** 4 **(D)** 3

19. The value of $a + b + R$ equals :

(A) 3 **(B)** 12 **(C)** 13 **(D)** None of these

20. The ordered pair $(\bar{x}, \bar{y})$ is :

(A) (9, 6) **(B)** (–9, 6) **(C)** (9, –5) **(D)** (9, 5)

Paragraph for Questions 21 - 23

The equation of an altitude of an equilateral triangle is $\sqrt{3}x + y = 2\sqrt{3}$ and one of its vertices is $(3, \sqrt{3})$ then

21. The possible number of triangle is :

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

22. Which of the following can't be the vertex of the triangle :

(A) (0, 0) **(B)** $(0, 2\sqrt{3})$ **(C)** $(3, -\sqrt{3})$ **(D)** None of these

23. Which of the following can be possible orthocenter of the triangle :

(A) $(1, \sqrt{3})$ **(B)** $(0, \sqrt{3})$ **(C)** (0, 2) **(D)** None of these

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

24. *ABCD* is rectangle with $A(-1, 2)$, $B(3, 7)$ and $AB : BC = 4:3$, if *d* is the distance of origin from the intersection point of diagonals of rectangle, then possible values of [*d*] is/are (where [.] denote greatest integer function) :

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6

25. Two straight lines $u = 0$ and $v = 0$ passes through the origin and the angle between them is $\tan^{-1}\left(\frac{7}{9}\right)$. If the ratio of slopes of $v = 0$ and $u = 0$ is $\frac{9}{2}$, then their equations are :

(A) $y = 3x$ and $3y = 2x$ **(B)** $2y = 3x$ and $3y = x$

(C) $y + 3x = 0$ and $3y + 2x = 0$ **(D)** $2y + 3x = 0$ and $3y + x = 0$

26. Let $B(1,-3)$ and $D(0,4)$ represent two vertices of rhombus *ABCD* in (*x*, *y*) plane, then coordinates of vertex *A* is $\angle BAD = 60°$ can be equal to :

(A) $\left(\frac{1-7\sqrt{3}}{2},\frac{1-\sqrt{3}}{2}\right)$ **(B)** $\left(\frac{1+7\sqrt{3}}{2},\frac{1+\sqrt{3}}{2}\right)$

(C) $\left(\frac{-1+7\sqrt{3}}{2},\frac{-1+\sqrt{3}}{2}\right)$ **(D)** $\left(\frac{-1-7\sqrt{3}}{2},\frac{-1-\sqrt{3}}{2}\right)$

27. The equations of two equal sides *AB* and *AC* of an isosceles triangle *ABC* are $x + y = 5$ and $7x - y = 3$ respectively. If the area of the triangle *ABC* is 5 square unit, then the possible equations of the side *BC* is(are) :

(A) $x - 3y + 1 = 0$ **(B)** $3x + y + 2 = 0$ **(C)** $x - 3y - 21 = 0$ **(D)** $3x + y - 12 = 0$

28. A line '*L*' is drawn from (4, 3) to meet the lines $L_1 \equiv 3x + 4y + 5 = 0$ and $L_2 \equiv 3x + 4y + 15 = 0$ at points *A* and *B* respectively. From point '*A*', a line perpendicular to *L* is drawn meeting the line 'L_2' at A_1. Similarly, from point '*B*' a line perpendicular to *L*, is drawn meeting the line L_1 at B_1. Thus a parallelogram AA_1BB_1 is formed. If the area of the parallelogram AA_1BB_1 is least, the equation of the line *L* is/are :

(A) $x - 7y + 17 = 0$ **(B)** $x + 7y + 1\ 7 = 0$ **(C)** $3x + y - 31 = 0$ **(D)** $7x + 2y - 31\ =0$

29. If the angle bisector *AD* of the angle *A* of the triangle *ABC* divides the side *BC* into two segments *BD* = 4, *DC* = 2, then:

(A) $2 < b < 6$ **(B)** $4 < c < 12$ **(C)** $3 < b < 6$

(D) Maximum value of altitude through *A* is 4

30. If the bisectors of the interior angle *A* of Δ *ABC* divides *BC* into segments *BD* = 4, *DC* = 2. If the length of the altitude $AE > \sqrt{10}$ and if *AB* and *AC* are integer. Then the possible length of the side *AC* is(are) :

(A) 3 **(B)** 4 **(C)** 6 **(D)** 5

31. The lines L_1 and L_2 denoted by $3x^2 + 10xy + 8y^2 + 14x + 22y + 15 = 0$ intersect at the point *P* and have gradients m_1 and m_2 respectively. The acute angle between them is θ. Which of following relations hold good :

(A) $m_1 + m_2 = \frac{5}{4}$ **(B)** $m_1 m_2 = \frac{3}{8}$ **(C)** $\theta = \sin^{-1}\left(\frac{2}{5\sqrt{5}}\right)$

(D) Sum of the abscissa and ordinate of point *P* is –1.

32. Line $\frac{x}{a}+\frac{y}{b}=1$ cuts the coordinate axes at *A*(*a*, 0) and *B*(0, *b*) and the line $\frac{x}{a'}+\frac{y}{b'}=-1$ at $A'(-a',0)$ and $B'(0,-b')$. If the points *A*, *B*, A', B' are concyclic then the orthocenter of the triangle ABA' is :

(A) $(0,0)$ **(B)** $(0,b')$ **(C)** $\left(0,\frac{aa'}{b}\right)$ **(D)** $\left(0,\frac{bb'}{a}\right)$

33. The bisector of angle between the straight lines $y-b=\frac{2m}{1-m^2}(x-a)$ and $y-b=\frac{2m'}{1-m'^2}(x-a)$ are :

(A) $(y-b)(m+m')+(x-a)(1-mm')=0$ **(B)** $(y-b)(m+m')-(x-a)(1-mm')=0$

(C) $(y-b)(m-m')+(x-a)(1+mm')=0$ **(D)** $(y-b)(1-mm')-(x-a)(m+m')=0$

34. All the points lying inside the triangle formed by the points (1, 3), (5, 6), and (–1, 2) satisfy :

(A) $3x+2y\ge 0$ **(B)** $2x+y+1\ge 0$ **(C)** $-2x+11\ge 0$ **(D)** $2x+3y-12\ge 0$

35. Possible values of θ for which the point $(\cos\theta,\sin\theta)$ lies inside the triangle formed by lines $x+y=2; x-y=1$ and $6x+2y=\sqrt{10}$ are :

(A) $\pi/8$ **(B)** $\pi/4$ **(C)** $3\pi/8$ **(D)** $\pi/2$

36. In a $\Delta ABC, A \equiv (\alpha, \beta), B(1,2), C(2,3)$ and point A lies on line $y = 2x + 3$, where $\alpha, \beta \in I$. If the area of ΔABC be such that area of triangle lies in interval [2, 3), then the possible value of $\alpha + \beta$ is/are :

(A) -18 **(B)** -15 **(C)** 9 **(D)** 12

37. The point $(a^2, a+1)$ lies in the angle between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin then the possible integral values of a is/are :

(A) $a = 2$ **(B)** $a = -2$ **(C)** $a = 0$ **(D)** $a = -1$

38. The medians AD and BE of a triangle ABC with vertices $A(0, b)$, $B(0, 0)$ and $c(a, 0)$ are perpendicular to each other if :

(A) $a = \sqrt{2}\, b$ **(B)** $a = -\sqrt{2}\, b$ **(C)** $b = \sqrt{3}\, a$ **(D)** $b = -\sqrt{3}\, a$

39. Two sides of a triangle have the joint equation $(x - 3y + 2)(x + y - 2) = 0$, the third side which is variable always passes through the point (–5, –1), then possible values of slope of third side such that origin is an interior point of triangle is/are :

(A) $\frac{-4}{3}$ **(B)** $\frac{-2}{3}$ **(C)** $\frac{-1}{3}$ **(D)** $\frac{1}{6}$

40. Let x_1 and y_1 be the roots of $x^2 + 8x - 2009 = 0$; x_2 and y_2 be the roots of $3x^2 + 24x - 2010 = 0$ and x_3 and y_3 be the roots of $9x^2 + 72x - 2011 = 0$. The points $A(x_1, y_1) B(x_2, y_2)$ and $C(x_3, y_3)$:

(A) can not lie on a circle **(B)** form a triangle of area 2 sq. units

(C) form a right-angled triangle **(D)** are collinear

41. Consider a variable line 'L' which passes through the point of intersection 'P' of the lines $3x + 4y - 12 = 0$ and $x + 2y - 5 = 0$ meeting the coordinate axes at points A and B :

(A) then the locus of middle point of the segment AB has the equation $3x + 4y = 4xy$

(B) then the locus of the feet of the perpendicular from the origin on the variable line 'L' has the equation $2(x^2 + y^2) - 4x - 3y = 0$

(C) Locus of the centroid of the variable triangle OAB has the equation (where 'O' is the origin) $3x + 4y - 6xy = 0$

(D) Locus of the centroid of the variable triangle OAB has the equation (where 'O' is the origin) $3x + 4y + 6xy = 0$

42. A variable line 'L' is drawn through $O(0, 0)$ to meet the lines $L_1 : y - x - 10 = 0$ and $L_2 : y - x - 20 = 0$ at points A and B respectively. A point P is taken on line 'L' :

(A) If $\frac{2}{OP} = \frac{1}{OA} + \frac{1}{OB}$, then locus of P is $3y - 3x = 40$

(B) If $OP^2 = (OA)(OB)$, then locus of P is $(y - x)^2 = 200$

(C) If $\frac{1}{OP^2} = \frac{1}{(OA)^2} - \frac{1}{(OB)^2}$, then locus of P is $(y - x)^2 = 80$

(D) If $\frac{1}{OP^2} = \frac{1}{(OA)^2} + \frac{1}{(OB)^2}$ then locus of P is $(y - x)^2 = 80$

43. Given ΔABC whose vertices are $A(x_1, y_1); B(x_2, y_2); C(x_3, y_3)$. Let there exists a point $P(a,\ b)$ such that $6a = 2x_1 + x_2 + 3x_3; 6b = 2y_1 + y_2 + 3y_3$:

(A) $P(a,b)$ lies inside the ΔABC **(B)** Area of triangle PBC is less than area of ΔABC

(C) $P(a,b)$ lies outside the ΔABC **(D)** Area of triangle PBC is greater than area of ΔABC

44. If one vertex of an equilateral triangle of side 'a' is at $(1,0)$, also another one lies on the line $\sqrt{3}x - y - \sqrt{3} = 0$, then co-ordinates of the third vertex may be

(A) $\left(1-\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$ **(B)** $\left(\frac{a}{2}+1, \frac{-\sqrt{3}a}{2}\right)$ **(C)** $(a+1, 0)$ **(D)** $(a-1, 0)$

45. A line passing through the origin (O) has point A and B in the same direction such that $OA = AB = r$. Through points A and B two lines are drawn making equal angle $tan^{-1}\left(\sqrt{3}\right)$ with the line AB. Then points which lies on the locus of point of intersection of the lines is/are:

(A) $\left(r, \sqrt{2}r\right)$ **(B)** $\left(\sqrt{2}r, r\right)$ **(C)** (r, r) **(D)** $\left(-\sqrt{2}r, r\right)$

46. A line 'L' passes through the point $(2,3)$ and making intercept of length 3 units between the lines $2x + y - 2 = 0$ and $4x + 2y - 10 = 0$ then which of the following may be true about the line L:

(A) Parallel to x axis **(B)** Parallel to y-axis

(C) having slope $-\frac{3}{4}$ **(D)** Perpendicular to $2x + y - 2 = 0$

47. For all values of θ, the lines represented by the equation

$(2\cos\theta + 3\sin\theta)x + (3\cos\theta - 5\sin\theta)y - (5\cos\theta - 2\sin\theta) = 0$

(A) pass through a fixed point

(B) Pass through the point $(1,1)$

(C) pass through a fixed point whose reflection in the line $x + y = \sqrt{2}$ is $\left(\sqrt{2}-1, \sqrt{2}-1\right)$

(D) pass through the origin

48. The centroid of an equilateral triangle is $(0,0)$. If two vertices of the triangle lies on $x + y - 2 = 0$, then:

(A) Area of triangle is $6\sqrt{3}$ square units

(B) vertex not lying on the line is $(-2,-2)$

(C) foot of the perpendicular from $(0,0)$ to the line is $(1,1)$

(D) vertices on the given line are $\left(1+\sqrt{3}, 1-\sqrt{3}\right)$ and $\left(1-\sqrt{3}, 1+\sqrt{3}\right)$

49. In a ΔABC equation of median and altitude from vertex C and B are $x + 2 = 0$ and $x + y - 2 = 0$ respectively, vertex A is at the origin, then:

(A) co-ordinate of point B is $(-4,6)$ **(B)** equation of AC is $x - y = 0$

(C) Area of ΔABC is 10 square units **(D)** vertex C is $(-2,-3)$

50. A ray of light is sent along the line $x-2y=8$. After refracting across the line $x+y=1$ it enters the opposite side after turning by 15° away from the line $x+y=1$. Then the equation of line along which refracted ray travels will:

(A) have slope $\frac{5\sqrt{3}-6}{3}$ **(B)** have slope $\frac{5\sqrt{3}-6}{13}$

(C) pass through $\left(0,\frac{13\sqrt{3}-50}{3\sqrt{3}}\right)$ **(D)** pass through $\left(0,\frac{-50\sqrt{3}-31}{39}\right)$

51. The four lines given by $12x^2+7xy-12y^2=0$ and $12x^2+7xy-12y^2-x+7y-1=0$ will make:

(A) Rectangle **(B)** Square **(C)** Rhombus **(D)** Parallelogram

52. Equations of the sides of the triangle having $(3,-1)$ as a vertex, $x-4y+10=0$ *and* $6x+10y-59=0$ being the equations of an angle bisector and a median respectively drawn from different vertices, is/are:

(A) $6x-7y=25$ **(B)** $18x+13y=41$ **(C)** $2x+9y=65$ **(D)** $13x+4y=8$

53. The equation of a pair of straight lines is $ax^2+2hxy+by^2=0$. If the angle by which the axes be rotated so the term containing *xy* in the equation may be removed is ϕ, then:

(A) $\phi=\frac{\pi}{4}$ if $a=b, h>0$ **(B)** $\phi=\frac{\pi}{8}$, *if* $2h=a-b$

(C) $\phi=\frac{\pi}{8}$, *if* $a=b, h\neq 0$ **(D)** $\phi=\frac{\pi}{4}$, *if* $2h=a-b$

54. The equation of straight lines passing through ordered pairs (a,b) satisfying equation $sec^2((a+1)b)+a^2-1=0$, and having slope $\frac{1}{2}$, is (are):

(A) $x-2y=0$ **(B)** $x+2y=1$ **(C)** $x-2y=2\pi$ **(D)** $x-2y+2\pi=0$

55. $A(1,2)$ and $B(7,10)$ are two points. If $P(x,y)$ is a point such that the angle *APB* is 60° and the area of the ΔAPB is maximum, then which of given is (are) true?

(A) *P* lies on any line perpendicular of *AB* **(B)** *P* lies on the right bisector of *AB*

(C) *P* lies on the straight line $3x+4y=36$

(D) *P* lies on the circle passing through the points $(1,2)$ and $(7,10)$ and having a radius of 10 units

56. A line which makes an acute angle θ with the positive direction of x-axis is drawn through the point $P(3,4)$ to meet the line $x=6$ at *R* and $y=8$ at *S*, then:

(A) $PR=3\sec\theta$ **(B)** $PS=4\ \text{cosec}\theta$

(C) $PR+PS=\frac{2(3\sin\theta+4\cos\theta)}{\sin 2\theta}$ **(D)** $\frac{9}{(PR)^2}+\frac{16}{(PS)^2}=1$

57. The equation of the bisectors of the angles between the two intersecting lines $\frac{x-3}{\cos\theta}=\frac{y+5}{\sin\theta}$ and $\frac{x-3}{\cos\phi}=\frac{y+5}{\sin\phi}$ are $\frac{x-3}{\cos\alpha}=\frac{y+5}{\sin\alpha}$ and $\frac{x-3}{\beta}=\frac{y+5}{\gamma}$, then:

(A) $\alpha=\frac{\theta+\phi}{2}$ **(B)** $\beta=-\sin\alpha$ **(C)** $\gamma=\cos\alpha$ **(D)** $\beta=\sin\alpha$

58. Two roads are represented by the equation $y-x=6$ and $x+y=8$. An inspection bunglow has to be so constructed that it is at a distance of 100 from each of the roads. Possible location of the bunglow is given by:

(A) $\left(100\sqrt{2}+1,7\right)$ **(B)** $\left(1-100\sqrt{2},7\right)$ **(C)** $\left(1,7+100\sqrt{2}\right)$ **(D)** $\left(1,7-100\sqrt{2}\right)$

59. Let $L_1 : 3x+4y=1$ *and* $L_2 : 5x-12y+2=0$ be two given lines. Let image of every point on L_1 with respect to a line *L* lies on L_2 then possible equation of *L* can be:

(A) $14x+112y-23=0$ **(B)** $64x-8y-3=0$

(C) $11x-4y=0$ **(D)** $52y-45x=7$

60. Let $A(1,1)$ *and* $B(3,3)$ be two fixed points and *P* be a variable point such that area of ΔPAB remains constant equal to 1 for all position of *P*, then locus of *P* is given by:

(A) $2y=2x+1$ **(B)** $2y=2x-1$ **(C)** $y=x+1$ **(D)** $y=x-1$

61. The vertices of a triangle are $(1,3),(5,0)$ *and* $(-1,2)$. Which of the following inequalities will be satisfied by all points lying inside the triangle?

(A) $3x+2y\ge 0$ **(B)** $2x-3y-12\le 0$ **(C)** $x+y-6\le 0$ **(D)** $2x-y\ge 0$

62. $A(x_1,y_1),B(x_2,y_2),(y_1<y_2)$ are two points on the line $x+y=4$ from which perpendicular *AQ* and *BP* are drawn on line $4x+3y=10$ where *P* and *Q* are the feet of perpendicular such that $AQ=BP=1$. Now considering *AB* as diameter, a circle is drawn which meets the line $4x+3y=10$ at *C* and *D* such that *C* is closer to *P*.
Then which of the following statement(s) is correct?

(A) the value of $\dfrac{y_1+y_2}{x_1+x_2}$ is equal to –3 **(B)** the length PQ is equal to 14

(C) length QD is equal to $5\sqrt{2}-7$ **(D)** radius of circle obtained is $5\sqrt{2}$ units

63. If the two lines represented by $x^2\left(tan^2\theta+cos^2\theta\right)-2xy\,tan\,\theta+y^2\,sin^2\theta=0$ make angles α,β with the x-axis, then:

(A) $tan\,\alpha+tan\,\beta=4\,cosec\,2\theta$ **(B)** $tan\,\alpha\,tan\,\beta=sec^2\theta+tan^2\theta$

(C) $tan\,\alpha-tan\,\beta=2$ **(D)** $\dfrac{tan\,\alpha}{tan\,\beta}=\dfrac{2+sin\,2\theta}{2-sin\,2\theta}$

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column I are labelled as (A), (B), (C) & (D) whereas statements in Column II are labeled as p, q, r, s & t. More than one choice from Column II can be matched with Column I.

64. MATCH THE FOLLOWING :

	Column 1		**Column 2**
(A)	The number of integral values of 'a' for which point (a, a^2) lies completely inside the triangle formed by lines $x=0, y=0, 2y+x=3$	**(p)**	0
(B)	A point on the line $x+y=4$ which lies at a unit distance from the line $4x+3y=10$ has the coordinates (α,β) then possible value of $\alpha+\beta$	**(q)**	1
(C)	If (α,β) be the Orthocenter of triangle made by lines $x+y=1, x-y+3=0, 2x+y=7$ then the value of $\alpha+\beta$ is	**(r)**	2
(D)	In a triangle *ABC*, the bisector of angles *B* and *C* lie along the lines $y=x$ and $y=0$. If *A* is (1, 2) then $\sqrt{10}\, d(A, BC)$ equals (where $d(A, BC)$ denotes the perpendicular distance of *A* from *BC*.)	**(s)**	4

65. MATCH THE FOLLOWING :

	Column 1		**Column 2**
(A)	Two perpendicular straight lines are drawn from the origin to make an iscosceles triangle together with the line $2x+y=5$ Then the area of triangle is	**(p)**	$\sqrt{5}$
(B)	Let the line $2x+y=4$ meet *x*-axis at *A* and *y*-axis at *B*, and the perpendicular bisector of *AB* meets the horizontal line through (0, –1) at *C*. Let *G* be the centroid of the triangle *ABC*. Then perpendicular distance from *G* to *AB* equals	**(q)**	5
(C)	The number of integral points inside the triangle made by the line $3x+4y-12=0$ with the coordinate axes which are equidistant from at least two sides is/are (an intergral point is a point both of whose coordinates are integers)	**(r)**	3
(D)	The line $x=c$ cuts the triangle with corners (0, 0), (1, 1) and (9, 1) into two regions. For the area of the two regions to be the same c must be equal to :	**(s)**	1

66. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	A straight line with negative slope through (1, 4) meets the co-ordinate axes at *A* and *B*. The minimum length of $OA+OB$, *O* being the origin is :	**(p)**	$5\sqrt{2}$
(B)	If the point *P* is symmetric to the point $Q(4,\ -1)$ with respect to the bisector of the first quadrant, then the length of *PQ* is :	**(q)**	$3\sqrt{2}$
(C)	On the portion of the straight line $x+y=2$ between the axes a square is constructed away from the origin, with this portion as one of its side. If '*d*' denotes the perpendicular distances of a side of this square from the origin, then the maximum value of '*d*' is :	**(r)**	$\frac{9}{2}$
(D)	If the parametric equation of a line is given by $x=4+\frac{\lambda}{\sqrt{2}}$ and $y=-1+\sqrt{2}\ \lambda$, where λ is the parameter, then the intercept made by the line on the *x*-axis is :	**(s)**	9

67. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	The pair of lines $6x^2-\alpha xy-3y^2-24x+3y+\beta=0$ intersect on *x*-axis, then the value of $20\alpha-\beta$.	**(p)**	5
(B)	Let *P* be any point on the line $x-y+3=0$ and *A* be a fixed point (3, 4). If the family of lines given by the equation $(3\sec\theta+5\operatorname{cosec}\theta)x+(7\sec\theta-3\operatorname{cosec}\theta)y+11(\sec\theta-\operatorname{cosec}\theta)=0$ are concurrent at a point *B* for all permissible values of θ and maximum value of $\lvert PA-PB\rvert=2\sqrt{2n}\,(n\in N)$, then find the value of *n*.	**(q)**	8
(C)	A square with centre at (3, 7) and side length 4 units has one of its diagonal parallel to the line *y* = *x*. If the vertices of the square be $(x_1,y_1),(x_2,y_2),(x_3,y_3)$ and (x_4,y_4) then the value of max (y_1, y_2, y_3, y_4) $-\min(x_1,x_2,x_3,x_4)$.	**(r)**	1
(D)	The equation of a line through the mid point of the sides *AB* and *AD* of rhombus *ABCD*, whose one diagonal is $3x-4y+5=0$ and one vertex is *A*(3, 1) is $ax+by+c=0$. Then the absolute value of (*a* + *b*+ *c*) where *a*, *b*, *c* are integers expressed in lowest form:	**(s)**	6

68. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	If $P\left(1+\frac{t}{\sqrt{2}}, 2+\frac{t}{\sqrt{2}}\right)$ be any point on a line then value of t for which the point P lies between parallel lines $x+2y=1$ and $2x+4y=15$ is	**(p)**	(1, 2)
(B)	If the point $\left(2x_1-x_2+t\left(x_2-x_1\right), 2y_1-y_2+t\left(y_2-y_1\right)\right)$ divides the join of $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ internally, then t lies in	**(q)**	$\left(\frac{-\sqrt{13}-1}{2}, -1\right) \cup \left(1, \frac{\sqrt{13}-1}{2}\right)$
(C)	If the point (1, t) always remains in the interior of the triangle formed by the lines $y = x$, $y = 0$ and $x+y=4$, then t lies in	**(r)**	$\left(\frac{-4\sqrt{2}}{3}, \frac{5\sqrt{2}}{6}\right)$
(D)	Set of values of 't' for which the point $P\left(t, t^2-2\right)$ lies inside the triangle formed by lines $x+y=1$, $y=x+1$ and $y=-1$ is	**(s)**	(0, 1)

69. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	$P(3, 1)$, $Q(6, 5)$ and $R(x, y)$ are three points such that angle PRQ is right angle and the area of ΔPRQ is 7, then number of such points R is.	**(p)**	2
(B)	Let $ABCD$ is a square with sides of unit length. Points E and F are taken on sides AB and AD respectively so that $AE = AF$. The maximum possible area of quadrilateral $CDFE$ is	**(q)**	$\frac{21}{4}$
(C)	Let $A \equiv (0,0), B \equiv (5,0), C \equiv (5,3)$ and $D \equiv (0,3)$ are the vertices of rectangle $ABCD$. If P is a variable point lying inside the rectangle $ABCD$ and $d(P, L)$ denote perpendicular distance of point P from line L. If $d(P, AB) \le \min\{d(P, BC), d(P, AD), d(P, CD)\}$, then area of the region in which P lies is:	**(r)**	0
(D)	The slope of one of lines given by $ax^2+2hxy+by^2=0$ be the square of the slope of the other, if $ab(a+b)+\alpha abh+\beta h^3=0$, then $\alpha+\beta$ is equals:	**(s)**	$\frac{5}{8}$

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

70. Let ABC be a triangle. Let A be the point (1, 2), $y = x$ is the perpendicular bisector of AB and $x-2y+1=0$ is the angle bisector of angle C. If the equation of BC is given by $ax+by-5=0$ then the value of $a+b$ is _________.

71. In a ΔABC, the equations of right bisectors of sides AB and CA are $3x+4y=20$ and $8x+6y=65$ respectively. If the vertex A be (10, 10), then the value of $\frac{1}{7}(ar\,\Delta\,ABC)$ is _________.

72. In a ΔABC, the vertex A is (1, 1) and orthocenter is (2, 4). If the sides AB and BC are members of the family of straight lines $ax+by+c=0$. Where a, b, c are in $A.P.$ then the coordinates of vertex C are (h, k). The value of $2h+9k$ is _________.

73. The slopes of three sides of a triangle ABC are $-1,-2,3$ respectively. If the orthocenter of triangle ABC is origin, then the locus of its centroid is $y=\frac{a}{b}x$ where a, b are relatively prime than $b-a$ is equal to _________.

74. The lines $x+y=0$, $x-4y=0$ and $2x-y=0$ are the altitudes of a triangle. If one of the vertices has coordinates of the form $(\lambda,\ -\lambda)$, if the locus of the centroid of such a triangle is $ax+by=0$, then the value of $a+b$ is ______.

75. Two equal sides OA and OB of an isosceles triangle lie in the first quadrant. If the slopes of OA and OB are $\frac{7}{17}$ and 1, respectively and the length of perpendicular from O to AB *is* $\sqrt{13}$, if the equation of the side AB is $ax+by=c$ then the value of $(c-a-b)$ is ___________.

76. A line intersects the x-axis at $A(7, 0)$ and y-axis $B(0, -5)$. A variable line PQ perpendicular to AB intersects the x-axis at P and the y-axis at Q. If AQ and BP intersect at R, show that the locus of R is $x^2+y^2-ax+by=0$. Then the value of $(a-b)$ is :

77. In triangle ABC if the median to side BC has length $\left(11-6\sqrt{3}\right)^{\frac{-1}{2}}$ and it divides angle $\angle A$ into angles 30º and 45º. Then length of side BC is ___________.

78. In a triangle ABC if $AC=3$, $BC=4$ and median AD and BE are perpendicular to each other, Δ be the area of the triangle ABC. Then the value of $[\Delta]$ is ______. (where [.] denotes the greatest integer) :

79. In a triangle ABC, the coordinates of A is (1, 2) and the equations to the medians through B and C are $x+y=5$ and $x=4$. If coordinate of B is (x_1, y_1) and co-ordinate of C is (x_2, y_2) then $x_1y_2+x_2y_1$ equals____.

80. A straight line through the point $A(-2,-3)$ cuts the lines $x+3y=9$ *and* $x+y+1=0$ at B and C respectively. If $AB.AC=20$, then product of slopes of line is____.

81. A right angled triangle $ABC\left(\angle C=\frac{\pi}{2}\right)$ is constructed so that its sides are parallel to coordinate axes and the medians through A and B lie on the lines $y=3x+1$ and $y=mx+2$ respectively. Then product of values of m for which such a triangle is possible is____.

82. If the distance of any point $P(x,y)$ from the origin is defined as $d(x,y)=\max\{|x|,|y|\}, d(x,y)=2$ then the area of curve represented by the locus of point P is S then $[S]$ is____.

83. Two equal sides AB and AC of an acute angle triangle ABC are formed by the equations $7x-y+3=0$ and $x+y-3=0$, side BC is $ax+by-31=0$ where $a-10b=31$. Value of $2a+10b+31=0$ is____.

84. A variable line 'L' of the form $y=mx$ is drawn to meet the lines $L_1: 2x+3y-5=0;\ L_2: x+2y-5=0$ and $L_3: 6x+4y-5=0$ at points A, B and C. A point $P(a,b)$ is taken on the line 'L' also $\frac{k(a+b)}{OP}=\frac{1}{OA}+\frac{1}{OB}+\frac{1}{OC}$ then value of k is______.

85. ABC is a triangle, whose vertex A is $(3,4)$ $L_1=0, L_2=0$ are the angle bisectors of angle B and C respectively where $L_1=x+2y-5=0$, $L_2=x-2y-3=0$ also $AB=KAI$ where I is the incentre then K is

86. If the equal sides PQ and PR (each equal to 2) of a right angled isosceles ΔPQR be produced to A and B so that $QA.RB=PR^2$ then the line AB passes through a fixed point which also satisfies the line $ax+by-6=0$ then $a+b$ is______.

87. If from point P (4, 4) perpendiculars to the straight lines $3x+4y+5=0$ and $y=mx+7$ meet at Q and R respectively and area of triangle PQR is maximum. Then the value of $12m$ must be____.

88. Let $(3,4)$ be a fixed point. A straight line passing through this point cuts the positive direction of the coordinate axes at the points P and Q. If λ sq unit be the minimum area of the triangle OPQ, O being the origin. Then the value of λ must be____.

89. If the slope of one of the line represented by $ax^2+2hxy+by^2=0$ is the square of the other, then the value of $\frac{a+b}{h}+\frac{8h^2}{ab}$ is______.

90. In a triangle ABC, the bisector of angles B and C lies along the lines $y=x$ and $y=0$. If A is $(1,2)$ then $\sqrt{10}d(A,BC)$ equal (where $d(A,BC)$ denotes the perpendicular distance of A from BC).

91. Let P be any point on the line $x-y+3=0$ and A be a fixed point $(3,4)$. If the family of lines given by the equation $(3\sec\theta+5\operatorname{cosec}\theta)x+(7\sec\theta-3\operatorname{cosec}\theta)y+11(\sec\theta-\operatorname{cosec}\theta)=0$ are concurrent at a point B for all permissible values of θ and maximum value of $|PA-PB|=2\sqrt{2n}\ (n\in N)$, then find the value of n.

92. The slopes of three sides of a triangle ABC are $-1,-2,3$ respectively. If the orthocenter of triangle ABC is origin, then the locus of its centroid is $y=\frac{a}{b}x$ where a, b are relatively prime than $b-a$ is equal to______.

93. A straight line through the origin O meets the parallel lines $3x-4y=6$ and $6x-8y+c=0$ at points Q and P respectively such that $\left|\frac{OP}{OQ}\right|=\frac{4}{3}$. If $c=2^k$, then k is equal to______.

94. The vertices B and C of a triangle ABC lie on the lines $3y=4x$ *and* $y=0$ respectively and the side BC passes through the point $\left(\frac{2}{3},\frac{2}{3}\right)$. If $ABOC$ is a rhombus, O being the origin and the coordinates of A are (h,k), then $\frac{h}{k}$ is equal to______.

95. If A line is passing through the point $P(1,2)$ cutting the lines $x+y-5=0$ and $2x-y=7$ at A and B respectively such that the harmonic mean of PA and PB is 10. If equation of line is $(y-2)=\tan\left[\pi-\sin^{-1}\left[\frac{a}{\sqrt{146}}\right]-\sin^{-1}\left[\frac{b}{\sqrt{146}}\right]\right](x-1)$. The $|a-b|\times\frac{25}{41}$ is ______.

96. If the straight line through the point $P(3,4)$ makes an angle $\frac{\pi}{6}$ with the x-axis and meets the line $12x+5y+10=0$ at Q then the value of $\frac{\left(12\sqrt{3}+5\right)}{11}$ PQ is______.

97. A line through $A(-5,-4)$ meets the lines $x+3y+2=0$, $2x+y+4=0$ and $x-y-5=0$ at B, C and D respectively. If $\left(\frac{15}{AB}\right)^2+\left(\frac{10}{AC}\right)^2=\left(\frac{6}{AD}\right)^2$ equation of line is $2x+by+c=0$ then value of $2+b+c$ is______.

98. If $6a^2-3b^2-c^2+7ab-ac+4bc=0$, then the family of lines $ax+by+c=0$ is concurrent at ordered pairs (A,B) and (C,D), $A>0$ then the value of $A-B-C-D$ is______.

JEE Advanced Revision Booklet	Circle

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. Two Circles of radii 36 units and 9 units touch each other externally, a third circle of radius r touches the two given circles externally and also their common tangent, then the value of r is :

(A) 4 **(B)** 5 **(C)** $\sqrt{17}$ **(D)** $\sqrt{18}$

2. If $A(0,\alpha)$ and $B(0,\beta)$, α, $\beta > 0$ are two vertices of a variable triangle ABC, where the vertex C $(x, 0)$ is variable. The value of x for which $\angle ACB$ is maximum is :

(A) $\frac{\alpha+\beta}{2}$ **(B)** $\sqrt{\alpha\beta}$ **(C)** $\frac{2\alpha\beta}{\alpha+\beta}$ **(D)** $\frac{\alpha\beta}{\alpha+\beta}$

3. Two tangents are drawn from a point P to the circle $x^2+y^2=1$. If the tangents make an intercept of 2 units on the line $x=1$, then locus of P is :

(A) parabola **(B)** pair of lines **(C)** circle **(D)** straight line

4. Let $ABCD$ be a quadrilateral in which $AB \parallel CD$, $AB \perp AD$ and $AB = 3CD$. If the area of the quadrilateral $ABCD$ is 4, then the radius of the circle touching all the four sides of the quadrilateral is

(A) $\sin\frac{\pi}{12}$ **(B)** $\sin\frac{\pi}{3}$ **(C)** $\sin\frac{\pi}{4}$ **(D)** $\sin\frac{\pi}{6}$

5. Three concentric circles of which the biggest is $x^2+y^2=1$, have their radii in *A.P.* If the line $y=x+1$ cuts all the circles in real and distinct points. The interval in which the common difference of the *A.P.* will lie is:

(A) $\left(0,\frac{1}{4}\right)$ **(B)** $\left(0,\frac{2-\sqrt{2}}{4}\right)$ **(C)** $\left(0,\frac{1}{2\sqrt{2}}\right)$ **(D)** None of these

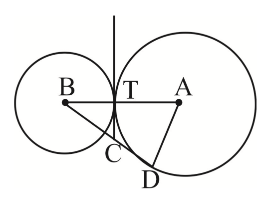
6. One circle has a radius of 5 units and its center is at (0, 5). A second circle has a radius of 12 and its centre is at (12, 0). The radius of a third circle which passes through the center of the second circle and both points of intersection of the first two circles, is equal to:

(A) 13/2 **(B)** 15/2 **(C)** 17/2 **(D)** None of these

7. A circle is inscribed in an equilateral triangle with side lengths 6 units. Another circle is drawn inside the triangle (but outside the first circle), tangent to the first circle and two of the sides of the triangle. The radius of the smaller circle is:

(A) $1/\sqrt{3}$ **(B)** $2/3$ **(C)** $1/2$ **(D)** 1

8. Two circles with centre at A and B, touch at T. BD is the tangent at D and TC is a common tangent. AT has length 3 and BT has length 2. The length of CD is:



(A) 4/3 **(B)** 3/2
(C) 5/3 **(D)** 7/4

9. In a triangle ABC, right angled at A, on the leg AC as diameter, a semicircle is described. The chord joining A with the point of intersection D of the hypotenuse and the semicircle, then the length AC equals to:

(A) $\frac{AB\cdot AD}{\sqrt{AB^2+AD^2}}$ **(B)** $\frac{AB\cdot AD}{AB+AD}$ **(C)** $\sqrt{AB\cdot AD}$ **(D)** $\frac{AB\cdot AD}{\sqrt{AB^2-AD^2}}$

10. In a circle with centre 'O' PA and PB are two chords. PC is the chord that bisects the angle APB. The tangent to the circle at C is drawn meeting PA and PB extended at Q and R respectively. If $QC = 3$, $QA = 2$ and $RC = 4$, then length of RB equals:

(A) 2 **(B)** 8/3 **(C)** 10/3 **(D)** 11/3

11. A circle is inscribed in a rhombus $ABCD$ with one angle 60°. The distance from the centre of the circle to the nearest vertex is equal to 1. If P is any point on the circle, then $|PA|^2+|PB|^2+|PC|^2+|PD|^2$ is equal to :

(A) 12 **(B)** 11 **(C)** 9 **(D)** None of these

12. Considering the circles, $x^2+y^2=25$ and $x^2+y^2=9$. From the point $A(0, 5)$ two segments are drawn touching the inner circle at the points B and C while intersecting the outer circle at the points D and E. If 'O' is the centre of both the circles then the length of the segment OF that is perpendicular to DE, is:

(A) 7/5 **(B)** 7/2 **(C)** 5/2 **(D)** 3

13. Points P and Q are 3 units apart. A circle centered at P with a radius of 3 units intersects a circle centered at Q with radius $\sqrt{3}$ units at point A and B. The area of the quadrilateral $APBQ$ is:

(A) $\sqrt{99}$ **(B)** $\frac{\sqrt{99}}{2}$ **(C)** $\sqrt{\frac{99}{2}}$ **(D)** $\sqrt{\frac{99}{16}}$

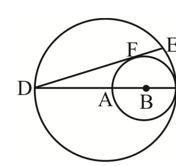
14. Circle $x^2+y^2+16x+12y+c=0$ is touched by a straight line with slope 2 and y-intercept 5 units at a point Q. Then the coordinates of Q are

(A) (–6, –7) **(B)** (–9, –13) **(C)** (–10, –15) **(D)** (–6, 11)

15. Tangents are drawn at the point of intersections of the circles $x^2+y^2=1$ and $x^2+y^2-(\lambda+6)x+(8-2\lambda)y-3=0$. ($\lambda$ being the variable). Then the locus of the point of intersection of these tangents is :

(A) $2x - y + 10 = 0$ **(B)** $x+2y-10=0$ **(C)** $x-2y+10=0$ **(D)** $2x+y-10=0$

16. In the diagram, DC is a diameter of the large circle centered at A, and AC is a diameter of the smaller circle centered at B. If DE is tangent to the smaller circle at F and $DC = 12$ units then the length of DE is:



(A) 13 **(B)** 16 **(C)** $8\sqrt{2}$ **(D)** $10\sqrt{2}$

17. Let C be a circle $x^2+y^2=1$. The line $y=mx+m$ intersects C at the point P other than $(-1,0)$, the number of rational choices for m for which both the coordinates of P are rational, is:

(A) 3 **(B)** 4 **(C)** 5 **(D)** infinitely many

18. The locus of the mid points of the chords of the circle $x^2+y^2-ax-by=0$ which subtend a right angle at $(a/2, b/2)$ is:

(A) $ax+by=0$ **(B)** $ax+by=a^2+b^2$

(C) $x^2+y^2-ax-by+\frac{a^2+b^2}{8}=0$ **(D)** $x^2+y^2-ax-by-\frac{a^2+b^2}{8}=0$

19. If the two circles $C_1: x^2+y^2=16$ and circle C_2 of radius 5 units intersect in such a manner that the common chord of maximum length has a slope equal to 3/4, then the coordinates of the centre of C_2 are:

(A) $\left(\pm\frac{9}{5},\pm\frac{12}{5}\right)$ **(B)** $\left(\pm\frac{9}{5},\mp\frac{12}{5}\right)$ **(C)** $\left(\pm\frac{12}{5},\pm\frac{9}{5}\right)$ **(D)** $\left(\pm\frac{12}{5},\mp\frac{9}{5}\right)$

Paragraph for Questions 20 - 22

Let S_1 and S_2 be two fixed circles touching each other externally with radius 2 and 3 respectively. Let S_3 be a variable circle touching internally both S_1 and S_2 at points A and B respectively. The tangents to S_3 at A and B meet at T, and $TA = 4$.

20. The radius of circle S_3 is :

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8

21. The area of circle circumscribing $x = \frac{-3h}{2}$ is:

(A) 10π **(B)** 20π **(C)** 40π **(D)** 80π

22. Let C_1, C_2, C_3 be centres of circles S_1, S_2, S_3 respectively, then which of the following must be true:

(A) $C_3C_1 + C_3C_2 = 5$ **(B)** $C_3C_1 - C_3C_2 = 3$ **(C)** $C_3C_1 + C_3C_2 = 3$ **(D)** $C_3C_1 - C_3C_2 = 1$

Paragraph for Questions 23 - 25

Let $f(x,y) = 0$ be the equation of a circle such that $f(0,y) = 0$ has equal real roots and $f(x,0) = 0$ has two distinct real roots. Let $g(x,y) = 0$ be the locus of points 'p' from where tangents to circle $f(x,y) = 0$ make angle $\frac{\pi}{3}$ between them and $g(x,y) = x^2 + y^2 - 5x - 4y + c, c \in R$

23. Let Q be a point from where tangents drawn to circle $g(x,y) = 0$ are mutually perpendicular. If A, B are the points of contact of tangent drawn from Q to circle $g(x,y) = 0$, then area of triangle QAB is:

(A) 25/12 **(B)** 25/8 **(C)** 25/4 **(D)** 25/2

24. The area of region bounded by circle $f(x,y) = 0$ with x-axis in the first quadrant is:

(A) $3 + \frac{25}{8}\left(\pi - \tan^{-1}\frac{1}{2}\right)$ **(B)** $3 + \frac{25}{8}\left(\tan^{-1}\frac{24}{11}\right)$

(C) $3 + \frac{25}{8}\left(2\pi - \tan^{-1}\frac{3}{4}\right)$ **(D)** $3 + \frac{25}{8}\left(2\pi - \tan^{-1}\frac{24}{7}\right)$

25. The number of points with positive integral coordinates satisfying $f(x,y) > 0, g(x,y) < 0; y > 3$ and $x < 6$ is:

(A) 7 **(B)** 8 **(C)** 10 **(D)** 11

Paragraph for Questions 26 - 28

If a circle C_0, with radius 1 unit touches both the axes and as well as line (L_1) through $P(0,4)$, L_1 cut the x-axis at $(x_1, 0)$. Again a circle C_1 is drawn touching x-axis, line L_1 and another line L_2 through point P. L_2 intersects x-axis at $(x_2, 0)$ and this process is repeated n times.

26. The value of x_2 is

(A) 3 **(B)** 4 **(C)** 15/2 **(D)** 17/2

27. The centre of circle C_2 is

(A) (3, 1) **(B)** $\left(\frac{15}{2}, 1\right)$ **(C)** $\left(\frac{31}{4}, 1\right)$ **(D)** none of these

28. The $\lim_{n\to\infty} \frac{x_n}{2^n}$ is

(A) 4 **(B)** 0 **(C)** 1 **(D)** 2

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

29. ABC is a right angled triangle right angled at A, with side $AC = 1$ and $AB = a$, a circle having AC as diameter cuts the side CB at D if $CD = b$ then :

(A) $ab > 1$ **(B)** $ab < 1$ **(C)** $\frac{b}{a} > \frac{1}{a^2 + \frac{1}{2}}$ **(D)** $\frac{b}{a} < \frac{1}{a^2 + \frac{1}{2}}$

30. The equation of the circle which touches the axes of coordinates and the line $\frac{x}{3} + \frac{y}{4} = 1$ and whose centre lies in the first quadrant is $x^2 + y^2 + 2rx + 2ry + r^2 = 0$, then r can be equal to:

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6

31. The equation of the largest circle passing through the points (1, 1) and (2, 2) and lying completely in the first quadrant is :

(A) $x^2 + y^2 - 4x - 2y + 4 = 0$ **(B)** $x^2 + y^2 - 2x - 4y + 4 = 0$

(C) $x^2 + y^2 - 3x - 3y + 4 = 0$ **(D)** $x^2 + y^2 + 3x + 3y - 4 = 0$

32. If $4l^2 - 5m^2 + 6l + 1 = 0$ and the line $lx + my + 1 = 0$ touches a fixed circle, then:

(A) centre of circle is at (3, 0) **(B)** the radius of circle is $\sqrt{5}$

(C) The radius of circle is $\sqrt{3}$ **(D)** the circle passes through (1, 1)

33. The equation of a circle in which the chord joining the points (1, 2) and (2, –1) subtends an angle of $\frac{\pi}{4}$ at any point on the circumference is

(A) $x^2 + y^2 - 5 = 0$ **(B)** $x^2 + y^2 - 6x - 2y + 5 = 0$

(C) $x^2 + y^2 + 6x + 2y - 15 = 0$ **(D)** $x^2 + y^2 + 7x - 2y + 14 = 0$

34. Equations of four circles are $(x \pm a)^2 + (y \pm a)^2 = a^2$, then:

(A) The radius of the greatest circle touching all the four circles is $(\sqrt{2} + 1)a$

(B) The radius of the smallest circle touching all the four circles is $(\sqrt{2} - 1)a$

(C) Area of region enclosed by four given circles with co-ordinate axes is $(4 - \pi)a^2$ sq.units

(D) The centres of four circles are the vertices of a square

35. If the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ is inscribed in a triangle whose two sides are co-ordinate axes and one side has negative slope cutting intercepts a and b on positive x and positive y axis, then :

(A) $1 - \frac{1}{a} - \frac{1}{b} = \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$ **(B)** $\frac{1}{a} + \frac{1}{b} < 1$

(C) $\frac{1}{a} + \frac{1}{b} > 1$ **(D)** $\frac{1}{a} + \frac{1}{b} + 1 = \sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$

36. The locus of points of intersection of the tangents to $x^2+y^2=a^2$ at the extremities of a chord of circle $x^2+y^2=a^2$ which touches the circle $x^2+y^2-2ax=0$ is(are):

(A) $y^2=a(a-2x)$ **(B)** $x^2=a(a-2y)$ **(C)** $x^2+y^2=(x-a)^2$ **(D)** $x^2+y^2=(y-a)^2$

36. A circle touches the line $x+y-2=0$ $at(1,1)$ and cuts the circle $x^2+y^2+4x+5y-6=0$ at P and Q. Then :

(A) PQ can never be parallel to the given line $x+y-2=0$

(B) PQ can never be perpendicular to the given line $x+y-2=0$

(C) PQ always passes through (6, – 4) **(D)** PQ always passes through (– 6, 4)

38. Let x, y be real variable satisfy the $x^2+y^2+8x-10y-40=0$. Let $a=\max\left(\sqrt{(x+2)^2+(y-3)^2}\right)$ and $b=\min\left(\sqrt{(x+2)^2+(y-3)^2}\right)$, then:

(A) $a+b=18$ **(B)** $a+b=4\sqrt{2}$ **(C)** $a-b=4\sqrt{2}$ **(D)** $ab=73$

39. A square is inscribed in the circle $x^2+y^2-2x+4y-93=0$ with the sides parallel to the coordinate axes. The coordinates of the vertices are :

(A) (8, 5) **(B)** (8, 9) **(C)** (– 6, 5) **(D)** (– 6, – 9)

40. Let A, B, C, D lie on a line such that $AB = BC = CD = 1$. The points A and C are also joined by a semicircle with AC as diameter and P is a variable point on this semicircle such that $\angle PBD=\theta, 0\le\theta\le\pi$. Let R is the region bounded by arc AP, the straight line PD and line AD

(A) The maximum possible area of region R is $\frac{2\pi+3\sqrt{3}}{6}$

(B) If 'L' is the perimeter of region 'R', then L is equal to $3+\pi-\theta+\sqrt{5-4\cos\theta}$

(C) The maximum possible area of region R is $\frac{2\pi-3\sqrt{3}}{6}$

(D) If 'L' is the perimeter of region 'R', then L is equal to $3+\pi-\theta+\sqrt{5+4\cos\theta}$

41. Consider two circles S_1 and S_2 (externally touching) having centres at points A and B whose radii are 1 and 2 respectively. A tangent to circle S_1 from point B touches the circle S_1 at point C. D is chosen on circle S_2 so that AC is parallel to BD and two segments BC and AD do not intersect. Segment AD intersect the circle S_1 at E. The line through B and E intersects the circle S_1 at another point F.

(A) The length of segment EF is $\frac{2\sqrt{3}}{3}$ **(B)** The area of triangle ABD is $2\sqrt{2}$

(C) The length of the segment DE is 2 **(D)** ABD is a triangle of perimeter $2\sqrt{3}$

42. The circle 'S' touches the sides AB and AD of the rectangle $ABCD$ and cuts the side DC at single point F and the side BC at a single point E. If $|AB|=32, |AD|=40$ and $|BE|=1$

(A) The angle between pair of tangents drawn form the point D to the circle 'S' is $\pi-\tan^{-1}\left(\frac{15}{8}\right)$

(B) The Area of trapezium AFCB is 1180 sq.units

(C) The radius of circle is 25 units

(D) The angle between pair of tangents drawn form the point D to the circle 'S' is $\pi-2\tan^{-1}\left(\frac{15}{8}\right)$

43. In the triangle ABC, the angle bisector AK is perpendicular to the median BM and $\angle ABC = 120°$, then :

(A) The value of ratio $\frac{BC}{AB}$ is equal to $\frac{\sqrt{13}-1}{2}$

(B) The value of ratio of radius of the circle circumscribing the triangle ABC to the side length AB is equal to $\frac{2}{\sqrt{3}}$

(C) The ratio of the area of ΔABC to the area of the circle circumscribing ΔABC is equal to $\frac{3\sqrt{3}}{32\pi}\left(\sqrt{13}-1\right)$

(D) The value of ratio of the sides AB to AC is equal to 1/2

44. There are two circles in a parallelogram. One of them of radius 3units is inscribed in the parallelogram, and the other touches two sides of the parallelogram and the first circle. The distance between the points of tangency which lie on the same side of the parallelogram is equal to 3 units.

(A) The radius of the other circle is ¾ units

(B) Area of the parallelogram is equal to 75/2 units

(C) Let d_1, d_2 denote the lengths of the diagonals of parallelogram, then the product $d_1.d_2$ is equal to 75

(D) Let d_1, d_2 denote the lengths of the diagonals of parallelogram, then the product $d_1.d_2$ is equal to 95

45. Point M moved on the circle $(x-4)^2 + (y-8)^2 = 20$. Then it broke away from it and move along a tangent to the circle, cuts the x-axis at the point $(-2, 0)$. The coordinates of a point on the circle at which the moving point broke away is:

(A) $\left(-\frac{3}{5}, \frac{46}{5}\right)$ **(B)** $\left(-\frac{2}{5}, \frac{44}{5}\right)$ **(C)** (6, 4) **(D)** (3, 5)

46. Two chords are drawn from the point $P(h, k)$ on the circle $x^2 + y^2 = hx + ky$. If the y-axis divides both the chords in the ratio 2:3, then which of the following may be correct?

(A) $k^2 > 15h^2$ **(B)** $15k^2 > h^2$ **(C)** $h^2 = 15k^2$ **(D)** $k^2 > 5h^2$

47. The equation(s) of the tangent at the point (0, 0) to the circle, making intercepts of lengths $2a$ and $2b$ units on the coordinates axes, is(are):

(A) $ax + by = 0$ **(B)** $ax - by = 0$ **(C)** $x - y = 0$ **(D)** $bx + ay = 0$

48. If the area of the quadrilateral formed by the tangents from the origin to the circle $x^2 + y^2 + 6x - 10y + c = 0$ and the radii corresponding to the points of contact is 15, then a value of c is :

(A) 9 **(B)** 4 **(C)** 5 **(D)** 25

49. Let C be a circle with centre 'O' and HK is the chord of contact of tangents drawn from a point A. OA intersects the circle 'C' at P and Q and B is the midpoint of HK, then :

(A) AB is the harmonic mean of AP and AQ **(B)** OA is the arithmetic mean of AP and AQ

(C) $(AK)^2 = (OA)(AB)$ **(D)** AB is the geometric mean of AP and AQ

50. Chords of the circle $x^2 + y^2 = 9$ are drawn such that segments intercepted from the chords by the curve $y^2 - 4x - 4y = 0$ subtend right angle at the origin. If the locus of the middle points of the chords with respect to circle is a curve S, then:

(A) S is a pair of line
(B) S is a circle
(C) S passes through the origin
(D) S meets the given circle $x^2 + y^2 = 9$ at A and B and the tangents at A and B to the circle $x^2 + y^2 = 9$ intersect at (4, 4)

51. A and B are two points in xy plane, which are $2\sqrt{2}$ unit distance apart and subtend an angle of 90° at $C(1, 2)$ on the line $x - y + 1 = 0$ which is larger than any angle subtend by the line segment AB at any other point on the line. The equation of the circle through the points A, B and C is:

(A) $x^2 + y^2 - 6x + 7 = 0$ **(B)** $x^2 + y^2 - 4x + 2y + 3 = 0$
(C) $x^2 + y^2 - 6y + 7 = 0$ **(D)** $x^2 + y^2 - 4x - 2y + 3 = 0$

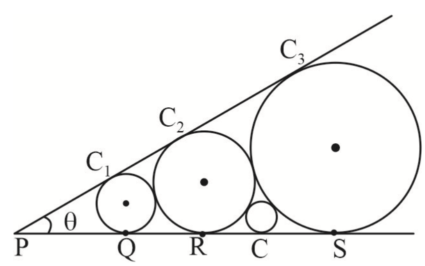
52. A variable circle passes through the origin O and cuts off portions OP and OQ from X-axis and Y-axis respectively such that $m(OP) + n(OQ)$ is equal to unity. If the circle passes through a fixed point (x_1, y_1) other than O, then:

(A) $x_1^2 + y_1^2 = m^2 + n^2$ **(B)** $x_1 + y_1 = m + n$ **(C)** $mx_1 + ny_1 = 1$ **(D)** $nx_1 - my_1 = 0$

53. If two points $A(-2, \alpha)$ and $B(4, \beta)$ are such that the triangle AOB is the right-angle triangle right angle at O. If $S = 0$ be the equation of the locus of the foot of the perpendicular P drawn to AB from the point O :

(A) (1, 3) lies on $S = 0$ **(B)** Minimum possible area of triangle AOB is 9
(C) Locus is a parabola **(D)** Locus is a circle

54. If the C_1, C_2, C_3 and C are four circles of radius r_1, r_2, r_3, r respectively as shown in figure:



(A) radius of the circle C is $2\sqrt{r_2 r_3}$
(B) $\tan\frac{\theta}{2} = \frac{r_2 - r_1}{2\sqrt{r_1 r_2}}$
(C) $PQ = \frac{2r_1^{3/2} r_2^{1/2}}{r_2 - r_1}$
(D) If $r_1 = 2$ and $r_1 + r_2 + r_3 = 14$ then $\frac{r_3}{r_2} = 2$

55. Let A, B, C and D be four distinct point on a line in that order. The circles with diameter AC is $x^2 + y^2 + ax + c = 0$ and BD is $x^2 + y^2 - by = 0$ intersect at X and Y the line XY meets BC at Z. Let P be a point on XY other than Z, the line CP intersects the circle with diameter AC at C and M, and line BP intersects the circle with diameter BD at B and N and the equation of line AM and DN are $bx + cy + a = 0$ and $cx + ay + b = 0$ respectively, then which of the following is true (where ω is a cube root of unity)

(A) $a + b + c = 1$ **(B)** $a + b\omega + c\omega^2 = 0$ **(C)** $a + b\omega^2 + c\omega = 0$ **(D)** $a + b + c = 0$

56. An isosceles right angled triangle ABC is such that $\angle B = 90°, AC = \sqrt{2}$ and A and C moves on positive coordinate axis, then

(A) locus of the point B is $y - x = 0$

(B) locus of the circum centre of ΔABC $x^2 + y^2 = \frac{1}{2}$

(C) centre of the circle circumscribing ΔABC will lie on the line $y - 3x = 0$

(D) centre of the circle circumscribing ΔOAC will lie on the $y - 4x = 0$

57. If the circle passing through the distinct points $(a,t),(t,a)$ and (t,t) for the all values of 't' passes through a fixed point then

(A) possible values of a is 4 **(B)** possible values of a is 8

(C) possible values of a is 12 **(D)** possible values of a is 15

58. A line L_1 intersect x and y axes at P and Q respectively. Another line L_2 , perpendicular to L_1 , cuts x and y axes at T and S respectively. The locus of the point of intersection of the lines PS and QT is a circle passing through the

(A) origin **(B)** point P **(C)** point Q **(D)** point T

59. The equation of a circle of radius 1 touching the circles $x^2 + y^2 - 2|x| = 0$ is :

(A) $x^2 + y^2 + 2\sqrt{3}y - 2 = 0$ **(B)** $x^2 + y^2 - 2\sqrt{3}y + 2 = 0$

(C) $x^2 + y^2 + 2\sqrt{3}y + 2 = 0$ **(D)** $x^2 + y^2 - 2\sqrt{3}y - 2 = 0$

60. A straight line through the vertex P of a triangle PQR intersect the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then:

(A) $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{SQ \times SR}}$ **(B)** $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{SQ \times SR}}$

(C) $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$ **(D)** $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

61. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, which of the following equations can represent L_1.

(A) $x + y = 0$ **(B)** $x - y = 0$ **(C)** $x + 7y = 0$ **(D)** $x - 7y = 0$

62. Two circles have centres at $(a, 0)$ and $(-a, 0)$ and radii r_1 and r_2 $(a > r_1 > r_2)$. Then the points of contact of the common tangents to two circles lies on the

(A) $x^2 + y^2 = a^2 + r_1 r_2$ **(B)** $x^2 + y^2 = a^2 - r_1 r_2$

(C) $x^2 + y^2 = a^2 - 2r_1 r_2$ **(D)** $x^2 + y^2 = a^2 + 2r_1 r_2$

63. A circle S of radius unity touches a line L at P. A point A lies on S and N is the foot of the perpendicular from A to L. The area of ΔPAN as A varies cannot be equal to:

(A) 1 **(B)** $\sqrt{3}$ **(C)** $\frac{3\sqrt{3}}{8}$ **(D)** $\frac{\sqrt{3}}{2}$

64. Consider the circle $x^2+y^2-10x-6y+30=0$. Let O be the centre of the circle and tangent at $A(7, 3)$ and $B(5, 1)$ meet at C. Let $S = 0$ represents family of circles passing through A and B, then :

(A) area of quadrilateral $OACB = 4$

(B) the radical axis for the family of circles $S = 0$ is $x + y = 10$

(C) the smallest possible circle of the family $S = 0$ is $x^2+y^2-12x-4y+38=0$

(D) the coordinates of point C are (7, 1)

65. Let C_1 *and* C_2 be centres of two circles whose radii are 2 and 4 respectively. Also $C_1C_2=10$ and direct common tangents of these circles touch them at P, Q, R, S. Another circle of radius 'λ' is drawn passing through *P, Q, R, S.* Then

(A) Midpoint of C_1C_2 is centre of the circle passing through *P, Q, R, S*

(B) Centre of the circle passing through *P, Q, R, S* divides $C_1.C_2$ in the ratio $1:2$

(C) $\lambda^2=33$

(D) $\lambda^2=35$

66. If a circle passes through the point $\left(3, \sqrt{\frac{7}{2}}\right)$ and touches $x+y=1$ and $x-y=1$, then the centre of the circle is:

(A) (4, 0) **(B)** (4, 2) **(C)** (6, 0) **(D)** (7, 9)

67. A point M divides A and B in the ratio $1:2$ where A and B diametrically opposite ends of a circle $x^2+y^2-5x-9y+22=0$ square AMCD and BMEF on the length AM and MB are constructed on the same side of line AB if co-ordinates of A is (1, 3) then find the orthocenter of ΔABE.

(A) (1, 6) **(B)** (1, 5) **(C)** (3, 3) **(D)** (4, 6)

68. If the conics equations are $S \equiv \sin^2\theta\, x^2 + 2h\tan\theta xy + \cos^2\theta\, y^2 + 32x + 16y + 19 = 0,$
$S' \equiv \cos^2\theta\, x^2 + 2h'\cot\theta xy + \sin^2\theta y^2 + 16x + 32y + 19 = 0$ intersect at four concyclic points, then: (where $\theta \in [0, \pi/2]$)

(A) $h + h' = 0$ **(B)** $h - h' = 0$ **(C)** $\theta = \pi/4$ **(D)** None of these

69. If largest and smallest value of $\frac{y-4}{x-3}$ is p and q where (x, y) satisfy $x^2+y^2-2x-6y+9=0$ then which of the following is true:

(A) $p+q=\frac{4}{3}$ **(B)** $q=1$ **(C)** $p=\frac{4}{3}$ **(D)** $pq=\frac{4}{3}$

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column 1 are labelled as (A), (B), (C) & (D) whereas statements in Column 2 are labeled as p, q, r, s & t. More than one choice from Column 2 can be matched with Column 1.

70. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	If the point $(c, c+2)$ is an interior point of smaller segment of the curve $x^2+y^2-4=0$ made by the chord of the curve whose equation is $3x+4y+12=0$,then the value of c is	**(p)**	-1
(B)	If the set represented by $\{(x,y) \mid x^2+y^2+2x \le 1\} \cap \{(x,y) \mid x-y+c \ge 0\}$ contains only one point then the value of c is	**(q)**	ϕ
(C)	*ABCD* is a square of unit area, if a circle touches two sides of *ABCD* and passes through exactly one of its vertices. Then the radius of this circle is	**(r)**	$3\sqrt{2}$
(D)	If tangents are drawn from(4, 4) to the circle $x^2+y^2-2x-2y-7=0$ to meet the circle at *A* and *B*, then length of the chord *AB* is	**(s)**	2
		(t)	$2-\sqrt{2}$

71. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	If the line $2x-y+1=0$ is tangent to the circle at the point (2, 5) whose centre lies on the line $x-2y=4$, then radius of this circle is	**(p)**	$6\sqrt{26}$
(B)	Triangle *ABC* is right angled at *A*. The circle with centre *A* and radius *AB* cuts *BC* and *AC* internally at *D* and *E* respectively. If *BD* = 20 and *DC* = 16 then the length *AC* equals	**(q)**	1
(C)	Let *C* be the circle of radius unity centred at the origin. If two positive numbers x_1 and x_2 are such that the line passing through $(x_1, -1)$ and $(x_2, 1)$ is tangent to *C* then x_1x_2 is:	**(r)**	$3\sqrt{5}$
(D)	If $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$ and $\left(d, \frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units then, *abcd* is equal to	**(s)**	2
		(t)	$2-\sqrt{2}$

72. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	Consider 3 non-collinear points A, B, C with coordinates (0, 6), (5, 5) and (–1, 1) respectively. If the equation of a line tangent to the circle circumscribing the triangle ABC and passing through the origin is $ax+by=0$ then $b-a$ is	**(p)**	–1
(B)	From (3, 4) chords are drawn to the circle $x^2+y^2-4x=0$. The locus of the mid points of the chords is $(x-a)(x-b)+y(y-c)=0$, then the value of $a+b+c$ is	**(q)**	9
(C)	A foot of the normal from the point (4, 3) to a circle is (2, 1) and a diameter of the circle has the equation $2x-y-2=0$. Then the equation of the circle is $x^2+y^2-ax-b=0$, then $a-b$ is	**(r)**	14
(D)	The equation of the circle symmetric to the circle $x^2+y^2-2x-4y+4=0$ about the line $x-y=3$ is $x^2+y^2-ax+by+28=0$, then $a+b$ is	**(s)**	2
		(t)	1

73. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	A circle of constant radius 'a' passes through origin 'O' and cuts the axes of coordinates in points P and Q, then the equation of the locus of the foot of perpendicular from O to PQ is $(x^2+y^2)^2\left(\frac{1}{x^2}+\frac{1}{y^2}\right)=ka^2$,then k is	**(p)**	7
(B)	Tangents are drawn from any point on the circle $x^2+y^2=R^2$ to the circle $x^2+y^2=r^2$. The line joining the points of intersection of these tangents with circle also touch the second. If R equals $k\,r$,then k is	**(q)**	0
(C)	A ray of light incident at the point (–2, –1) gets reflected from the tangent at (0, –1) to the circle $x^2+y^2=1$. The reflected ray touches the circle. If the equation of the line along which the incident ray moves is $ay+bx=11$,then the value of $a+b$ is	**(r)**	4
(D)	The equation of a line inclined at an angle $\frac{\pi}{4}$ to the axis X, such that the two circle $x^2+y^2=4, x^2+y^2-10x-14y+65=0$ intercept equal lengths on it is $ax+by-3=0$,then the value of $a+b$ is	**(s)**	2
		(t)	$2-\sqrt{2}$

74. MATCH THE FOLLOWING :

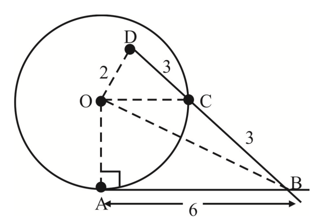
	Column 1		Column 2
(A)	P is a point (a, b) in the first quadrant. If the two circles which pass through P and touch both the coordinate axes cut at right angles, then the relation between a and b is given by $a^2 + b^2 - kab = 0$,then the value of k is	**(p)**	3
(B)	If two chords of the circle $x^2 + y^2 - ax - by = 0$, drawn from the point (a, b) is divided by the x-axis in the ratio 2:1 if $a^2 - kb^2 > 0$,then the value of k is	**(q)**	1
(C)	AB is a diameter of a circle. CD is a chord parallel to AB and $2CD = AB$. The tangent at B meets the line AC product at E then AE is equal to (kAB), then the value of k is	**(r)**	4
(D)	The lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to the same circle whose radius is r, then $4r$ is equal to	**(s)**	2
		(t)	$2-\sqrt{2}$

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

75. From a point $A(2, 2)$ two chords AB and AC of 1 unit length are drawn to the circle $x^2 + y^2 = 8$. If the equation of the chord BC is given by $ax + by = 15$, then the value of $a + b$ is___________.

76. In the given figure AB is tangent at A to the circle with centre at O; point D is interior to circle and DB intersects the circle at C. If $BC = DC = 3$, $OD = 2$ and $AB = 6$, then find the value of $[r]$ (where r is the radius of circle and [.] represent G.I.F.)



77. If r_1 and r_2 are the radius of two circles passing through (–1, 1) and touching the lines $x + y = 2$, $x - y = 2$, and $r_1 + r_2 = a\sqrt{2}$, then a is equal to _________.

78. If a circle touches the hypotenuse of a right-angled triangle at its middle point and passes through the middle point of shorter side. If a and b ($a < b$) be the length of the sides and the radius of the circle is $\frac{b}{ka}\sqrt{a^2 + b^2}$, then the value of k is __________.

79. On the side AC of an acute angled triangle ABC a point D is taken, such that $AD = 1$, $DC = 2$ and BD is an altitude of ΔABC. A circle of radius 2, which passes through points A and D and touches a circle at the point D circumscribed about the ΔBDC. If the area of ΔABC is Δ then the value of $\frac{1}{11}[\Delta]$ is equal to __________. (Where [.] represents G.I.F.)

80. The centre of *a* circle *C* lies on the line $2x - 2y + 9 = 0$ and this circle cuts $x^2 + y^2 = 4$ orthogonally. If this circle passes through two fixed points (a, b) and (c, d), then the value of $a + b + c + d$ is __________.

81. Let $P(a, b)$ be a variable point satisfying $4 \le a^2 + b^2 \le 9$ and $b^2 - 4ab + a^2 \le 0$. Let *R* be the complete region represented in *x-y* plane in which *P* can lie, if *m* be the minimum value of $|a+b|$ for all position of *P* lying in region *R*. Then $[m]$ is __________. (Where [.] represents G.I.F.)

82. *ABCD* is rectangle a circle passing through *C* touches *AB* and *AD* at *M* and *N* respectively. If the perpendicular distance of *MN* from *C* is 5 then the area of rectangle is___.

83. Through the point of intersection *P* of the circle $x^2 + y^2 = 1$ and $x^2 + y^2 + 2x + 4y + 1 = 0$ a common chord *APB* is drawn terminating on the two circles such that the chords *AP* and *BP* of the given circles subtend equal angles at the respective centres. If the coordinates of *P* are integral and the equation of the chord is $y = 2mx + 1$ then the value of *m* is _____.

84. A circle *S*, whose radius is 1 unit, touches the *X*-axis at point *A*. The centre *Q* of *S* lies in the first quadrant. The tangent from the origin *O* to the circle touches it at *T* and a point *P* lies on it such that the triangle *OAP* is a right-angled triangle at *A* and its perimeter is 8 unit. The length of *QP* is _____.

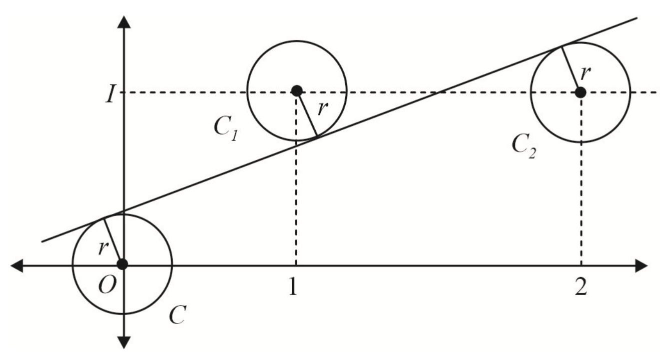
85. *CD* is the common chord of the two circles of equal radii touching a line *L* at *A* and *B*. Let C be closer to the line *L* than *D*. The ratio of the radii of circumcircles of the triangle *ACB* and *ADB* is _____.

86. The centres of two circles C_1 and C_2 each of unit radius are at a distance 6 unit from each other. Let P be the mid-point of the line segment joining the centres of C_1 and C_2. If a common tangent to C_1 and C_2 passing through P is also a common tangent to C_2 and C, then the radius of the circle C is _____.

87. The circle $x^2 + y^2 + 6x - 24y + 72 = 0$ and $x^2 - y^2 + 6x + 16y - 46 = 0$ intersect at four points. The sum of distances from these four points to the point (–3, 2) is 10 *k*. Then value of *k* is equal to _____.

88. Consider a series of '*n*' concentric circles $C_1, C_2, C_3, \ldots, C_n$ with radii $r_1, r_2, r_3, \ldots, r_n$ respectively, such that $r_1 > r_2 \ldots\ldots > r_n$ and $r_1 = 20$. If the tangents drawn from any point on to C_{i+1} are such that the chord of contact is a tangent to $C_{i+2}\,(i = 1, 2, 3, \ldots)$ and the angle between the tangents from any point on C_1 to C_2 is $\frac{\pi}{3}$, then find the values of $\lim_{n\to\infty} \sum_{i=1}^{n} r_i$.

89. As shown in the figure, three circles which have the same radius r have centers at $(0,0), (1,1), and\ (2,1)$. If they have a common tangent line, as shown, then the value of $10\sqrt{5}r$ is___.



90. If the circles $x^2 + y^2 + (3 + \sin\beta)x + (2\cos\alpha)y = 0$ and $x^2 + y^2 + (2\cos\alpha)x + 2cy = 0$ touch each other, then the maximum value of *c* is ______.

91. Let BD be the internal angle bisector of angle B in triangle ABC with D on side AC. The circumcircle of triangle BDC meets AB at E, while the circumcircle of triangle ABD meets BC at F., if $AE = 3$, then CF is equal to ______.

92. Six points $(x_i, y_i); i = 1,2,3,4,5,6$ are taken on the circle $x^2 + y^2 = 4$ such that $\sum_{i=1}^{6} x_i = 8$ and $\sum_{i=1}^{6} y_i = 4$. The line segment joining orthocenter of a triangle made by any three points and the centroid of the triangle made by other three points passes through a fixed point (h, k). The value of $h + k$ is ______.

93. The number of points of intersection of curve $\sin x = \cos y$ and circle $x^2 + y^2 = 1$.

94. Three circles touch one another externally. The tangents at their points of contact meet at a point whose distance from point of contact is 4. Find the ratio of the product of the radii to the double of the sum of the radii of the circles.

95. A circle passes through the point (3, 4) and cuts the circle $x^2 + y^2 = a^2$ orthogonally. The locus of its centre is a straight line. If the distance of the straight line from the origin is 817, then find the value of $a^2 - 8140$.

96. Let S_1 and S_2 denote the circles $x^2 + y^2 + 10x - 24y - 87 = 0$ and $x^2 + y^2 - 10x - 24y + 153 = 0$ respectively. (Let m be the smallest positive value of 'a' for which the line $y = ax$ contains the centre of a circle which touches S_2 externally and S_1 internally). Given that $m^2 = \frac{p}{q}$, where p *and* q are relatively prime integers, if $(p + q)$ is equal to 13^k, then the value of k is equal to ______.

97. A circle touches the hypotenuse of a right-angled triangle at its middle point and passes through the middle point of the shorter side. If 3 units and 4 units be the length of the sides and 'r' be the radius of the circle, then find the value of '$3r$'.

98. From a point 'P' on the normal $y = x + c$ of the circle $x^2 + y^2 - 2x - 4y + 5 - \lambda^2 = 0$, two tangents are drawn to the same circle touching it at points B and C. If the area of the quadrilateral $OBPC$ (where O is the centre of the circle), is 36 sq. units, the possible positive value of λ, (if it is given that the point P is at a distance of $|\lambda|(\sqrt{2} - 1)$ from the circle) is ______.

99. The number of possible integral values of m for which the circle $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y + m^2 = 0$ have exactly two common tangents is ___________.

100. Let PT be a tangent from the point $P(5, 3+\sqrt{3})$ to the circle $x^2 + y^2 + 4x - 6y - 3 = 0$, with centre C, at T and AB is secant which passes through P such that BT is the normal at T. If $Ar(\Delta CAB) + Ar(\Delta CAT) = \frac{\lambda}{25}$, then find the value of $\left(\left[\sqrt{\lambda}\right] - 15\right)$ ([.] denotes G.I.F).

101. If the two circles $(x-1)^2 + (y-3)^2 = r^2$ $(r > 0)$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then the number of odd positive integral values of r is____ .

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. The locus of a point on the variable parabola $y^2 = 4ax$, whose distance from focus is constant k, is equal to: (a is parameter)

(A) $4x^2 + y^2 - 4kx = 0$ **(B)** $x^2 + y^2 - 4kx = 0$
(C) $x^2 + 2y^2 - 4kx = 0$ **(D)** $4x^2 - y^2 + 4kx = 0$

2. Let S be the focus of y^2 = 4x and a point P is moving on the curve such that its abscissa is increasing at the rate of 4 units/sec, then the rate of increase of projection of SP on $x + y = 1$ when P is at (4, 4) is :

(A) $-\sqrt{2}$ **(B)** -1 **(C)** $\sqrt{2}$ **(D)** $-\frac{3}{\sqrt{2}}$

3. An equilateral triangle SAB is inscribed in the parabola $y^2 = 4ax$ having its focus at 'S'. If chord AB lies towards the left of S, then side length of this triangle is :

(A) $2a(2-\sqrt{3})$ **(B)** $4a(2-\sqrt{3})$ **(C)** $a(2-\sqrt{3})$ **(D)** $8a(2-\sqrt{3})$

4. $\min\left[(x_1 - x_2)^2 + \left(12 - \sqrt{1-x_1^2} - \sqrt{4x_2}\right)^2\right] \forall x_1, x_2 \in R$ is :

(A) $4\sqrt{5}+1$ **(B)** $4\sqrt{5}-1$ **(C)** $\sqrt{5}+1$ **(D)** $\sqrt{5}-1$

5. The straight line joining any point P on the parabola $y^2 = 4ax$ to the vertex and perpendicular from the focus to the tangent at P, intersect at R, then the equation of the locus of R is :

(A) $x^2 + 2y^2 - ax = 0$ **(B)** $2x^2 + y^2 - 2ax = 0$
(C) $2x^2 + 2y^2 - ay = 0$ **(D)** $2x^2 + y^2 - 2ay = 0$

6. If the ellipse $\frac{x^2}{4} + y^2 = 1$ meets the ellipse $x^2 + \frac{y^2}{a^2} = 1$ in four distinct points and $a = b^2 - 5b + 7$, then b does not lie in

(A) [4, 5] **(B)** $(-\infty, 2) \cup (3, \infty)$ **(C)** $(-\infty, 0)$ **(D)** [2, 3]

7. There are exactly two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose distance from its centre is same and is equal to $\sqrt{\frac{a^2 + 2b^2}{2}}$. Then the eccentricity of the ellipse is :

(A) $\frac{1}{2}$ **(B)** $\frac{1}{\sqrt{2}}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{3\sqrt{2}}$

8. Let S and S' be two foci of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If a circle described on *SS'* as diameter intersects the ellipse in real and distinct points, then the eccentricity *e* of the ellipse satisfies.

(A) $e=\frac{1}{\sqrt{2}}$ **(B)** $e\in(1/\sqrt{2},1)$ **(C)** $e\in(0,1/\sqrt{2})$ **(D)** None of these

9. From any point *P* lying in first quadrant on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, *PN* is drawn perpendicular to the major axis and produced to *Q* so that *NQ* equals to *PS*, where *S* is the focus (–3, 0). Then the locus of *Q* is :

(A) $5y-3x-25=0$ **(B)** $3x+5y+25=0$ **(C)** $3x-5y-25=0$ **(D)** None of these

10. Tangents are drawn to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$ and the circle $x^2+y^2=a^2$ at the points where a common ordinate cuts them (on the same side of the x-axis). Then the greatest acute angle between these tangents is given by :

(A) $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$ **(B)** $\tan^{-1}\left(\frac{a+b}{2\sqrt{ab}}\right)$ **(C)** $\tan^{-1}\left(\frac{2ab}{\sqrt{a-b}}\right)$ **(D)** $\tan^{-1}\left(\frac{2ab}{\sqrt{a+b}}\right)$

11. If the eccentricity of the hyperbola $x^2-y^2\sec^2\alpha=5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\alpha+y^2=25$, then a value of α is :

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{4}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{\pi}{2}$

12. If a ray of light incident along the line $3x+(5-4\sqrt{2})y=15$ gets reflected from the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ at the point $(4\sqrt{2},3)$, then its reflected ray goes along the line :

(A) $\sqrt{2}x-y+5=0$ **(B)** $\sqrt{2}y-x+5=0$

(C) $\sqrt{2}y-x-5=0$ **(D)** None of these

13. If two distinct tangents can be drawn from the point $(\alpha,2)$ on different branches of the hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$, then

(A) $|\alpha|<\frac{3}{2}$ **(B)** $|\alpha|>\frac{2}{3}$ **(C)** $|\alpha|>3$ **(D)** None of these

14. Area of the triangle formed by the asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and any tangent to the hyperbola is $a^2\tan\lambda$ in magnitude then its eccentricity is :

(A) $\sec\lambda$ **(B)** $\operatorname{cosec}\lambda$ **(C)** $\sec^2\lambda$ **(D)** $\operatorname{cosec}^2\lambda$

15. $(x-1)(y-2)=5$ and $(x-1)^2+(y+2)^2=r^2$ intersect at four points *A, B, C, D* and if centroid of ΔABC lies on line $y=3x-4$, then locus of *D* is :

(A) $y=3x$ **(B)** $x^2+y^2+3x+1=0$ **(C)** $3y=x+1$ **(D)** $y=3x+1$

16. If S_1 and S_2 are the foci of the hyperbola whose transverse axis length is 4 and conjugate axis length is 6, S_3 and S_4 are the foci of the conjugate hyperbola, then the area of the quadrilateral $S_1S_2S_3S_4$ is :

(A) 24 **(B)** 26 **(C)** 22 **(D)** None of these

For Questions 17 - 19

Two tangents to a parabola are $x-y=0$ and $x+y=0$. If (2, 3) is focus of the parabola, then :

17. The equation of tangent at vertex is :

(A) $4x-6y+5=0$ **(B)** $4x-6y+3=0$ **(C)** $4x-6y+1=0$ **(D)** $4x-6y+3/2=0$

18. Length of latus rectum of the parabola is :

(A) $\frac{6}{\sqrt{13}}$ **(B)** $\frac{10}{\sqrt{13}}$ **(C)** $\frac{2}{\sqrt{13}}$ **(D)** None of these

19. If *P, Q* are ends of focal chord of the parabola, then $\frac{1}{SP}+\frac{1}{SQ}=$

(A) $\frac{2\sqrt{13}}{3}$ **(B)** $2\sqrt{13}$ **(C)** $\frac{2\sqrt{13}}{5}$ **(D)** None of these

For Questions 20 - 22

A curve is represented by $C \equiv 21x^2-6xy+29y^2+6x-58y-151=0$.

20. Eccentricity of curve is :

(A) 1/3 **(B)** $1/\sqrt{3}$ **(C)** 2/3 **(D)** $2/\sqrt{5}$

21. The lengths of axes are :

(A) $6, 2\sqrt{6}$ **(B)** $5, 2\sqrt{5}$ **(C)** $4, 4\sqrt{5}$ **(D)** None of these

22. The centre of the conic C is :

(A) (1, 0) **(B)** (0, 0) **(C)** (0, 1) **(D)** None of these

For Questions 23 - 25

Let $P(x, y)$ is a variable point such that $\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}\right|=3$ which represents hyperbola.

23. The eccentricity e' of the corresponding conjugate hyperbola is :

(A) 5/3 **(B)** 4/3 **(C)** 5/4 **(D)** $3/\sqrt{7}$

24. Locus of intersection of two perpendicular tangents to the hyperbola is :

(A) $(x-3)^2+\left(y-\frac{7}{2}\right)^2=\frac{55}{4}$ **(B)** $(x-3)^2+\left(y-\frac{7}{2}\right)^2=\frac{25}{4}$

(C) $(x-3)^2+\left(y-\frac{7}{2}\right)^2=\frac{7}{4}$ **(D)** None of these

25. If origin is shifted to point $\left(3, \frac{7}{2}\right)$ and the axes are rotated through an angle θ in clockwise sense so that equation of given hyperbola changes to the standard form $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then θ is :

(A) $\tan^{-1}\left(\frac{4}{3}\right)$ **(B)** $\tan^{-1}\left(\frac{3}{4}\right)$ **(C)** $\tan^{-1}\left(\frac{5}{3}\right)$ **(D)** $\tan^{-1}\left(\frac{3}{5}\right)$

For Questions 26 - 28

Let $A\left(\frac{1}{2},0\right), B\left(\frac{3}{2},0\right), C\left(\frac{5}{2},0\right)$ be the given points and P be a point satisfying max $(PA+PB, PB+PC) < 2$

26. All points P are points common to:

(A) two ellipse **(B)** two hyperbola

(C) a circle and an ellipse **(D)** a circle and an hyperbola

27. The locus of P is symmetric about:

(A) origin **(B)** the line $y = x$ **(C)** y-axis **(D)** x-axis

28. The area of region of the point P is:

(A) $\sqrt{2}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)$ **(B)** $\sqrt{3}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)$ **(C)** $2\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)$ **(D)** $3\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)$

For Questions 29 - 31

Consider a point *P*, such that 2 of the normal drawn from it to the parabola are at right angles, then:

29. If the equation of parabola is $y^2 = 8x$, then locus of P is:

(A) $x^2 = 4(y-6)$ **(B)** $y^2 = 2(x-6)$ **(C)** $y^2 = 8(x-6)$ **(D)** $2x^2 = (y-6)$

30. The ratio of latus rectum of given parabola and that of made by locus of point P is:

(A) 4:1 **(B)** 2:1 **(C)** 16:1 **(D)** 1:1

31. If $P \equiv (x_1, y_1)$ the slope of third normal is:

(A) $\frac{y_1}{8}$ **(B)** $\frac{y_1}{2}$ **(C)** $-\frac{y_1}{8}$ **(D)** $-\frac{y_1}{2}$

For Questions 32 - 34

Let a hyperbola whose centre is at origin. A line $x + y = 2$ touches this hyperbola at $P(1,1)$ and intersects the asymptotes at A and B such that $AB = 6\sqrt{2}$ units. (you can use the concept that incase of hyperbola portion of tangent intercepted between asymptotes is bisected at the point of contact).

32. Equation of asymptotes are

(A) $5xy + 2x^2 + 2y^2 = 0$ **(B)** $3x^2 + 2y^2 + 6xy = 0$

(C) $2x^2 + 2y^2 - 5xy = 0$ **(D)** none of these

33. Angle subtended by AB at centre of the hyperbola is

(A) $sin^{-1}\frac{4}{5}$ **(B)** $sin^{-1}\frac{2}{5}$ **(C)** $sin^{-1}\frac{3}{5}$ **(D)** none of these

34. Equation of the tangent to the hyperbola at $\left(-1, \frac{7}{2}\right)$ is

(A) $5x + 2y = 2$ **(B)** $3x + 2y = 4$ **(C)** $3x + 4y = 11$ **(D)** none of these

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

35. The equation of the directrix of the parabola with vertex at the origin and having the axis along the x-axis and a common tangent of slope 2 with the circle $x^2 + y^2 = 5$ is/are :

(A) $x = 10$ **(B)** $x = 20$ **(C)** $x = -10$ **(D)** $x = -20$

36. Tangent is drawn at any point (x_1, y_1) other than vertex on the parabola $y^2 = 4ax$. If tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ such that all the chords of contact pass through a fixed point (x_2, y_2), then :

(A) x_1, a, x_2 are in G.P. **(B)** $\frac{y_1}{2}, a, y_2$ are in G.P.

(C) $-4, \frac{y_1}{y_2}, \frac{x_1}{x_2}$ are in G.P. **(D)** $x_1x_2 + y_1y_2 = a^2$

37. If the focus of the parabola $x^2 - ky + 3 = 0$ is (0, 2), then a value of k is/are:

(A) 4 **(B)** 6 **(C)** 3 **(D)** 2

38. If $y = 2$ be the directrix and (0, 1) be the vertex of the parabola $x^2 + \lambda y + \mu = 0$ then :

(A) $\lambda = 4$ **(B)** $\mu = 8$ **(C)** $\lambda = -8$ **(D)** $\mu = -4$

39. The extremities of latus rectum of a parabola are (1, 1) and (1, –1), then the equation of the parabola can be :

(A) $y^2 = 2x - 1$ **(B)** $y^2 = 1 - 2x$ **(C)** $y^2 = -2x + 3$ **(D)** $y^2 = 2x - 3$

40. Parabola $y^2 = 4x$ and the circle having its centre at (6, 5) intersect at right angle. Possible point of intersection of these curves can be :

(A) (9, 6) **(B)** $(2, \sqrt{8})$ **(C)** (4, 4) **(D)** $(3, 2\sqrt{3})$

41. A normal drawn to parabola $y^2 = 4ax$ meet the curve again at Q such that angle subtended by PQ at vertex is 90º, then coordinates of P can be

(A) $(8a, 4\sqrt{2}a)$ **(B)** $(8a, 4a)$ **(C)** $(2a, -2\sqrt{2}a)$ **(D)** $(2a, 2\sqrt{2}a)$

42. The locus of the midpoint of the focal distance of a variable point moving on the parabola, $y^2 = 4ax$ is a parabola whose

(A) Latus rectum is half the latus rectum of the original parabola **(B)** Vertex is $(a/2, 0)$

(C) Directrix is y-axis **(D)** Focus is $(a, 0)$

43. $\frac{x^2}{r^2 - r - 6} + \frac{y^2}{r^2 - 6r + 5} = 1$ will represents the ellipse, if r lies in the interval

(A) $(-\infty, -2)$ **(B)** $(3, \infty)$ **(C)** $(5, \infty)$ **(D)** $(1, \infty)$

44. The co-ordinates (2, 3) and (1, 5) are the foci of an ellipse which passes through the origin, then the equation of

(A) Tangent at the origin is $(3\sqrt{2} - 5)x + (1 - 2\sqrt{2})y = 0$

(B) Tangent at the origin is $(3\sqrt{2} + 5)x - (1 + 2\sqrt{2})y = 0$

(C) Normal at the origin is $(3\sqrt{2} + 5)x - (2\sqrt{2} + 1)y = 0$

(D) Normal at the origin is $(3\sqrt{2} - 5)x + (1 - 2\sqrt{2})y = 0$

45. If a pair of variable straight lines $x^2+4y^2+\alpha xy=0$ (where α is a real parameter) cut the ellipse $x^2+4y^2=4$ at two points A and B, then the locus of the point of intersection of tangents at A and B is :

(A) $x-2y=0$ **(B)** $2x-y=0$ **(C)** $x+2y=0$ **(D)** $2x+y=0$

46. Which of the following is/are true?

(A) There are infinite positive integral values of a for which $(13x-1)^2+(13y-2)^2=\left(\frac{5x+12y-1}{a}\right)^2$ represents an ellipse

(B) The minimum distance of a point (1, 2) from the ellipse $4x^2+9y^2+8x-36y+4=0$ is 1

(C) If from a point $P(0, \alpha)$ (P is not the origin) two normals other than axes are drawn to the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, then $|\alpha|<\frac{9}{4}$

(D) If the length of latus rectum of an ellipse is one-third of its major axis, then its eccentricity is equal to $\frac{1}{\sqrt{3}}$

47. Let E_1 and E_2 be two ellipses $\frac{x^2}{a^2}+y^2=1$ and $x^2+\frac{y^2}{a^2}=1$ (where a is a parameter). Then the locus of the points of intersection of the ellipses E_1 and E_2 is a set of curves comprising

(A) Two straight lines **(B)** One straight line

(C) One circle **(D)** One parabola

48. Consider the ellipse $\frac{x^2}{f(k^2+2k+5)}+\frac{y^2}{f(k+11)}=1$ and $f(x)$ is a positive decreasing function, then :

(A) The set of values of k, for which the major axis is x-axis is (–3, 2)

(B) The set of values of k, for which the major axis is y-axis is $(-\infty, 2)$

(C) The set of values of k, for which the major axis is y-axis is $(-\infty,-3)\cup(2,\infty)$

(D) The set of values of k, for which the major axis is y-axis is $(-3, \infty)$

49. If two concentric ellipses are such that the foci of each one are on the other and their major axes are equal. Let e and e' be their eccentricities, then

(A) The quadrilateral formed by joining the foci of the two ellipses is a parallelogram

(B) The angle θ between their axes is given by $\theta=\cos^{-1}\sqrt{\frac{1}{e^2}+\frac{1}{e'^2}-\frac{1}{e^2e'^2}}$

(C) If $e^2+e'^2=1$, then the angle between the axis of the two ellipses is 90º

(D) If $e+e'=1$, then the angle between the axis of the two ellipses is 90º

50. If the tangent drawn at point $(t^2, 2t)$ on the parabola $y^2=4x$ is same as the normal drawn at point $(\sqrt{5}\cos\theta, 2\sin\theta)$ on the ellipse $4x^2+5y^2=20$. Then :

(A) $\theta=\cos^{-1}\left(-\frac{1}{\sqrt{5}}\right)$ **(B)** $\theta=\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ **(C)** $t=-\frac{2}{\sqrt{5}}$ **(D)** $t=-\frac{1}{\sqrt{5}}$

51. The equation $\left|\sqrt{x^2+(y-1)^2}-\sqrt{x^2+(y+1)^2}\right|=K$ will represent a hyperbola for

(A) $K\in(0,2)$ **(B)** $K\in(-2,1)$ **(C)** $K\in(1,\infty)$ **(D)** $K\in(2,\infty)$

52. If $x, y\in R$ then the equation $3x^4-2(19y+8)x^2+(361y^2+2(100+y^4)+64)=2(190y+2y^2)$ represents in rectangular Cartesian system:

(A) parabola **(B)** hyperbola **(C)** circle **(D)** ellipse

53. For which of the following hyperbolas, we can have more than one pair of perpendicular tangents?

(A) $\frac{x^2}{4}-\frac{y^2}{9}=1$ **(B)** $\frac{x^2}{4}-\frac{y^2}{9}=-1$ **(C)** $x^2-y^2=4$ **(D)** $xy=44$

54. For the hyperbola $9x^2-16y^2-18x+32y-151=0$

(A) one of the directrix is $x = 21/5$ **(B)** length of latus rectum = 9/2

(C) foci are (6, 1) and (–4, 1) **(D)** eccentricity is 5/4

55. Circles are drawn on chords of the rectangular hyperbola $xy=4$ parallel to the line $y=x$ as diameters. All such circles pass through two fixed points whose coordinates are

(A) (2, 2) **(B)** (2, –2) **(C)** (–2, 2) **(D)** (–2, –2)

56. The equation $(x-\alpha)^2+(y-\beta)^2=k(lx+my+n)^2$ represents

(A) a parabola for $k<(l^2+m^2)^{-1}$ **(B)** an ellipse for $0<k<(l^2+m^2)^{-1}$

(C) a hyperbola for $k>(l^2+m^2)^{-1}$ **(D)** a point circle for $k=0$

57. If P is a point on a hyperbola, then

(A) Locus of excentre of the circle described opposite to $\angle P$ for $\Delta PSS'(S,S'$ are foci), is tangent at vertex

(B) Locus of excentre of the circle described opposite to $\angle S'$ is hyperbola

(C) Locus of excentre of the circle described opposite to $\angle P$ for $\Delta PSS'(S,S'$ are foci), is hyperbola

(D) Locus of excentre of the circle described opposite to $\angle S'$, is tangent at vertex

58. From point (2, 2) tangents are drawn to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ then point of contact lies in

(A) I quadrant **(B)** II quadrant **(C)** III quadrant **(D)** IV quadrant

59. For hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, let n be the number of points on the plane through which perpendicular tangents are drawn.

(A) if $n = 1$, then $e=\sqrt{2}$ **(B)** if n> 1, then $1<e<\sqrt{2}$

(C) if n = 0, then $e>\sqrt{2}$ **(D)** if $n>1$, then $e>\sqrt{2}$

60. From the points (x_1,y_1) *and* (x_2,y_2) tangents are drawn to the hyperbola $xy=c^2$, such that a circle passes through these points and the four points of contact, then:

(A) $x_1y_1=x_2y_2$ **(B)** $x_1x_2=y_1y_2$

(C) $x_1y_2+x_2y_1=4c^2$ **(D)** $x_1y_1+x_2y_2=4c^2$

61. Equations of the asymptotes of the hyperbola whose equation is given by $x=a\tan(\theta+\alpha)$ *and* $y=b\tan(\theta+\beta)$, θ being a parameter, is/are:

(A) $by=ax\tan(\alpha-\beta)$ **(B)** $y=ax\tan(\alpha-\beta)$

(C) $x+a\cot(\alpha-\beta)=0$ **(D)** $y-b\cot(\alpha-\beta)=0$

62. If equation of tangent at *P, Q* and vertex *A* of a parabola are $3x+4y-7=0, 2x+3y-10=0$ and $x-y=0$ respectively, then:

(A) Focus is (4, 5) **(B)** Length of latus rectum is $2\sqrt{2}$

(C) Axis is $x+y-9=0$ **(D)** Vertex is $\left(\frac{9}{2},\frac{9}{2}\right)$

63. Let PQ be a chord of the parabola $y^2 = 4x$. A circle drawn with PQ as a diameter passes through the vertex V of the parabola. If area $(\Delta PVQ) = 20\, unit^2$ then the coordinates of P is/are

(A) $(16,8)$ **(B)** $(16,-8)$ **(C)** $(-16,8)$ **(D)** $(-16,-8)$

64. The chord AB of the parabola $y^2 = 4ax$ cuts the axis of the parabola at C (C is internal to AB). If $A \equiv (at_1^2, 2at_1)$ and $B \equiv (at_2^2, 2at_2)$ and $AC : AB = 1 : 3$, then:

(A) $t_2 = 2t_1$ **(B)** $t_2 + 2t_1 = 0$ **(C)** $t_1 + 2t_2 = 0$ **(D)** $6t_1^2 = t_2(t_1 + 2t_2)$

65. Variable circle is described to pass through point $(1,0)$ and tangent to the curve $y = tan(tan^{-1} x)$. The locus of the centre of the circle is a parabola whose:

(A) length of the latus rectum is $2\sqrt{2}$ **(B)** axis of symmetry has the equation $x + y = 1$

(C) vertex has the co-ordinates $\left(\frac{3}{4}, \frac{1}{4}\right)$ **(D)** length of the latus rectum is $\sqrt{2}$

66. The range of α for which the points $(\alpha, 2+\alpha)$ and $\left(\frac{3}{2}\alpha, \alpha^2\right)$ lie on opposite sides of the line $2x + 3y = 6$ can lie in intervals:

(A) $(-\infty,-2)$ **(B)** $(-2,0)$ **(C)** $(0,1)$ **(D)** $(2,4)$

67. Let $y^2 = 4ax$ be a parabola and PQ be a focal chord. Let R be the point of intersection of the tangents at P and Q, then:

(A) area of circumcircle of ΔPQR is $\frac{\pi (PQ)^2}{4}$ **(B)** orthocenter of ΔPQR lies at the directrix

(C) incentre of ΔPQR lies at the vertex **(D)** minimum area of the circumcircle a ΔPQR is $4\pi a^2$

68. The normal drawn at the extermities P and Q of a focal chord meet the parabola again in P' and Q' respectively. Then**:**

(A) PQ and $P'Q'$ are perpendicular **(B)** PQ and $P'Q'$ are parallel

(C) $P'Q' = 3PQ$ **(D)** $P'Q' = 2\sqrt{3}PQ$

69. The parabolas $y^2 = 4ax$ and $y^2 = 4c(x-d)$ have a common normal other than the X-axis if and only if:

(A) $c > a$ and $2a > d + 2c$ **(B)** $c < a$ and $2a > d + 2c$

(C) $c > a$ and $2a < d + 2c$ **(D)** $c < a$ and $2a < d + 2c$

70. Let O be the vertex of a parabola and Q be any point on the axis of the parabola. If PQR be any chord passing through Q and PM and RN be the ordinates of P and R, then:

(A) $OM.ON = OQ^2$ **(B)** $OM.ON = 4aOQ$

(C) $PM.RN = 4aOQ$ **(D)** $PM.RN = OQ^2$

71. Coordinates of the feet of normal drawn from the point (7, 14) to the parabola $x^2 - 8x - 16y = 0$ is/are:

(A) $(0,0)$ **(B)** $(-4,3)$ **(C)** $(4,-1)$ **(D)** $(16,8)$

72. The values of a for which $y = ax^2 + ax + \frac{1}{24}$, $x = ay^2 + ay + \frac{1}{24}$ touch each other is/are

(A) $\frac{2}{3}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{13+\sqrt{601}}{12}$ **(D)** $\frac{13-\sqrt{601}}{12}$

73. If P is any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose foci are S_1 *and* S_2. Let $\angle PS_1S_2=\alpha$ *and* $\angle PS_2S_1=\beta$ then,

(A) $PS_1+PS_2=2a,\ if\ a>b$ **(B)** $PS_1+PS_2=2b,\ if\ a<b$

(C) $tan\frac{\alpha}{2}.tan\frac{\beta}{2}=\frac{1-e}{1+e}$ **(D)** $tan\frac{\alpha}{2}.tan\frac{\beta}{2}=\frac{\sqrt{a^2-b^2}}{b^2}\left[a-\sqrt{a^2-b^2}\right]$

74. A line $3x+y=8$ touches a hyperbola $H=0$ at $P(1,5)$ meets its asymptotes at A and B. If $AB=2\sqrt{10}, C(1,1)$ be the centre of hyperbola, e and l are eccentricity and latus rectum of hyperbola then

(A) $e=\frac{\sqrt{7}}{2}$ **(B)** $e=\frac{\sqrt{5}}{2}$ **(C)** $l=\sqrt{2}$ **(D)** $l=2\sqrt{2}$

75. Two tangents $2x+y=2$ *and* $x-2y=3$ to a parabola touching it at $A(2,-2)$ and $B(5,1)$ respectively. If focus of parabola is $S(\alpha,\beta)$ and latus rectum length is L, then:

(A) $\alpha-\beta=3$ **(B)** $\alpha-\beta=4$ **(C)** $L=\frac{27\sqrt{3}}{25}$ **(D)** $L=\frac{27\sqrt{2}}{25}$

76. Let $P(x_1,y_1)$ and $Q(x_2,y_2), y_1<0, y_2<0$ be the end points of the latus rectum of the ellipse $x^2+4y^2=4$. The equation of the parabolas with latus rectum PQ are:

(A) $x^2+2\sqrt{3}y=3+\sqrt{3}$ **(B)** $x^2-2\sqrt{3}y=3+\sqrt{3}$

(C) $x^2+2\sqrt{3}y=3-\sqrt{3}$ **(D)** $x^2-2\sqrt{3}y=3-\sqrt{3}$

77. Tangents are drawn from the point $(-2,0)$ to the parabola $y^2=8x$, radius of circle (s) that would touch these tangents and the corresponding chord of contact, can be equal to

(A) $4(\sqrt{2}+1)$ **(B)** $4(\sqrt{2}-1)$ **(C)** $8\sqrt{2}$ **(D)** none of these

78. If two distinct chords of the parabola $y^2=4ax$, passing through $(a,2a)$ are bisected on the line $x+y=1$, then length of the latus-rectum can be

(A) 2 **(B)** 1 **(C)** 4 (D) 5

79. The locus of point of intersection of any tangent to the parabola $y^2=4a(x-2)$ with a line perpendicular to it and passing through the focus, is

(A) the tangent to the parabola at the vertex **(B)** $x=2$

(C) $x=0$ **(D)** none of these

80. The tangent to the circle $x^2+y^2=4$ at a point P intersect the parabola $y^2=4x$ at points Q and R. Tangents to the parabola at Q and R intersect at S. If Q lies in the first quadrant such that $PQ=1$ units, then:

(A) $PR=8\,units$ **(B)** $SR=\frac{35}{4}units$

(C) $SQ=\frac{7}{2}units$ **(D)** area of $\Delta SQR=\frac{343}{16}sq.units$

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column I are labelled as (A), (B), (C) & (D) whereas statements in Column II are labeled as p, q, r, s & t. More than one choice from Column II can be matched with Column I.

81. Consider the parabola $(x-1)^2+(y-2)^2=\frac{(12x-5y+3)^2}{169}$:

	Column 1		Column 2
(A)	Locus of point of intersection of perpendicular tangent	**(p)**	$12x-5y-2=0$
(B)	Locus of foot of perpendicular from focus upon any tangent	**(q)**	$5x+12y-29=0$
(C)	Line along which minimum length of focal chord occurs	**(r)**	$12x-5y+3=0$
(D)	Line about which parabola is symmetrical	**(s)**	$24x-10y+1=0$

82. **MATCH THE FOLLOWING:**

	Column 1		Column 2
(A)	Points from which perpendicular tangents can be drawn to parabola $y^2=4x$	**(p)**	(–1, 2)
(B)	Points from which only one normal can be drawn to parabola $y^2=4x$	**(q)**	(3, 2)
(C)	Points at which chord $x-y-1=0$ of parabola $y^2=4x$ is bisected	**(r)**	(–1,–5)
(D)	Points from which tangents cannot be drawn to parabola $y^2=4x$	**(s)**	(5, –2)

83. **MATCH THE FOLLOWING:**

	Column 1		Column 2
(A)	Distance between the points on the curve $4x^2+9y^2=1$, where tangent is parallel to the line $8x=9y$, is less than	**(p)**	1
(B)	Sum of distances of the foci of the curve $25(x+1)^2+9(y+2)^2=225$ from $(-1,0)$ is more than	**(q)**	4
(C)	Sum of distances from the x-axis of the points on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$, where the normal is parallel to the line $2x+y=1$, is less than	**(r)**	**7**
(D)	Tangents are drawn from points on the line $x-y+2=0$ to the ellipse $x^2+2y^2=2$, then all the chords of contact pass through the point whose distance from (2, ½) is more than	**(s)**	5

84. **MATCH THE FOLLOWING :**

	Column 1		Column 2
(A)	If vertices of a rectangle of maximum area inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are extremities of latus rectum. Then eccentricity of ellipse is	**(p)**	$\frac{2}{\sqrt{5}}$
(B)	If extremities of diameter of the circle $x^2+y^2=16$ are foci of a ellipse, then eccentricity of the ellipse, if its size is just sufficient to contain the circle, is	**(q)**	$\frac{1}{\sqrt{2}}$
(C)	If normal at point (6, 2) to the ellipse passes through its nearest focus (5, 2), having centre at (4, 2) then its eccentricity is	**(r)**	$\frac{1}{3}$
(D)	If extremities of latus rectum of the parabola $y^2=24x$ are foci of ellipse and if ellipse passes through the vertex of the parabola, then its eccentricity is	**(s)**	$\frac{1}{2}$

85. If e_1 and e_2 are the roots of the equation $x^2-ax+2=0,$ then match the following.

	Column 1		Column 2
(A)	If e_1 and e_2 are the eccentricities of the ellipse and hyperbola, respectively then the values of a are	**(p)**	6
(B)	If both e_1 and e_2 are the eccentricities of the hyperbolas, then values of a are	**(q)**	$2\sqrt{2}+10^{-3}$
(C)	If e_1 and e_2 are eccentricities of hyperbola and conjugate hyperbola, then values of a are	**(r)**	$2\sqrt{2}$
(D)	If e_1 is the eccentricity of the hyperbola for which no such points exist from which perpendicular tangents can be drawn, then the values of a are	**(s)**	5

86. Match the following:

	List 1		List 2
P.	The normal at an end of a latusrectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through an end of the minor axis if e^4 is equal to	**1.**	$\frac{1}{9}$
Q.	PQ is a double ordinate of a parabola $y^2=4ax$. If the locus of its point of trisection is another parabola length of whose latus rectum is k times the length of the latus rectum of the given parabola then k is equal to	**2.**	$\frac{1}{a}$
R.	If e and e' are the distances of the extremities of any focal chord from the focus f of the parabola $y^2=4ax$, then $\frac{1}{e}+\frac{1}{e'}$ is equal to	**3.**	1
S	If e and e' be the eccentricities of a hyperbola and its conjugate, then $\frac{1}{e}+\frac{1}{e'^2}$ is equal to	**4.**	$1-e^2$

Codes:

	P	Q	R	S		P	Q	R	S
(A)	4	1	2	3	**(B)**	2	3	4	1
(C)	1	2	3	4	**(D)**	2	4	3	1

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

87. If the normals to the curve $y = x^2$ at the points P, Q & R passes through the point $(0, 3/2)$, find the radius of the circle circumscribing ΔPQR.

88. At any point P on the parabola $y^2 - 2y - 4x + 5 = 0$, a tangent is drawn which meets the directrix at Q. If the locus of R which divides QP externally in the ratio 1 : 2 is $(y-1)^2(x+1) + \lambda = 0$, then find λ.

89. The chord AC of the parabola $y^2 = 4ax$ subtends an angle of 90º at points B & D on the parabola. If A, B, C and D are represented by t_1, t_2, t_3 & t_4, then find the value of $\left|\frac{t_2 + t_4}{t_1 + t_3}\right|$.

90. The straight line $ax + by + c = 0$ cuts the locus of point of intersection of the lines $\frac{tx}{4} - \frac{y}{3} + t = 0$, $\frac{x}{4} + \frac{ty}{3} - t = 0$ at A & B such that line AB subtends a right angle at the origin, then $\left[\frac{3a - 4b}{c}\right]$ is _____. ([.] represents greatest integer function)

91. Let P, Q be two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose eccentric angles differ by a right angle. The tangents at P and Q meet at R. If the chord PQ divides the line segment CR in m : n, then find *m*/*n* (where C is the centre of ellipse).

92. The length of the major axis of the ellipse $(5x-10)^2 + (5y+15)^2 = \frac{(3x-4y+7)^2}{4}$ is A. Find [A]. ([.] represents greatest integer function).

93. Find the number of distinct normals that can be drawn to the ellipse $\frac{x^2}{169} + \frac{y^2}{25} = 1$ from the point *P*(0, 6).

94. If *e* be the eccentricity of a hyperbola and *f*(*e*) be the eccentricity of its conjugate hyperbola, then the value of $\int_1^3 \underbrace{f\,f\,f\,..............f(e)}_{n\ times}\, de$ is (*n* is even)

95. A normal to the hyperbola $\frac{x^2}{4} - \frac{y^2}{1} = 1$ has equal intercepts on positive *x* and *y*-axes. If this normal touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then find $[a^2 + b^2]$ ([.] represents greatest integer function).

96. If *k* be the length of the latus rectum of the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, then find 3k/8.

97. From a point *P* three normal are drawn to the parabola $y^2 = 4ax$, such that the product of slopes of two of the normal is *p*. If the locus of *P* is a part of the parabola, then $|p|$ equal to

98. Tangent is drawn at any fixed point (x_1, y_1) on the parabola $y^2 = 4ax$. Now tangents are drawn from any point on this tangent to the circle $x^2 + y^2 = a^2$ so that all the chords of contact pass through a fixed point (x_2, y_2).

If $4\left(\frac{x_1}{x_2}\right)+\left(\frac{y_1}{y_2}\right)^2 = ka^2$, then k equals to

99. A chord cut the same branch of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ in P, P' and the asymptotes in Q, Q', then the value of $(PQ+PQ')-(P'Q'+P'Q)$ is_____ .

100. A normal to the hyperbola $\frac{x^2}{4}-\frac{y^2}{1}=1$, has equal intercepts on positive x and positive y axis. If this normal touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then the value of $\frac{9}{25}\left(a^2+b^2\right)$ is____ .

101. Variable pairs of chords at right angles are drawn through any point P (with eccentric angle $\frac{\pi}{4}$) on the ellipse $\frac{x^2}{4}+y^2=1$, to meet the ellipse at two points say A and B. If the line joining A and B passes through a fixed point $Q(a, b)$ such that a^2+b^2 has the value equal to $\frac{m}{n}$, where m , n are relatively prime positive integers, then the value of $\frac{m+n}{3}$ is____ .

102. A normal is drawn to the ellipse $\frac{x^2}{\left(a^2+2a+2\right)^2}+\frac{y^2}{\left(a^2+1\right)^2}=1$, $a>0$ whose centre is at O. If maximum radius of the circle, centered at the origin and touching the normal, is 5 then the positive value of 'a' is….

103. If the normal at the points where the straight line $lx+my=1$ meet the parabola $y^2=4ax$, meet at the point (h, k) on the parabola $y^2=4ax$, then $\frac{kl}{am}$ is equal to____ .

104. 'O' is the vertex of parabola $y^2=4x$ and L is the upper end of latus rectum. If LH is drawn perpendicular to OL meeting x-axis in H, then length of double ordinate through H is $\sqrt{N}$, then $N=$

105. The straight line $\frac{lx}{a}+\frac{my}{b}=n$ meet the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at in the points P and Q. If OP and OQ are along a pair of semi-conjugate diameters, O being the centre of the ellipse, then $\frac{l^2}{n^2}+\frac{m^2}{n^2}$ equals____.

106. From a point P, perpendicular tangents are drawn to the ellipse $x^2+2y^2=2$. If the chords of contact are tangents to a family of concentric circles, having the centres same as that of the ellipse then the ratio of the areas of the largest circle to the smallest circle is____.

107. The tangent to the hyperbola $xy=1$ at the point *P* intersects the *X*-axis in *T* and the *Y*–axis in T'. The normal to the hyperbola at *P* intersects the *X*-axis in *N* and the *Y*-axis in N'. Let the areas of the triangles *PNT* and $PN'T'$ are Δ and Δ' respectively, If $\frac{1}{\Delta}+\frac{1}{\Delta'}$ is constant for all positions of *P* then the value of constant is _____.

108. The tangents at a point *P* to the rectangular hyperbola $xy=1$ meets the lines $x-y=0\, and\, x+y=0$ at *Q* and *R* respectively and Δ_1 is the area of the triangle *OQR*, where *O* is the origin. The normal at *P* meets X-axis at *M* and the *Y*-axis at *N* and Δ_2 is the area of the triangle *OMN*, then the value of $\Delta_1^2\Delta_2$ is ….

109. Transverse and conjugate axes of a rectangular hyperbola are along *X*-axis and *Y*-axis respectively and the distance between the foci is $10\sqrt{14}$. Number of the points (*x*, *y*) on the curve such that *x* and *y* are positive integers, is equal to _____.

110. The normal at four points *A, B, C* and *D* on the rectangular hyperbola $xy=c^2$ meet in $P(h,k)$ and $PA^2+PB^2+PC^2+PD^2=n\left(h^2+k^2\right)$, where '*n*' is equal to

111. If the point $\left(\lambda^2,\lambda-2\right)$ is a point lying interior of the region bounded by the parabola $y^2=2x$ and the chord joining the point (2, 2) and (8, –4), then the number of the integral values of λ is ____.

112. The straight line $\frac{x}{4}+\frac{y}{3}=1$ intersects the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ at two points *A* and *B*, there is a point *P* on this ellipse such that the area of ΔPAB is equal to $6\left(\sqrt{2}-1\right)$. Then the number of such points *P* is____.

113. The length of the sub-tangent to the hyperbola $x^2-4y^2=4$ corresponding to the normal having slope unity is $\frac{1}{\sqrt{k}}$, then *k* is equal to _____.

114. If the normals to curve $y=x^2$ at the points *P, Q* and *R* pass through the point $\left(0,\frac{3}{2}\right)$, then the radius of the circle circumscribing ΔPQR is___.

115. Consider a parabola $4y=x^2$ and point $B(0,1)$. Let $A_1(x_1,y_1), A_2(x_2,y_2),..............,A_n(x_n,y_n)$ are n points on the parabola such that $x_r>0$ and $\angle OBA_r=\frac{r\pi}{2n}(r=1,2,3,.........,n)$ then $\pi\left(\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}BA_r\right)$ is equal to ____.

116. If the equation on reflection of $\frac{(x-4)^2}{16}+\frac{(y-3)^2}{9}=1$ about the line $x-y-2=0$ is $16x^2+9y^2+k_1x-36y+k_2=0$ then $\frac{k_1+k_2}{100}$ is___ .

JEE Advanced Revision Booklet	Functions

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. The reflection about the line $x + y = 0$ of the inverse function $f^{-1}(x)$ of a function $f(x)$ is :

(A) $-f(x)$ **(B)** $-f(-x)$ **(C)** $-f^{-1}(x)$ **(D)** $-f^{-1}(-x)$

2. The range of $f(x) = tan^{-1} x + \frac{1}{2} sin^{-1} x$ is :

(A) $(-\pi, \pi)$ **(B)** $\left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right]$ **(C)** $\left(-\frac{3\pi}{4}, \frac{3\pi}{4}\right)$ **(D)** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

3. Let $f(x)$ be any function. The graphs of $y = f(x-1)$ and $y = f(-x+1)$ are symmetric about the line :

(A) $y = 0$ **(B)** $x = 0$ **(C)** $x = 1$ **(D)** $x = -1$

4. Let $f(x)$ be such that $f(x+2) = f(x)$ and $f(-x) = f(x)$ for any real number x. On the interval $[2, 3]$, $f(x) = x$. Then the formula of $f(x)$ given on $[-2, 0]$ is :

(A) $x+4$ **(B)** $2-x$ **(C)** $3-|x+1|$ **(D)** $2+|x+1|$

5. Suppose $f(x) = x^3 + log_2\left(x + \sqrt{x^2+1}\right)$. For any $a, b \in R$, to satisfy $f(a) + f(b) \geq 0$, the condition $a + b \geq 0$ is:

(A) Necessary and sufficient **(B)** Necessary but not sufficient
(C) Not necessary but sufficient **(D)** Neither necessary nor sufficient

6. Let $f(x) = sin^4 x - sin x\, cos x + cos^4 x$, then range of $f(x)$ is :

(A) $\left[0, \frac{9}{8}\right]$ **(B)** $\left[-\frac{9}{8}, 0\right]$ **(C)** $\left(0, \frac{9}{8}\right)$ **(D)** $\left(-\frac{9}{8}, 0\right)$

7. Let $f(x) = 1 + 2cos x + 3 sin x$. If real numbers a, b, c are such that $a\,f(x) + b\,f(c-x) = 1$ holds for any $x \in R$ then $\frac{b cos\, c}{a} =$

(A) 1 **(B)** – 1 **(C)** $\frac{1}{2}$ **(D)** $-\frac{1}{2}$

8. A function $f : R \to R$ has property $f(x+y) = f(x) \cdot e^{f(y)-1}$, for every $x, y \in R$ then positive value of $f(4)$ is :

(A) 1 **(B)** 2 **(C)** 4 **(D)** 8

9. The number of positive integers x that satisfy $3^x = x^3 + 3x^2 + 2x + 1$ is :

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4

10. It is given that the polynomial $P(x) = x^3 + ax^2 + bx + c$ has three distinct positive integer roots and $P(22) = 21$. Let $Q(x) = x^2 - 2x + 22$. It is also given that $P(Q(x))$ has no real roots then a is equal to :

(A) – 45 **(B)** – 55 **(C)** 45 **(D)** 60

11. The graph of the function $f(x)=\frac{9x+7}{3x+12}$ is symmetric to the point :

(A) $(-4,3)$ **(B)** $(-3,4)$ **(C)** $(3,4)$ **(D)** $(-3,-4)$

12. The only real solution to the equation $(x^2+100)^2=(x^3-100)^3$ has how many digits in base 10 representation?

(A) one **(B)** two **(C)** three **(D)** four

13. The equation $\sin(\cos x)=x$ has only one root x_1 in $(0,\pi/2)$ and the equation $\cos(\sin x)=x$ has also only one root x_2 in $(0,\pi/2)$. Then :

(A) $x_1>x_2$ **(B)** $x_1<x_2$ **(C)** $x_1=x_2$ **(D)** $x_1=2x_2$

14. Least value of the expression $\frac{1}{2bx-(x^2+b^2+\sin^2 x)}$, $x\in[-1,0]$, $b\in[2,3]$ is :

(A) $\frac{1}{4}$ **(B)** $-\frac{1}{4}$ **(C)** $\frac{-1}{8+\sin^2 1}$ **(D)** None of these

15. Let $f(x)=\left([a]^2-5[a]+4\right)x^3-\left(6\{a\}^2-5\{a\}+1\right)x-(\tan x)\operatorname{sgn} x$, be an even function for all $x\in-\left\{(2n+1)\frac{\pi}{2};\, n\in Z\right\}$, then sum of all possible values of a is : (where [.] {.} denotes greatest integer function and fractional part functions, respectively)

(A) $\frac{17}{6}$ **(B)** $\frac{53}{6}$ **(C)** $\frac{35}{3}$ **(D)** None of these

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

16. Define functions $f,g:R\to R$, $f(x)=3x-1+|2x+1|$ and $g(x)=\frac{1}{5}(3x+5-|2x+5|)$ then

(A) $f(g(x))=g(f(x))$ **(B)** $(f(f(x)))=g(g(x))$

(C) $f(g(x))=x$ **(D)** for $x<-\frac{5}{2}, f(g(x))=x$

17. The function $f(x)$ satisfies $f(10+x)=f(10-x)$ and $f(20-x)=-f(20+x)$. Which of the following statements about $f(x)$ is true?

(A) Periodic function **(B)** Not periodic **(C)** odd function **(D)** Even function

18. Given the system of equations $[3x]+\{y\}+x-y=1$ and $[-y]-\{x\}-x+y=1$, [x] denotes greatest integer $\le$ x, and {x} denotes fractional part of x then

(A) $x=-1$ **(B)** $y=-5$ **(C)** $\{x\}=\{y\}$ **(D)** $x+y=4$

19. Consider the function $f(x)=\frac{\log x}{x}$

(A) $f(x)$ has horizontal tangent at $x=e$ **(B)** $f(x)$ cuts the x-axis at only one point

(C) $f(x)$ is many-one function **(D)** $f(x)$ has one vertical tangent

20. Which of the following is periodic?

(A) $\operatorname{sgn}\left(e^{-x}\right)$ **(B)** $\sin x+|\sin x|$

(C) minimum $(|x|, \sin x)$ **(D)** $2[-x]+\left[x+\frac{1}{2}\right]+\left[x-\frac{1}{2}\right]$

21. The function $f(x)=\frac{|x-1|}{x^2}$ is one – one in

(A) $(2,\infty)$ **(B)** $(1,2)$ **(C)** $(0,1)$ **(D)** $(-\infty,0)$

22. Consider $f(x)=x\left[x^2\right]+\frac{1}{\sqrt{1-x^2}}$

(A) $f(x)$ is an even function **(B)** $f(x)$ is an odd function

(C) $f(x)$ is periodic **(D)** Range of $f(x)$ has only positive values

23. $f:R\to R,\ f(x)=a_1x+a_3x^3+a_5x^5+..........+a_{2n+1}x^{2n+1}-\cot x$ where $0<a_1<a_3<.........<a_{2n+1}$ then $f(x)$ is

(A) one-one **(B)** many-one **(C)** onto **(D)** into

24. Consider $f(x)=\log_2\left(\frac{1-x}{1+x}\right)-\log_2\left(x+\sqrt{x^2+1}\right)\cdot$ Then :

(A) $f'(0)=0$ **(B)** $f''(0)=0$ **(C)** $f'''(0)=0$ **(D)** $f^{IV}(0)=0$

25. The range of the function $f(x)=x\{x\}-x[-x]$ does not contain, [x] denotes greatest integer $\le$ x, and {x} denotes fractional part of x :

(A) – 1 **(B)** – 2 **(C)** 1 **(D)** 2

26. Let $f(x)=\frac{\sin\pi x}{x}, x\in(0,1)$ and $g(x)=f(x)+f(1-x)$ then :

(A) $g(x)=\frac{1}{2}f\left(\frac{x}{2}\right)f\left(\frac{1-x}{2}\right)$ **(B)** $g(x)=f\left(\frac{x}{2}\right)f\left(\frac{1-x}{2}\right)$

(C) $g(x)$ is symmetric about the y-axis **(D)** $g(x)$ is symmetric about the line $2x-1=0$

27. Let $f:R-\{0\}\to R, f(x)=\frac{x}{e^x-1}+\frac{x}{2}+1$

(A) $f(x)$ is even **(B)** $f(x)$ is many-one

(C) $f(x)$ is into **(D)** $\lim_{x\to 0} f(x)=2$

28. Let $f(x)=2+\sqrt{x}$ and $g(x)=\frac{2x}{x^2+1}$, then :

(A) Domain $(f+g+2)=(-1,\infty)$ **(B)** Domain $(f+g+2)=[0,\infty)$

(C) Range $f\cap$ range $(g+2)=[2,3]$ **(D)** Range $f\cup$ range $(g+2)=[1,\infty)$

29. Let $f(x)=\sin x+\sin\left(x\sqrt{3}\right)$. Then, which of the following are false ?

(A) Maximum value of $f(x)$ cannot be 2 **(B)** Maximum value of $f(x)$ cannot be -2

(C) $f(x)$ is periodic function with period $2\sqrt{3}\pi$ **(D)** $f(x)>0\ \forall\ x\in R$

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column 1 are labelled as (A), (B), (C) & (D) whereas statements in Column 2 are labeled as p, q, r, s & t. More than one choice from Column 2 can be matched with Column 1.

30. Equation $e^x = x^n,\ n \in I^+$

	Column 1		**Column 2 (Number of real roots)**
(A)	$n = 1$	**(p)**	3
(B)	$n = 2$	**(q)**	2
(C)	odd $n \geq 3$	**(r)**	1
(D)	even $n \geq 4$	**(s)**	0

31. MATCH THE FOLLOWING

	Column 1		**Column 2 (Range of f (x))**
(A)	$f(x) = \dfrac{(x+3)^2}{x^2+1}$	**(p)**	[0, 3]
(B)	$A = \left\{(x, y); x, y \in R, x^2 + y^2 \leq 25\right\}$		
	$B = \left\{(x, y); x, y \in R, y \geq \dfrac{4x^2}{9}\right\}$	**(q)**	[3, 9]
	and let $(x, f(x)) = A \cap B$		
(C)	$f(x) = \dfrac{9}{2 - \cos 3x}$	**(r)**	[0, 10]
(D)	$f(x) = 3\sqrt{2} \cdot \sin\left(\sqrt{\dfrac{\pi^2}{16} - x^2}\right)$	**(s)**	[0, 5]

32. Match the following columns :

	Column I		Column II
(i)	Range of $sgn\{x\}$ is: (where {.} represents fractional part function	**(a)**	{1}
(ii)	Domain of $sin^{-1} x + sin^{-1}(1-x)$ is:	**(b)**	[0, 1)
(iii)	Range of $\sqrt{\dfrac{2\,tan^{-1} x}{\pi}}$ is:	**(c)**	0, 1}
(iv)	Range of $\dfrac{2}{\pi} sin^{-1}\left[x^2 + x + 1\right]$ is: (where [.] represent greatest integer function)	**(d)**	[0, 1]

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

33. Let 'a' be the real root of the equation $x^3-3x^2+5x-17=0$ and 'b' be the real root of the equation $x^3-3x^2+5x+11=0$. Then $a+b=$ ____ .

34. The number of real numbers x such that $\dfrac{x}{x+4}=\dfrac{5[x]-7}{7[x]-5}$ is ______. [x] denotes greatest integer $\le x$,

35. Let $f(x)=144^{\sin^2 x}+144^{\cos^2 x}$. The number of integral values that $f(x)$ can take is ______.

36. The number of real solutions to the equation $3x-7=\left[x^2-3x+2\right]$ is ___. [x] denotes greatest integer $\le x$,

37. Compute : $\left[\dfrac{2106^3}{2104\times 2105}-\dfrac{2104^3}{2105\times 2106}\right]$. [$x$] denotes greatest integer $\le x$,

38. The range of the function $f(x)=\dfrac{x+m}{x^2+1}$, $(m\in R)$ contains the interval [0, 1]. If $m\ge\dfrac{3}{k}$, then find k.

39. Let $f(x)=(x+1)(x+2)(x+3)(x+4)+5$; where $x\in[-6, 6]$. If the range of the function is [a, b]; where $a, b\in N$, then find the value of (a + b).

40. Find the minimum number of roots of $f(x)=f\left(\dfrac{x+4}{x-2}\right)$.

41. If $f(2x+1)=4x^2+14x$, then find the sum of the squares of roots of the equation $f(x)=0$.

42. If $\alpha=e^{2\pi i/13}$ and $f(x)=7\sum_{k=1}^{50}A_i x^k$, then find the value of $\left(\dfrac{1}{13}\right)\sum_{r=0}^{12}f(a^r x)$.

43. Let $f(x, y)$ be a periodic function satisfying $f(x, y)=f(2x+2y, 2y-2x)$ for all x, y. Define $g(x)=f(2^x, 0)$, the find the period of function g.

44. Let $g(x)=\dfrac{e^x-e^{-x}}{2}$ and $g(f(x))=x$, then evaluate $f\left(\dfrac{e^{22}-1}{2e^{11}}\right)$.

JEE Advanced Revision Booklet	Differential Calculus-1

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. If $f(x)=\begin{cases}\dfrac{((a-n)nx-\tan x)\sin nx}{x^2} & \text{at } x\neq 0\\ 0 & \text{at } x=0\end{cases}$, where n is a non-zero real number, and f is continuous at $x = 0$, then a is equal to :

(A) 0 **(B)** $\dfrac{n}{n+1}$ **(C)** n **(D)** $n+\dfrac{1}{n}$

2. Let f be a function defined on $(-\pi/2,\pi/2)$ as follows : $f(x)=\begin{cases}\dfrac{2^{|x|}e^{|x|}-|x|-|x|\ln 2-1}{x\tan x} & ,x\neq 0\\ k & ,x=0\end{cases}$. The value of k so that f is continuous at x = 0 is :

(A) $\frac{1}{2}(\ln 2)^2+\frac{1}{2}\ln 2+1$ **(B)** $(\ln 2)^2+\frac{1}{2}(\ln 2)+1$

(C) $(\ln 2)^2+(\ln 2)+\frac{1}{2}$ **(D)** $\frac{1}{2}(\ln 2)^2+\ln 2+\frac{1}{2}$

3. The function $f(x)=[x]+\sqrt{\{x\}}$, where [.] denotes the greatest integer function and {.} denotes the fractional part function respectively, is discontinuous at

(A) all x **(B)** all integer points

(C) no x **(D)** x which is not an integer.

4. Define $f:[0,\pi]\to R$ by $f(x)=\begin{cases}\tan^2 x\left[\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right], & x\neq \pi/2\\ k & , x=\pi/2\end{cases}$ is continuous at $x=\dfrac{\pi}{2}$, then k is equal to :

(A) 1/12 **(B)** 1/6 **(C)** 1/24 **(D)** 1/32

5. $\lim\limits_{x\to\infty}\left(\sqrt[3]{(x+a)(x+b)(x+c)}-x\right)=$

(A) $\sqrt{abc}$ **(B)** $\dfrac{a+b+c}{3}$ **(C)** abc **(D)** $(abc)^{1/3}$

6. Given $f(x) = \dfrac{e^x - \cos 2x - x}{x^2}$ for $x \in R - \{0\}$, $\{x\}$ is fractional part function

$$g(x) = \begin{cases} f\{x\} & n < x < n + \dfrac{1}{2} \\ f(1 - \{x\}) & n + \dfrac{1}{2} < x \le n + 1 \\ \dfrac{5}{2} & \text{otherwise} \end{cases}$$

Then $g(x)$ is :

(A) Discontinuous at all integral values of x only **(B)** Continuous everywhere except for $x = 0$

(C) Discontinuous at $x = n + \dfrac{1}{2}$; $n \in I$ **(D)** Continuous everywhere

7. $\lim\limits_{n\to\infty}\left(\left(\dfrac{n}{n+1}\right)^{\alpha} + \sin\dfrac{1}{n}\right)^n$ when $\alpha \in Q$ is equal to :

(A) $e^{-\alpha}$ **(B)** $-\alpha$ **(C)** $e^{1-\alpha}$ **(D)** $e^{1+\alpha}$

8. $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\dfrac{r}{n^2+n+r}$ equals :

(A) 0 **(B)** 1/3 **(C)** 1/2 **(D)** 1

9. $\lim\limits_{n\to\infty}\left(\sqrt{n^2+n+1} - \left[\sqrt{n^2+n+1}\right]\right)$ $(n \in I)$ where [ ] denotes the greatest integer function is :

(A) 0 **(B)** 1/2 **(C)** 2/3 **(D)** 1/4

10. If $f(x+y) = f(x) + f(y) + |x|y + xy^2$, $\forall\ x, y \in R$ and $f'(0) = 0$, then :

(A) f need not be differentiable at every non zero x **(B)** f is differentiable for all $x \in R$

(C) f is twice differentiable at $x = 0$ **(D)** None

11. $\lim\limits_{n\to\infty}\dfrac{1^2\, n + 2^2\,(n-1) + 3^2\,(n-2) + \ldots + n^2 . 1}{1^3 + 2^3 + 3^3 + \ldots + n^3}$ is equal to :

(A) 1/3 **(B)** 2/3 **(C)** 1/2 **(D)** 1/6

12. The value of $\underset{x\to\infty}{\text{Limit}}\dfrac{\cot^{-1}\left(x^{-a}\log_a x\right)}{\sec^{-1}\left(a^x \log_x a\right)}$ (a > 1) is equal to :

(A) 1 **(B)** 0 **(C)** $\pi/2$ **(D)** Does not exist

13. $\lim\limits_{n\to\infty}\left(\dfrac{\sqrt[n]{p}+\sqrt[n]{q}}{2}\right)^n$, p, q > 0 equals :

(A) 1 **(B)** $\sqrt{pq}$ **(C)** pq **(D)** $\dfrac{pq}{2}$

14. Let $a = \min(x^2 + 2x + 3, x \in R)$ and $b = \lim\limits_{x\to 0}\dfrac{\sin x \cos x}{e^x - e^{-x}}$. Then the value of $\sum\limits_{r=0}^{n} a^r b^{n-r}$ is :

(A) $\dfrac{2^{n+1}+1}{3\cdot 2^n}$ **(B)** $\dfrac{2^{n+1}-1}{3\cdot 2^n}$ **(C)** $\dfrac{2^n-1}{3\cdot 2^n}$ **(D)** $\dfrac{4^{n+1}-1}{3\cdot 2^n}$

15. $\underset{x\to\infty}{Limit}\dfrac{\cot^{-1}\left(\sqrt{x+1}-\sqrt{x}\right)}{\sec^{-1}\left\{\left(\dfrac{2x+1}{x-1}\right)^{x}\right\}}$ is equal to :

(A) 1 **(B)** 0 **(C)** $\pi/2$ **(D)** Non existent

16. If f (x) = $\dfrac{e^{2x}-(1+4x)^{1/2}}{\ln(1-x^2)}$ for x ≠ 0, then f has :

(A) An irremovable discontinuity at x = 0
(B) A removable discontinuity at x = 0 and f (0) = – 4
(C) A removable discontinuity at x = 0 and f (0) = – 1/4
(D) A removable discontinuity at x = 0 and f (0) = 4

Paragraph for Question 17 - 19

Let $f(x)$ is a function continuous for all $x \in R$ except at $x = 0$. Such that $f'(x) < 0$ $x \in (-\infty, 0)$ and $f'(x) > 0$ $x \in (0, \infty)$. Let $\underset{x\to 0^+}{Lim} f(x) = 2$, $\underset{x\to 0^-}{Lim} f(x) = 3$ and $f(0) = 4$.

17. The value of λ for which $2\left(\underset{x\to 0}{Lim} f(x^3-x^2)\right) = \lambda\left(\underset{x\to 0}{Lim} f(2x^4-x^5)\right)$ is :

(A) 4/3 **(B)** 2 **(C)** 3 **(D)** 5

18. The values of $\underset{x\to 0^+}{Lim}\dfrac{f(-x)\,x^2}{\left\{\dfrac{1-\cos x}{[f(x)]}\right\}}$ where [·] denote greatest integer function and {·} denote fraction part function.

(A) 6 **(B)** 12 **(C)** 18 **(D)** 24

19. $\underset{x\to 0^-}{Lim}\left(\left[3f\left(\dfrac{x^3-\sin^3 x}{x^4}\right)\right]-f\left(\left[\dfrac{\sin x^3}{x}\right]\right)\right)$ where [·] denote greatest integer function.

(A) 3 **(B)** 5 **(C)** 7 **(D)** 9

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

20. Let f be a function defined on (–1, 1) by $f(x)=\dfrac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}\left(1-\{x\}\right)}{\{x\}-\{x\}^3}, x \neq 0$.{.} is the fractional part function. Which of the following statements is correct?

(A) $\underset{x\to 0+}{lim} f(x)$ exists and equals $\dfrac{\pi}{\sqrt{2}}$ **(B)** $\underset{x\to 0-}{lim} f(x)$ exists and equals $\pi/4$

(C) f is continuous **(D)** $\underset{x\to 0^-}{lim} f(x)$ exists and equals $\underset{x\to 0+}{lim} f(x)$

21. Let $f(x)=\begin{cases}\dfrac{x^{2^{32}}-2^{32}x+4^{16}-1}{(x-1)^2} & ,x\neq 1\\ =k & ,x=1\end{cases}$, the value of k so that the function is continuous at $x = 1$ is :

(A) $2^{63}-2^{31}$ **(B)** $2^{65}-2^{33}$

(C) $\left(2^{16}+1\right)\left(2^{8}+1\right)\left(2^{4}+1\right)\left(2^{2}+1\right)\left(2^{32}+2^{31}\right)$ **(D)** $\left(2^{32}+1\right)\left(2^{16}+1\right)\left(2^{8}+1\right)\left(2^{4}+1\right)\left(2^{2}+1\right)\left(2^{33}+2^{31}\right)$

22. Which of the following statements are true?

(A) If f is differentiable at x = c, then $\lim\limits_{h\to 0}\dfrac{f(c+h)-f(c-h)}{2h}$ exists and equals $f'(c)$.

(B) Given a function f and a point c in the domain of f, if the $\lim\limits_{h\to 0}\dfrac{f(c+h)-f(c-h)}{h}$ exists, then the function is differentiable at x = c

(C) Let $g(x)=\begin{cases} x^2 \sin\dfrac{1}{x^2}, & x\neq 0 \\ 0, & x=0 \end{cases}$, then g' exists

(D) Let $g(x)=\begin{cases} x^2 \sin\dfrac{1}{x^2}, & x\neq 0 \\ 0, & x=0 \end{cases}$, then g' exists and is continuous.

23. Let f be a function given by $f(x)=\begin{cases} \dfrac{1}{x\ell n2}-\dfrac{1}{2^x-1}; & x\neq 0 \\ \dfrac{1}{2} & ; x=0 \end{cases}$, Then :

(A) f is continuous on R

(B) f is differentiable on R and $f'(0)$ equals $\dfrac{-\ell n2}{12}$

(C) f is not differentiable at $x = 0$

(D) f is differentiable on R and $f'(0)$ equals $\dfrac{-\ell n2}{6}$

24. Let f be a function with two continuous derivatives and $f(0)=0$, $f'(0)=0$, $f''(0)=0$. Function g is defined by

$$g(x)=\begin{cases} \dfrac{f(x)}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$$

Then which of the following statements are correct?

(A) g has a continuous first derivative

(B) $g'(x)$ exist at $x=0$

(C) $g(x)$ is continuous but $g'(x)$ do not exist

(D) $g(x)$ is continuous, but the first derivative of g is not continuous

25. The function $\left.\begin{aligned} f(x) &= \dfrac{x^2}{a} &, \quad & 0\le x<1 \\ &= a &, \quad & 1\le x<\sqrt{2} \\ &= \dfrac{2b^2-4b}{x^2} &, \quad & \sqrt{2}\le x<\infty \end{aligned}\right\}$ is continuous for $0\le x<\infty$. Then which of the following statements is correct?

(A) The number of all possible ordered pairs (a, b) is 3

(B) The number of all possible order pairs (a, b) is 4

(C) The product of all possible values of b is – 1

(D) The product of all possible values of b is 1.

26. Let $f: R \to R$ be defined by $f(x) = \begin{cases} x + 2x^2 \sin\frac{1}{x} & for \quad x \neq 0 \\ 0 & for \quad x = 0 \end{cases}$ then

(A) $f(x)$ is differentiable for all $'x'$ but $f'(x)$ is not continuous at $x = 0$

(B) $f'(0) = 1$ **(C)** $f(x)$ is increasing at $x = 0$

(D) Both $f(x)$ and $f'(x)$ are differentiable for all $'x'$

27. Let $f(x) = \min(x^3, x^2)$ and $g(x) = [x]^2 + \sqrt{\{x\}^2}$, where $[x]$ denotes the greatest integer and $\{x\}$ denotes the fractional part function. Then which of the following holds?

(A) f is continuous for all x. **(B)** g is discontinuous for all $x \in I$.

(C) f is differentiable for all $x \in (1, \infty)$ **(D)** g is not differentiable for all $x \in$ I

28. Let $f(x) = \underset{n \to \infty}{Lim} \dfrac{2x^{2n} \sin\frac{1}{x} + x}{1 + x^{2n}}$ then which of the following alternative(s) is/are correct?

(A) $\underset{x \to \infty}{Lim}\, x f(x) = 2$ **(B)** $\underset{x \to 1}{Lim}\, f(x)$ does not exist.

(C) $\underset{x \to 0}{Lim}\, f(x)$ does not exist **(D)** $\underset{x \to -\infty}{Lim}\, f(x)$ is equal to zero.

29. Assume that $\underset{\theta \to -1}{Lim} f(\theta)$ exists and $\dfrac{\theta^2 + \theta - 2}{\theta + 3} \le \dfrac{f(\theta)}{\theta^2} \le \dfrac{\theta^2 + 2\theta - 1}{\theta + 3}$ holds for certain interval containing the point $\theta = -1$ then $\underset{\theta \to -1}{Lim} f(\theta)$ and $\underset{\theta \to -1}{Lim} \dfrac{f(\theta)}{\theta^2}$ is :

(A) is equal to $f(-1)$ **(B)** is equal to 1 **(C)** is non existent **(D)** is equal to – 1

30. Let $f(x) = \begin{cases} \dfrac{\tan^2 \{x\}}{x^2 - [x]^2} & for \quad x > 0 \\ 1 & for \quad x = 0 \\ \sqrt{\{x\} \cot \{x\}} & for \quad x < 0 \end{cases}$ where $[x]$ is the step up function and $\{x\}$ is the fractional part function of x, then :

(A) $\underset{x \to 0^+}{Lim} f(x) = 1$ **(B)** $\underset{x \to 0^-}{Lim} f(x) = 1$

(C) $\cot^{-1}\left(\underset{x \to 0^-}{Lim} f(x)\right)^2 = 1$ **(D)** f is continuous at $x = 1$

31. The function, $f(x) = [|x|] - [|x|]$, where [ ] denotes greatest integer function:

(A) is continuous for all positive integers **(B)** is discontinuous for all non-positive integers

(C) has finite number of elements in its range **(D)** is such that its graph does not lie above the x-axis

32. The function $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$

(A) has its domain $-1 \le x \le 1$ **(B)** has finite one sided derivates at the point x = 0

(C) is continuous and differentiable at x = 0 **(D)** is continuous but not differentiable at x = 0

33. f is a continuous function in [a, b]; g is a continuous function in [b, c]

A function h (x) is defined as :

$$h(x) = f(x) \quad \text{for } x \in [a, b)$$
$$= g(x) \quad \text{for } x \in (b, c]$$

if f (b) = g (b), then

(A) h(x) has a removable discontinuity at x = b
(B) h(x) may or may not be continuous in [a, c]
(C) $h(b^-) = g(b^+)$ and $h(b^+) = f(b^-)$
(D) $h(b^+) = g(b^-)$ and $h(b^-) = f(b^+)$

34. In which of the following cases the given equations has atleast one root in the indicated interval?

(A) $x - \cos x = 0$ in $(0, \pi/2)$
(B) $x + \sin x = 1$ in $(0, \pi/6)$
(C) $\frac{a}{x-1} + \frac{b}{x-3} = 0$, a, b > 0 in (1, 3)
(D) f (x) – g (x) = 0 in (a, b) where f and g are continuous on [a, b] and f (a) > g (a) and f (b) < g (b)

35. If $f(x) = \begin{cases} \frac{x \,.\, \ell n(\cos x)}{\ell n\left(1 + x^2\right)} & x \neq 0 \\ 0 & x = 0 \end{cases}$ then :

(A) f is continuous at x = 0
(B) f is continuous at x = 0 but not differentiable at x = 0
(C) f is differentiable at x = 0
(D) f is not continuous at x = 0

36. $\underset{x \to c}{Lim} f(x)$ does not exist when :

(A) f(x) = [[x]] – [2x – 1], c = 3
(B) f(x) = [x] – x, c = 1
(C) $f(x) = \{x\}^2 - \{-x\}^2$, c = 0
(D) $f(x) = \frac{\tan(\text{sgn}\, x)}{\text{sgn}\, x}$, c = 0 .

where [x] denotes step up function and {x} fractional part function.

37. Which of the following limits vanish?

(A) $\underset{x \to \infty}{Limit} \sin \frac{1}{\sqrt{x}}$
(B) $\underset{x \to \pi/2}{Limit} (1 - \sin x) . \tan x$
(C) $\underset{x \to \infty}{Limit} \frac{2x^2 + 3}{x^2 + x - 5} . \text{sgn}(x)$
(D) $\lim_{x \to 3} \frac{[x]^2 - 9}{x^2 - 9}$

where [ ] denotes greatest integer function

38. Let $f(x) = |x-1|([x]-[-x])$, then which of the following statement(s) is/are correct. (where [.] denotes greatest integer function.)

(A) $f(x)$ is continuous at $x = 1$
(B) $f(x)$ is derivable at $x = 1$
(C) $f(x)$ is non-derivable at $x = 1$
(D) $f(x)$ is discontinuous at $x = 1$

39. If $y = f(x)$ defined parametrically by $x = 2t - |t-1|$ and $y = 2t^2 + t|t|$, then:

(A) $f(x)$ is continuous for all $x \in R$
(B) $f(x)$ is continuous for all $x \in R - \{2\}$
(C) $f(x)$ is differentiable for all $x \in R$
(D) $f(x)$ is differentiable for all $x \in R - \{2\}$

40. $f: R \to R$ is one-one, onto and differentiable function and $f(4+x)+f(4-x)=0 \forall x \in R$ then:

(A) $f^{-1}(2010)+f^{-1}(-2010)=8$

(B) $\int_{-2010}^{2018} f(x)dx = 0$

(C) If $f'(-100)>0,$ then roots of $x^2 - f'(10)x - f'(10) = 0$ are non-real

(D) If $f'(10)=20,$ then $f'(-2)=20$

41. Let f be a real valued function defined on the interval $(0,\infty),$ by $f(x)=\ln x + \int_0^x \sqrt{1+\sin t}\,dt.$ Then, which of the following statement(s) is/are true?

(A) $f''(x)$ exists for all $x \in (0,\infty)$

(B) $f'(x)$ exists for all $x \in (0,\infty)$ and f' is continuous on $(0,\infty)$ but not differentiable on $(0,\infty)$

(C) There exists $\alpha > 1$ such that $|f'(x)| < |f(x)|$ for all $x \in (0,\infty)$

(D) There exists $\beta > 0$ such that $|f(x)| < |f'(x)| \le \beta$ for all $x \in (0,\infty)$

42. If $f(x)=\begin{cases} 3-\left[\cot^{-1}\left(\dfrac{2x^3-3}{x^2}\right)\right], & for\ x>0 \\ \{x^2\}\cos\left(e^{1/x}\right), & for\ x<0 \end{cases}$, where $\{x\}$ and $[x]$ denote fractional part and the greatest integer function respectively, then which of the following statements does not hold good?

(A) $f(0^-)=0$ **(B)** $f(0^+)=0$

(C) $f(0)=0 \Rightarrow$ continuous at $x=0$ **(D)** Irremovable discontinuity at $x=0$

43. If $f(x)=\begin{cases} b([x]^2+[x])+1, & for\ x\ge -1 \\ \sin(\pi(x+a)), & for\ x<-1 \end{cases}$, where $[x]$ denotes the integral part of $x,$ then for what values of a and $b,$ the function is continuous at $x=-1$?

(A) $a=2n+\frac{3}{2}; b\in R, n\in I$ **(B)** $a=4n+2; b\in R, n\in I$

(C) $a=4n+\frac{3}{2}; b\in R^+, n\in I$ **(D)** $a=4n+1; b\in R^+, n\in I$

44. Let $[x]$ be the greatest integer function, then $f(x)=\dfrac{\sin\frac{1}{4}\pi[x]}{[x]}$ is:

(A) not continuous at any point **(B)** continuous at $x=\frac{3}{2}$

(C) discontinuous at $x=2$ **(D)** differentiable at $x=\frac{4}{3}$

45. Let $f(x) = \cos x$ and $H(x) = \begin{cases} \min(f(t) : 0 \le t < x), & \text{for } 0 \le x \le \pi/2 \\ \frac{\pi}{2} - x, & \text{for } \frac{\pi}{2} < x \le 3 \end{cases}$, then:

(A) $H(x)$ is continuous and derivable in [0, 3]

(B) $H(x)$ is continuous but not derivable at $x = \frac{\pi}{2}$

(C) $H(x)$ is neither continuous nor derivable at $x = \frac{\pi}{2}$

(D) Maximum value of $H(x)$ in [0, 3] is 1

46. Let $f(x) = \begin{cases} \frac{\tan^2\{x\}}{x^2 - [x]^2}, & \text{for } x > 0 \\ 1, & \text{for } x = 0 \\ \sqrt{\{x\}\cot\{x\}}, & \text{for } x < 0 \end{cases}$, where $[x]$ is greatest integer function and $\{x\}$ is the fractional part function of x, then:

(A) $\lim_{x \to 0^+} f(x) = 1$ **(B)** $\lim_{x \to 0^-} f(x) = 1$ **(C)** $\cot^{-1}\left(\lim_{x \to 0^-} f(x)\right)^2 = 1$ **(D)** None of these

47. Let $f(x) = \lim_{n \to \infty} \sum_{r=0}^{n} \frac{x}{(rx+1)\{(r+1)x+1\}}$. Then:

(A) $f(0) = 0$ **(B)** $f(0) = 1$ **(C)** $f(2) = 1$ **(D)** $f(3) = 1$

48. For $\lim_{x \to 0} \frac{\cot^{-1}\left(\frac{1}{x}\right)}{x}$.

(A) RHL exists

(B) LHL does not exist

(C) Limit does not exist as RHL is 1 and LHL is -1

(D) Limit does not exist as RHL and LHL both are non-existent

49. For $a > 0$, let $l = \lim_{x \to \frac{\pi}{2}} \frac{a^{\cot x} - a^{\cos x}}{\cot x - \cos x}$ and $m = \lim_{x \to -\infty} \left(\sqrt{x^2 + ax} - \sqrt{x^2 - ax}\right)$, then:

(A) $l > m$, for all $a > 0$ **(B)** $l > m$, when $a \ge 1$

(C) $l > m$, for all $a > e^{-a}$ **(D)** $e^l + m = 0$

50. Consider the function $f(x) = \left(\frac{ax+1}{bx+2}\right)^x$, where $a, b > 0$ the $\lim_{x \to \infty} f(x)$ is:

(A) exists for all values of a and b **(B)** zero for $a < b$

(C) non-existent for $a > b$ **(D)** $e^{-(1/a)}$ or $e^{-(1/b)}$, if $a = b$

51. $\lim_{x\to c} f(x)$ does not exist when (where [x] denotes the greatest integer less than or equal to x)

(A) $f(x)=[[x]]-[2x-1], c=3$ **(B)** $f(x)=[x]-x, c=1$

(C) $f(x)=\{x\}^2-\{-x\}^2, c=0$ **(D)** $f(x)=\dfrac{\tan(\text{sgn}\, x)}{(\text{sgn}\, x)}, c=0$

52. The function $f(x)=\left[x^2\left[\dfrac{1}{x^2}\right]\right], x\neq 0$ is ($[x]$ represents the greatest integer $\le x$)

(A) continuous at $x=1$ **(B)** discontinuous at $x=-1$

(C) discontinuous at infinitely many points **(D)** discontinuous at finite number of points

53. A function is defined as $f(x)=[\tan x]+\sqrt{\tan x-[\tan x]}$ $0\le x<\dfrac{\pi}{2}$ (where [.] denotes the greatest integer function) then:

(A) $f(x)$ is continuous in $\left[0,\dfrac{\pi}{2}\right)$ **(B)** $f(x)$ is not continuous at $x=0$

(C) $f(x)$ is continuous at $x=0,\dfrac{\pi}{4}$ **(D)** $f(x)$ has infinite points of discontinuities

54. Let $f(x)=\sec^{-1}\left(\left[1+\sin^2 x\right]\right)([.])$ denotes the greatest-integer function). Then set of points, where $f(x)$ is not continuous is:

(A) $\left\{\dfrac{n\pi}{2}, n\in I\right\}$ **(B)** $\left\{\{2n-1\}\dfrac{\pi}{2}, n\in I\right\}$ **(C)** $\left\{(2n+1)\dfrac{\pi}{2}, n\in I\right\}$ **(D)** $\{n\pi, n\in I\}$

55. Let $f(x)=\begin{cases}\int_0^x \{1+|1-t|\}dt & \text{if } x>2\\ 5x-7 & \text{if } x\le 2\end{cases}$, then:

(A) f is not continuous at $x=2$

(B) f is continuous but not differentiable at $x=2$

(C) f is differentiable every where

(D) $\lim_{x\to 2^+} f'(x)=2$

56. If $F(x)=f(x)g(x)$ and $f'(x)g'(x)=c$, then (where f and g are thrice differentiable)

(A) $F'=c\left[\dfrac{f}{f'}+\dfrac{g}{g'}\right]$ **(B)** $\dfrac{F''}{F}=\dfrac{f''}{f}+\dfrac{g''}{g}+\dfrac{2c}{fg}$

(C) $\dfrac{F'''}{F}=\dfrac{f'''}{f}+\dfrac{g'''}{g}$ **(D)** $\dfrac{F'''}{F''}=\dfrac{f'''}{f''}+\dfrac{g'''}{g''}$

57. $f(x)$ is defined for $x\ge 0$ and has a continuous derivative. It satisfies $f(0)=1, f'(0)=0$ and $(1+f(x))f''(x)=1+x$. The values $f(1)$ can't take is/are:

(A) 2 **(B)** 1.75 **(C)** 150 **(D)** 1.35

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column 1 are labelled as (A), (B), (C) & (D) whereas statements in Column 2 are labeled as p, q, r, s & t. More than one choice from Column 2 can be matched with Column 1.

58. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	$f(x)=\begin{cases} e^{-1/x^2}, & x \neq 0 \\ 0, & x=0 \end{cases}$	**(p)**	First derivative exists
(B)	$f(x)=\begin{cases} e^{-1/x}, & x>0 \\ 0, & x \leq 0 \end{cases}$	**(q)**	First derivative is continuous
(C)	$f(x)=\begin{cases} e^{\frac{-1}{x-e}+\frac{1}{x-\pi}}, & x \in (e,\pi) \\ 0, & x \notin (e,\pi) \end{cases}$	**(r)**	Second derivative exists
(D)	$f(x)=\begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x=0 \end{cases}$	**(s)**	Second derivative is continuous

59. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	$f(x)=\begin{cases} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots\ldots\left[\frac{8}{x}\right]\right), & x \neq 0 \\ 9k, & x=0 \end{cases}$ The value of k such that f is continuous at x = 0 is ([.] denotes the greatest integer function)	**(p)**	1
(B)	$f(x)=\begin{cases} \left(1+xe^{-1/x^2}\sin\frac{1}{x^4}\right)^{e^{1/x^2}}, & x \neq 0 \\ k, & x=0 \end{cases}$ The value of k such that f is continuous at x = 0 is	**(q)**	2
(C)	f: $[0,\infty) \to R$; $f(x)=\begin{cases} \left(2\sin\sqrt{x}+\sqrt{x}\sin\frac{1}{x}\right)^x, & x>0 \\ k, & x=0 \end{cases}$ The value of k such that f is continuous at x = 0 is	**(r)**	3
(D)	$f:(0,\pi) \to R$; $f(x)=\begin{cases} \frac{1-\sin x}{(\pi-2x)^2}\cdot\frac{\ell n \sin x}{\ell n\left(1+\pi^2-4\pi x+4x^2\right)}; & x \neq \frac{\pi}{2} \\ k; & x=\frac{\pi}{2} \end{cases}$ The value of $8\sqrt{\lvert k \rvert}$ such that f is continuous at x = $\frac{\pi}{2}$ is	**(s)**	4

60. Let f be a polynomial of degree 4 having real coefficients satisfying

$$f'(0) = f'(1) = f'(-1) = 0 \text{ and } f(0) = 4, f''\left(\frac{1}{2}\right) = -1$$

MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	$f(x) = 0$ has	**(p)**	root at $x = 2$
(B)	$4 - f(x) = 0$ has	**(q)**	root at $x = 1$
(C)	$f'(x) + x - 1 = 0$ has	**(r)**	2 equal real roots
(D)	$x f'(x) - 4f(x) = 0$ has	**(s)**	no real roots

61. Let $f(x) = x^2 + ax + b, \forall x \in R, f(0) > 0$ & $f(x)$ has integral roots. Tangent at $\left(\frac{5}{2}, p\right)$ to $y = f(x)$ is parallel to x-axis & $g(x) = f(x+1)$.

	Column 1		Column 2
(A)	$(a+b)$ can be	**(p)**	–1
(B)	Value of [p] can be (where [.] represents greatest integer function)	**(q)**	1
(C)	Number of points where $g(\|x\|)$ is non differentiable can be	**(r)**	3
(D)	Number of points where $\|g(\|x\|)\|$ is non differentiable can be	**(s)**	–3
		(t)	5

SUBJECTIVE INTEGER TYPE

Each of the following question has an integer answer between 0 and 9.

62. $f(x) = \begin{cases} \dfrac{\sin\left(\pi \cos^2\left(\tan\left(\sin x\right)\right)\right)}{\pi x^2} &, x \neq 0 \\ k &, x = 0 \end{cases}$ .The value of k such that f is continuous at $x = 0$, is

63. If the independent variable x is changed to y, then the expression $x\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 - \dfrac{dy}{dx} = 0$ is transformed to $x\dfrac{d^2x}{dy^2} + \left(\dfrac{dx}{dy}\right)^2 = \lambda\dfrac{dx}{dy}$, then λ equals.

64. Let f and g be continuously differentiable functions such that $f(0) = 0, f'(0) = 2$ and $g(x) = f(-x + f(f(x)))$. The value of $g'(0)$ equals.

65. Let $f(x) = \begin{cases} x^2 \displaystyle\sum_{r=0}^{\left[\frac{1}{|x|}\right]} r & ; x \neq 0 \\ \dfrac{k}{2} ; & otherwise \end{cases}$ ([.] denotes the greatest integer function)

The value of k such that f become continuous at $x = 0$ is ________.

66. Let $f : R \to R$ be a continuous function such that $f(x+y) = f(x) + f(y) + f(x).f(y), \forall\ x, y \in R$. Also $f'(0) = 1$. Then $\left[\dfrac{f(4)}{f(2)}\right]$ equals ($[\bullet]$ represents greatest integer function)

67. Let $f(x) = tan^{-1} x, |x| \le 1$

$= \dfrac{\pi}{4} sgn\, x + \dfrac{x-1}{2}, |x| > 1$, (where sgn$x$ denotes signum function)

Then the value of $4f'\left(1^+\right)$ equals.

68. Let $K > 0$ and $\lambda = \lim\limits_{x \to 0} \dfrac{K\left(1 - 4\sqrt{K^2 - x^2}\right)}{x^2 \sqrt{K^2 - x^2}}$ is finite then the value of λK is______.

69. Let $f(x)$ be a differentiable function, $f(1) = 0,\ f'(1) = 2$ then the value of $\lim\limits_{x \to 1} \displaystyle\int_1^{x^2} \dfrac{x^2 sin(f(t))dt}{(x-1)^2}$ is____.

70. Let $f : [-1, 1] \to \left[\dfrac{-\pi}{4} - \tan 1, \dfrac{\pi}{4} + \tan 1\right]$ defined by $f(x) = \tan x + \tan^{-1} x$ and the derivative of $f^{-1}(x)$ at $x = 0$ is $'k'$ then the value of $\dfrac{4}{k}$ is ____.

71. The number of point where $|xf(x)| + ||x-2| - 1|$ is non-differentiable in $x \in (0, 3\pi)$, where $f(x) = \displaystyle\prod_{k=1}^{\infty} \left(\dfrac{1 + 2\cos\left(\dfrac{2x}{3^k}\right)}{3}\right)$, is_______.

72. If $f\left(\dfrac{xy}{2}\right) = \dfrac{f(x).f(y)}{2}; x, y \in R, f(1) = f'(1)$. Then, $\dfrac{f(3)}{f'(3)}$ is_______.

73. Let $f : R \to R$ be a differentiable function satisfying $f(x) = f(y) f(x-y), \forall x, y \in R$ and $f'(0) = \displaystyle\int_0^4 \{2x\} dx$, where $\{.\}$ denotes the fractional part function and $f'(-3) = \alpha e^{\beta}$. Then, $|\alpha + \beta|$ is equal to_______.

74. Let $f(x)$ is a polynomial function and $(f(\alpha))^2 + (f'(\alpha))^2 = 0$, then find $\lim\limits_{x \to \alpha} \dfrac{f(x)}{f'(x)} \left[\dfrac{f'(x)}{f(x)}\right]$, (where $[.]$ denotes greatest integer function) is_______.

75. Let $f : R \to R$ is a function satisfying $f(10 - x) = f(x)$ and $f(2-x) = f(2+x), \forall x \in R$. If $f(0) = 101$. Then, the minimum possible number of values of x satisfying $f(x) = 101, x \in [0, 25]$ is_______.

76. If $f(x)=\begin{cases}\dfrac{\left(\dfrac{\pi}{2}-\sin^{-1}\left(1-\{x\}^2\right)\right)\sin^{-1}\left(1-\{x\}\right)}{\sqrt{2}\left(\{x\}-\{x\}^3\right)} & , \quad x>0\\ k & , \quad x=0\\ \dfrac{A\sin^{-1}\left(1-\{x\}\right)\cos^{-1}\left(1-\{x\}\right)}{\sqrt{2\{x\}}\left(1-\{x\}\right)} & , \quad x<0\end{cases}$

is continuous at $x=0$, then the value of A is_______. (where $\{.\}$ denotes fractional part of x).

77. In a ΔABC, angles A,B,C are in AP. If $f(C)=\lim\limits_{A\to C}\dfrac{\sqrt{3-4\sin A\sin C}}{|A-C|}$, then $f'\left(\dfrac{\pi}{12}\right)$ is equal to_______.

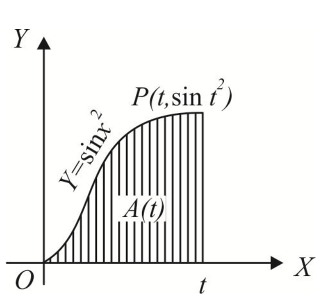
78. Let $f_1(x)$ and $f_2(x)$ be twice differentiable function, Where $F(x)=f_1(x)+f_2(x)$ and $G(x)=f_1(x)-f_2(x),\forall x\in R, f_1(0)=2$ and $f_2(0)=1$. If $f_1'(x)=f_2(x)$ and $f'_2(x)=f_1(x),\forall x\in R$, then the number of solutions of the equation $(F(x))^2=\dfrac{9x^4}{G(x)}$ is_______.

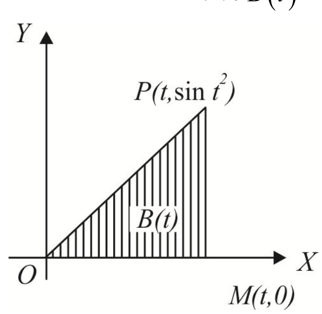
79. Let $f(x)=x\tan^{-1}\left(x^2\right)+x^4$. Let $f^k\ (x)$ denotes kth derivative of $f(x)\, w.r.t. x, k\in N$, .
If $f^{2m}(0)\neq 0, m\in N$, then m equals to_______.

80. Let $f(n)=\left[\sqrt{n}+\dfrac{1}{2}\right]$, where [.] denotes greatest integer function, $\forall n\in N$. Then $\sum\limits_{n=1}^{\infty}\dfrac{2^{f(n)}+2^{-f(n)}}{2^n}$ is equal to_______.

81. The value of $\lim\limits_{x\to\frac{\pi}{2}}\sqrt{\dfrac{\tan x-\sin\left\{\tan^{-1}(\tan x)\right\}}{\tan x+\cos^2(\tan x)}}$ is_______.

82. The figure shows two regions in the first quadrant. $A(t)$ is the area under the curve $y=\sin x^2$ from 0 to t and $B(t)$ is the area of the triangle with vertices 0, P and $M(t,0)$. If $\lim\limits_{t\to 0}\dfrac{A(t)}{B(t)}=\dfrac{1}{k}$, then k is_______.





83. Consider a parabola $y=\frac{x^2}{4}$ and the point $F(0,1)$.

Let $A_1(x_1,y_1),A_2(x_2,y_2),A_3(x_3,y_3),,..............,A_N(x_n,y_n)$, are '$n$' points on the parabola such that $x_k>0$ and $\angle OFA_k=\frac{k\pi}{2n}(k=1,2,.....,n)$. If the value of $\lim_{n\to\infty}\frac{1}{n}.\sum_{k=1}^{n}FA_k=\frac{m}{\pi}$, then m is_______.

84. The value of $\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2}}+\frac{1}{\sqrt{n^2+1}}+....+\frac{1}{\sqrt{n^2+2n}}\right)$ is_______.

85. Suppose $x_1=\tan^{-1}2>x_2>x_3>.......$ are the real numbers satisfying $\sin(x_{n+1}-x_n)+2^{-(n+1)}.\sin x_n.\sin x_{n+1}=0$ for all $n>1$ and the sequence is convergent and , $l=\lim_{n\to\infty}x_n$, the value of $4l$ is_______.

86. A function $f(x)$ satisfies the relation $f(x+y)=f(x)+f(y)+xy(x+y),\forall x,y\in R$. If $f'(0)=-1$, then $f'(3)=$

87. If $\lim_{x\to\infty}4x\left(\frac{\pi}{4}-\tan^{-1}\frac{x+1}{x+2}\right)=y^2+4y+5$, then the product of all possible value of y is_______.

88. If $\lim_{x\to 0}\frac{a\sin x-bx+cx^2+x^3}{2x^2\ln(1+x)-2x^3+x^4}$ exists and is equal to l then $a+b+c+l=$

89. If $\lim_{n\to\infty}\left(\frac{(n^3+1)(n^3+2^3)(n^3+3^3)..........(n^3+n^3)}{n^{3n}}\right)^{1/n}=4e^{\frac{\pi}{\sqrt{a}}}.e^{-b}$ (where $a,b\in N$) then $a+b$ is_______.

90. Let H_n denotes the harmonic mean of n positive integers $n+1,n+2,n+3,.....,n+n$, if $\lim_{n\to\infty}\left(\frac{H_n}{n}\right)=\frac{1}{k}$ then the value of e^k is_______.

JEE Advanced Revision Booklet	Differential Calculus-2

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. The equation $(x^2+3x+4)^2+3(x^2+3x+4)+4=x$ has :

(A) All its solutions real but not all positive **(B)** Only two of its solutions real

(C) Two of its solutions positive, two negative **(D)** None of its solution real

2. Let $f(x)$ be a differentiable function in the interval (0, 2), then the value of $\int_0^2 f(x)dx$ is :

(A) $f(c)$ where $c \in (0, 2)$ **(B)** $2f(c)$ where $c \in (0, 2)$

(C) $f'(c)$ where $c \in (0, 2)$ **(D)** None of these

For Questions 3 - 5

Consider $f(x)=x+\cos x-a$

3. Which of the following holds good for the above $f(x)$

(A) $a>1$ for which $f(x)$ has exactly one positive root

(B) $a>2$ for which $f(x)$ has exactly one –ve root

(C) $1<a<2$ $f(x)$ will have an imaginary set of roots

(D) None of these

4. Which of the following is true for $f(x)$

(A) $\sin\alpha > \dfrac{\cos\alpha-\cos\beta}{\beta-\alpha} > \cos\beta$; For $0<\alpha<\beta<\dfrac{\pi}{2}$

(B) $\sin\alpha > \dfrac{\cos\alpha-\cos\beta}{\beta-\alpha} > \sin\beta$; For $0<\alpha<\beta<\dfrac{\pi}{2}$

(C) $\tan\alpha < \dfrac{\cos\alpha-\cos\beta}{\beta-\alpha} < \tan\beta$; For $0<\alpha<\beta<\dfrac{\pi}{2}$

(D) None of these

5. The equation of tangent to $f(x)$ at $x=\dfrac{\pi}{2}$ is :

(A) $y=\dfrac{\pi}{2}-a$ **(B)** $y=\dfrac{\pi}{4}+a$ **(C)** $x+y=\dfrac{\pi}{4}$ **(D)** $x-y=\dfrac{\pi}{2}$

6. Point 'A' lies on the curve $y=e^{-x^2}$ and has the coordinate $(x,\ e^{-x^2})$ where $x>0$. Point B has the coordinates $(x, 0)$. If 'O' is the origin then the maximum area of the triangle AOB is :

(A) $\dfrac{1}{\sqrt{2e}}$ **(B)** $\dfrac{1}{\sqrt{4e}}$ **(C)** $\dfrac{1}{\sqrt{e}}$ **(D)** $\dfrac{1}{\sqrt{8e}}$

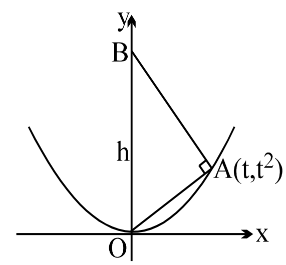
7. If a variable tangent to the curve $x^2y=c^3$ makes intercepts a, b on x and y axis respectively, then the value of a^2b is :

(A) $27c^3$ **(B)** $\dfrac{4}{27}c^3$ **(C)** $\dfrac{27}{4}c^3$ **(D)** $\dfrac{4}{9}c^3$

8. The function $f : [a, \infty) \to R$ where R denotes the range corresponding to the given domain, with rule $f(x) = 2x^3 - 3x^2 + 6$, will have an inverse provided :

(A) $a \ge 1$ **(B)** $a \ge 0$ **(C)** $a \le 0$ **(D)** $a \le 1$

9. The figure shows a right triangle with its hypotenuse OB along the y-axis and its vertex A on the parabola $y = x^2$. Let h represents the length of the hypotenuse which depends on the x-coordinate of the point A. The value of $\lim_{x \to 0} (h)$ equals



(A) 0 **(B)** 1/2
(C) 1 **(D)** 2

10. The least value of 'a' for which the equation, $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = a$ has atleast one solution on the interval $(0, \pi/2)$ is:

(A) 3 **(B)** 5 **(C)** 7 **(D)** 9

11. If $f(x) = 4x^3 - x^2 - 2x + 1$ and $g(x) = \begin{cases} Min\ \{f(t) : 0 \le t \le x\} & ; 0 \le x \le 1 \\ 3 - x & ; 1 < x \le 2 \end{cases}$ then $g\left(\frac{1}{4}\right) + g\left(\frac{3}{4}\right) + g\left(\frac{5}{4}\right)$ has the value equal to :

(A) 7/4 **(B)** 9/4 **(C)** 13/4 **(D)** 5/2

12. Given: $f(x) = 4 - \left(\frac{1}{2} - x\right)^{2/3}$, $g(x) = \begin{cases} \frac{\tan[x]}{x} & , x \ne 0 \\ 1 & , x = 0 \end{cases}$, $h(x) = \{x\}$

$k(x) = 5^{\log_2(x+3)}$, then in [0, 1], Lagranges Mean Value Theorem is NOT applicable to :

(A) f, g, h **(B)** h, k **(C)** f, g **(D)** g, h, k

where $[x]$ and $\{x\}$ denotes the greatest integer and fraction part function.

13. A curve is represented by the equations, $x = \sec^2 t$ and $y = \cot t$ where t is a parameter. If the tangent at the point P on the curve where $t = \pi/4$ meets the curve again at the point Q then | PQ | is equal to :

(A) $\frac{5\sqrt{3}}{2}$ **(B)** $\frac{5\sqrt{5}}{2}$ **(C)** $\frac{2\sqrt{5}}{3}$ **(D)** $\frac{3\sqrt{5}}{2}$

14. Let $f(x) = x^3 - 3x^2 + 2x$. If the equation $f(x) = k$ has exactly one positive and one negative solution then the value of k equals

(A) $-\frac{2\sqrt{3}}{9}$ **(B)** $-\frac{2}{9}$ **(C)** $\frac{2}{3\sqrt{3}}$ **(D)** $\frac{1}{3\sqrt{3}}$

15. Let h be a continuous twice differentiable positive function on an open interval J. Let $g(x) = ln(h(x))$ for each $x \in J$

Suppose $(h'(x))^2 > h''(x)\, h(x)$ for each $x \in J$. Then :

(A) g is increasing on J **(B)** g is decreasing on J
(C) g is concave up on J **(D)** g is concave down on J

16. Let $F(x) = \int_{\sin x}^{\cos x} e^{(1+\arcsin t)^2} dt$ in $\left[0, \frac{\pi}{2}\right]$ then :

(A) $F''(c) = 0$ for all $c \in \left(0, \frac{\pi}{2}\right)$ **(B)** $F''(c) = 0$ for some $c \in \left(0, \frac{\pi}{2}\right)$

(C) $F'(c) = 0$ for some $c \in \left(0, \frac{\pi}{2}\right)$ **(D)** $F(c) \ne 0$ for all $c \in \left(0, \frac{\pi}{2}\right)$

17. P and Q are two points on a circle of centre C and radius α, the angle PCQ being 2θ then the radius of the circle inscribed in the triangle CPQ is maximum when

(A) $\sin\theta = \frac{\sqrt{3}-1}{2\sqrt{2}}$ **(B)** $\sin\theta = \frac{\sqrt{5}-1}{2}$ **(C)** $\sin\theta = \frac{\sqrt{5}+1}{2}$ **(D)** $\sin\theta = \frac{\sqrt{5}-1}{4}$

18. Let a function f be defined as $f(x) = \begin{cases} \frac{|x-1|}{x^2+1} & \text{if } x > -1 \\ x^2 & \text{if } x \le -1 \end{cases}$

Then the number of critical point(s) on the graph of this function is/are :

(A) 4 **(B)** 3 **(C)** 2 **(D)** 1

19. Let $f(x) = ax^2 - b|x|$, where *a* and *b* are constants. Then at x = 0, f (x) has :

(A) a maxima whenever a > 0, b > 0 **(B)** a maxima whenever a > 0, b < 0

(C) minima whenever a > 0, b > 0 **(D)** neither a maxima nor minima whenever a > 0, b < 0

20. Consider $f(x) = \int_1^x \left(t + \frac{1}{t}\right) dt$ and $g(x) = f'(x)$ for $x \in \left[\frac{1}{2}, 3\right]$

If P is a point on the curve y = g(x) such that the tangent to this curve at P is parallel to a chord joining the points $\left(\frac{1}{2}, g\left(\frac{1}{2}\right)\right)$ and (3, g(3)) of the curve, then the coordinates of the point P

(A) can't be found out **(B)** $\left(\frac{7}{4}, \frac{65}{28}\right)$ **(C)** (1, 2) **(D)** $\left(\sqrt{\frac{3}{2}}, \frac{5}{\sqrt{6}}\right)$

21. $f : R \to R$

Statement 1 : $f(x) = 12x^5 - 15x^4 + 20x^3 - 30x^2 + 60x + 1$ is monotonic and surjective on R.

Statement 2 : A continuous function defined on R, if strictly monotonic has its range R.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1

(C) Statement-1 is true, statement-2 is false

(D) Statement-1 is false, statement-2 is true

For Questions 22 - 24

Consider the cubic $f(x) = 8x^3 + 4ax^2 + 2bx + a$ where a, b ∈ R.

22. For a = 1 if $y = f(x)$ is strictly increasing ∀ x ∈ R then maximum range of values of b is :

(A) $\left(-\infty, \frac{1}{3}\right]$ **(B)** $\left(\frac{1}{3}, \infty\right)$ **(C)** $\left[\frac{1}{3}, \infty\right)$ **(D)** $(-\infty, \infty)$

23. For b = 1, if $y = f(x)$ is non monotonic then the sum of all the integral values of a ∈ [1, 100], is :

(A) 4950 **(B)** 5049 **(C)** 5050 **(D)** 5047

24. If the sum of the base 2 logarithms of the roots of the cubic $f(x) = 0$ is 5 then the value of '*a*' is :

(A) – 64 **(B)** – 8 **(C)** – 128 **(D)** – 256

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

25. The points on the ellipse $4x^2+9y^2=13$ such that the rate of decrease of ordinate is equal to rate of increase in abscissa are :

(A) $\left(\frac{3}{2},\frac{2}{3}\right)$ **(B)** $\left(-\frac{3}{2},-\frac{2}{3}\right)$ **(C)** $\left(\frac{3}{2},-\frac{2}{3}\right)$ **(D)** $\left(-\frac{3}{2},\frac{2}{3}\right)$

26. If Rolle's theorem is applicable to the function *f* defined by $f(x)=\begin{cases} ax^2+b, & |x|<1 \\ 1, & |x|=1 \\ \dfrac{\lambda}{|x|} & |x|>1 \end{cases}$ $for\ x\in[-2,2]$ then :

(A) $a+b=1$ **(B)** $\lambda=a+b$ **(C)** $b=\frac{3}{2}$ **(D)** $3a+b=0$

27. $f(x)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ decreases in the region :

(A) $(-1, 0)$ **(B)** $(0, 1)$ **(C)** $(-\infty, -1)$ **(D)** $(-\infty, 1)$

28. The function $f(x)=x^{1/3}(x-1)$

(A) has 2 inflection points
(B) is strictly increasing for $x > 1/4$ and strictly decreasing for $x < 1/4$
(C) is concave down in $(-1/2, 0)$
(D) Area enclosed by the curve lying in the fourth quadrant is 9/28

29. If $\frac{x}{a}+\frac{y}{b}=1$ is a tangent to the curve $x = Kt$, $y=\frac{K}{t}$, $K > 0$ then :

(A) $a > 0,\ b > 0$ **(B)** $a > 0, b < 0$ **(C)** $a < 0, b > 0$ **(D)** $a < 0, b < 0$

30. The function $f(x)=\int_0^x \sqrt{1-t^4}\ dt$ is such that :

(A) it is defined on the interval $[-1, 1]$ **(B)** it is an increasing function
(C) it is an odd function **(D)** the point (0, 0) is the point of inflection

31. The co-ordinates of the point(s) on the graph of the function, $f(x)=\frac{x^3}{3}-\frac{5x^2}{2}+7x-4$ where the tangent drawn cut off intercepts from the co-ordinate axes which are equal in magnitude but opposite in sign, is :

(A) (2, 8/3) **(B)** (3, 7/2) **(C)** (1, 5/6) **(D)** None of these

32. If $f(x)=a^{\{a^{|x|}\operatorname{sgn} x\}}$; $g(x)=a^{[a^{|x|}\operatorname{sgn} x]}$ for $a>0,\ a\neq 1$ and $x\in R$, where { }and [ ] denote the fractional part and integral part functions respectively, then which of the following statements can hold good for the function $h(x)$, where $(\ln a)h(x)=(\ln f(x)+\ln g(x))$.

(A) 'h' is even and increasing **(B)** 'h' is odd and decreasing
(C) 'h' is even and decreasing **(D)** 'h' is odd and increasing.

33. On which of the following intervals, the function $x^{100}+\sin x-1$ is strictly increasing.

(A) $(-1, 1)$ **(B)** $(0, 1)$ **(C)** $(\pi/2, \pi)$ **(D)** $(0, \pi/2)$

34. If $f(x)=\begin{cases}3x^2+12x-1 & , -1\le x\le 2\\ 37-x & , 2<x\le 3\end{cases}$ then :

(A) $f(x)$ is increasing on [– 1, 2] **(B)** $f(x)$ is continuous on [– 1 , 3]

(C) $f'(2)$ does not exist **(D)** $f(x)$ has the maximum value at $x=2$

35. Consider the function $f(x) = x^2 - x\sin x - \cos x$ then the statements which holds good, are

(A) $f(x) = k$ has no solution for $k < -1$ **(B)** f is increasing for $x < 0$ and decreasing for $x > 0$

(C) $\lim_{x\to\pm\infty} f(x)\to\infty$ **(D)** The zeros of $f(x) = 0$ lie on the same side of the origin

36. Assume that inverse of the function f is denoted by g. Then which of the following statement hold good?

(A) If f is increasing then g is also increasing **(B)** If f is decreasing then g is increasing.

(C) The function f is injective **(D)** The function g is onto

37. For the function $f(x) = \ln(1 - \ln x)$ which of the following do not hold good?

(A) Increasing in (0, 1) and decreasing in (1, e) **(B)** Decreasing in (0, 1) and increasing in (1, e)

(C) $x = 1$ is the critical point for $f(x)$. **(D)** f has two asymptotes

38. Consider the function $f(x)=\left[\cos\left(\tan^{-1}\left(\sin(\cot^{-1}x)\right)\right)\right]^2$. Which of the following is correct?

(A) Range of f is (0, 1) **(B)** f is even

(C) $f'(0) = 0$ **(D)** The line y = 1 is asymptotes to the graph y = f(x)

39. Equation of a line which is tangent to both the curves $y = x^2+1$ and $y=-x^2$ is :

(A) $y=\sqrt{2}x+\frac{1}{2}$ **(B)** $y=\sqrt{2}x-\frac{1}{2}$ **(C)** $y=-\sqrt{2}x+\frac{1}{2}$ **(D)** $y=-\sqrt{2}x-\frac{1}{2}$

40. A function f is defined by $f(x)=\int_0^{\pi}\cos t\cos(x-t)dt$, $0\le x\le 2\pi$ then which of the following hold(s) good?

(A) $f(x)$ is continuous but not differentiable in (0, 2π)

(B) Maximum value of f is π

(C) There exists atleast one c ∈ (0, 2π) s.t. $f'(c) = 0$

(D) Minimum value of f is $-\frac{\pi}{2}$

41. Let $f(x)=\frac{x-1}{x^2}$ then which of the following is correct.

(A) f(x) has minima but no maxima

(B) f(x) increases in the interval (0, 2) and decreases in the interval (– ∞, 0) ∪ (2, ∞)

(C) f(x) is concave down in (– ∞, 0) ∪ (0, 3)

(D) $x = 3$ is the point of inflection

42. $f''(x) > 0$ for all $x \in [-3, 4]$, then which of the following are always true?

(A) $f(x)$ has a relative minimum on (–3, 4) **(B)** $f(x)$ has a minimum on [–3, 4]

(C) $f(x)$ is concave upwards on [–3, 4] **(D)** If $f(3) = f(4)$ then $f(x)$ has a critical point on [–3, 4]

43. Which of the following statements is/are **TRUE**?

(A) If f has domain [0, ∞) and has no horizontal asymptotes $\lim_{x\to\infty} f(x)\to\infty$ or $\lim_{x\to\infty} f(x)\to-\infty$

(B) If f is continuous on [–1, 1], $f(-1) = 4$ and $f(1) = 3$ then there exist a number r such that $|r| < 1$ and $f(r) = \pi$

(C) $\lim_{x\to\infty} \arcsin\left(\frac{x+1}{x}\right)$ does not exist

(D) For all values of $m \in R$ the line $y - mx + m - 1 = 0$ cuts the circle $x^2+y^2-2x-2y+1=0$ orthogonally

44. If a function f is continuous $\forall x$ and if f has a relative maximum at $(-1,4)$ and a relative minimum at $(3,-2)$, then which of the following statements can be incorrect ?

(A) The graph of f has a point of inflection somewhere between $x=-1$ and $x=3$.

(B) $f'(-1)=0$

(C) The graph of f has a horizontal tangent line at $x=3$

(D) The graph of f intersect both co-ordinate axes.

45. Which of the following function(s) cannot exist?

(A) $f''(x)>0$ for all $x \in R, f'(0)=1$ and $f'(1)=1$

(B) $f''(x)>0$ for all $x \in R, f'(0)=1$ and $f'(1)=2$

(C) $f''(x)\geq 0$ for all $x \in R, f'(0)=1$ and $f(x)\leq 100$ for all $x>0$.

(D) $f''(x)>0$ for all $x \in R, f'(0)=1$ and $f(x)\leq 1$ for all $x<0$

46. Possible integral value (s) of 'a' for which $x+\frac{a}{x^2}>2$ for every $x>0$, is (are)

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

47. Set $f(x)=ax^4+bx^3+cx^2+dx+e,\ a\neq 0$, is a function such that $x=1, x=2$ and $x=3$ are normals to the curve $y=f(x)$ such that $f(2)$ is always greater than $f(0)$, then which of the following are true for $f(x)$?

(i) $f(x)$ has 2 local maxima

(ii) there exist only one value of k such that Rolle's theorem is applicable to $f(x)$ on the interval $[0,k]$

(iii) $f(x)=0$ has two imaginary roots.

(A) only (iii) and (i) are true **(B)** (ii) is true and (iii) is false

(C) Only (i) & (ii) are true **(D)** All are true

48. Given a natural number n. Consider the function $f(x)=\frac{1}{(1-x)^n}-\frac{1}{(1+x)^n}-2nx$, then for all $0<x<1$, which of the following is / are correct

(A) $f''(x)>0$ **(B)** $f'(x)>0$ **(C)** $f(x)>0$ **(D)** None of these

49. Which of the following statement(s) is/are correct ?

(A) If $f'(x)$ and $g'(x)$ exist, $f'(x) > g'(x)$ for all real x, then the graph of $y=f(x)$ and the graph of $y=g(x)$ intersect atmost once.

(B) Let $f:(-2,2)\rightarrow R$ be defined as $f(x)=\frac{x^3}{4}-\sin \pi x+3$, then $f(c)=\frac{\pi}{2}$ for some $c\in(-2,2)$.

(C) If $f(x)=(x-\alpha)^p(x-\beta)^q, p,q>0$ the $x=c$ in Rolle's theorem divides the segment $\alpha\leq x\leq\beta$ in the ratio $p:q$.

(D) The equation $3^{x-1}+x=105$, where x is an integer, has exactly one solution.

50. For any real valued function satisfying $f'(x)-\sin x(f(x)-1)\leq 0\ \forall x\in R^+$ and $f(0)=1$ then range of $f(x)$ is $\left(-\infty,\frac{a}{2}\right]$ then a is :

(A) prime number **(B)** even number **(C)** odd number **(D)** a perfect square

51. Which of the following are true?

(A) There is no cubic curve for which the tangent lines at two distinct points coincide.

(B) If a is the number of horizontal tangents and b is the number of vertical tangents to the curve $y^2 - 3xy + 2 = 0$, then $a + b = 1$

(C) Number of integral values of a for which cubic $f(x) = x^3 + ax + 2$ is non monotonic and has exactly one real root is 2.

(D) If $f(x)$ is strictly increasing, then it is one-one.

52. If $\frac{f(x+y)+f(x-y)}{2} = f(x)f(y)$ for $\forall x, y \in R$ and $f(0) \neq 0$ then

(A) $f(2) = f(-2)$ **(B)** $f(3) + f(-3) = 0$

(C) $f'(2) + f'(-2) = 0$ **(D)** $f'(3) = f'(-3)$

53. If $f(x) = \left(\lim_{x\to\infty} \frac{\log x^n - [x]}{[x]}\right)^{[x^5+2]}$ Where [ ] represents greater integer function, then

(A) $f(x)$ is not differentiable for $x = n^{1/5}$, $n \in I$ **(B)** $f'(x) \neq 0$ for $-1 < x < 1$

(C) $f(2) = 1$ **(D)** $f'(-2) = -1$

54. If P is a point on the curve $5x^2 + 3xy + y^2 = 2$ and O is the origin, then OP has

(A) minimum value $\frac{1}{2}$ **(B)** minimum value $\frac{2}{\sqrt{11}}$

(C) maximum value $\sqrt{11}$ **(D)** maximum value 2

55. The interval to which b may belong so that the derivative of function $f(x) = \left(1 - \sqrt{\frac{21-4b-b^2}{b+1}}\right)x^3 + 5x + \sqrt{6}$ is always positive :

(A) $[-7,-1)$ **(B)** $[-6,-2]$ **(C)** $[2,5/2]$ **(D)** $[2,3]$

56. Let $g: R \to R$ be a differentiable function satisfying $g(x) = g(y)g(x-y) \forall x, y \in R$ and $g'(0) = a$ and $g'(3) = b$ then $g'(-3)$ is

(A) $\frac{a^2}{b}$ **(B)** $\frac{g'(3)a}{g'(0)}$ **(C)** $\frac{b}{a}$ **(D)** $\frac{g'(0)a}{g'(3)}$

57. If $1 \geq a \geq b \geq c \geq 0$ and 'λ' is a real root of the equation $x^3 + ax^2 + bx + c = 0$ then the value of $|\lambda|$ can be

(A) 1 **(B)** $\frac{3}{2}$ **(C)** 2 **(D)** $\frac{1}{2}$

58. If $x^2 + y^2 = 1, |x|, |y| \leq 1$ then minimum value of $\frac{kx^2}{y^2} + \frac{1}{k}\left(\frac{y^2 + kx^2}{x^2}\right)$, where $k > 0$ is

(A) 2 **(B)** 3 **(C)** 4 **(D)** None of these

59. If $\left|f''(x)\right| \le 1, \forall x \in R$ and $f(0) = 0 = f'(0)$, then which of the following can be true ?

(A) $f\left(-\frac{1}{2}\right) = -\frac{1}{5}$ **(B)** $f(-2) = -5$ **(C)** $f\left(\frac{1}{2}\right) = -\frac{1}{5}$ **(D)** $f(1) = 0$

60. If $f(x) = \begin{cases} x^2 + 3 + \log_{0.5}\log_2[k+3], & -1 \le x < 0 \\ x^2 + 3x + 2, & 0 \le x \le 1 \end{cases}$, (where $[.]$ denotes the greatest integer function) has minimum value at $x = 0$, then

(A) $k \in [2, 5)$ **(B)** $k \in [-2, 1)$ **(C)** $k \in [-1, 2]$ **(D)** $k \in [-1, 2)$

61. $f(x)$ and $g(x)$ are quadratic polynomials and $|f(x)| \ge |g(x)|, \forall x \in R.$ Also $f(x) = 0$ have real roots. Then the equation $h(x)h''(x) + (h'(x))^2 = 0$ (where $\left(\text{where } h(x) = f(x)g(x)\right)$.

(A) Has 6 roots **(B)** Has 4 distinct roots

(C) Has exactly one pair of equal roots **(D)** Has two pair of equal roots

62. If x_1 and x_2 are positive numbers between 0 and 1, then which of the following is/are true?

(A) $\sin\left(\frac{x_1 + x_2}{2}\right) \le \frac{\sin x_1 + \sin x_2}{2}$ **(B)** $\tan\left(\frac{x_1 + x_2}{2}\right) \le \frac{\tan x_1 + \tan x_2}{2}$

(C) $\log\left(\frac{x_1 + x_2}{2}\right) \le \frac{\log x_1 + \log x_2}{2}$ **(D)** $\left(\frac{x_1 + x_2}{2}\right)^2 \le \frac{x_1^2 + x_2^2}{2}$

63. If $f'(x) > f(x)$ for all $x \ge 1$ and $f(1) = 0$, then

(A) $e^x f(x)$ is a decreasing function **(B)** $e^{-x} f(x)$ is an increasing function

(C) $f(x) > 0$ for all $x \in (1, \infty)$ **(D)** $f(x) < 0$ for all $x \in [1, \infty)$

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column 1 are labelled as (A), (B), (C) & (D) whereas statements in Column 2 are labeled as p, q, r, s & t. More than one choice from Column 2 can be matched with Column 1.

64. Inequalities with intervals given such that inequalities are valid :

	Column 1		**Column 2**
(A)	$\frac{x}{1+x} < ln(1+x)$	**(p)**	$(0, \infty)$
(B)	$x - \frac{x^2}{2} < ln(1+x)$	**(q)**	$(-1, 0)$
(C)	$ln(1+x) < x$	**(r)**	$(1, \infty)$
(D)	$\frac{1}{ln(1+x)} - \frac{1}{x} < 1$	**(s)**	$(-1, 0) \cup (0, 1)$

65. $h(x)=3f\left(\frac{x^2}{3}\right)+f(3-x^2)\forall\ x\in(-3,4)$ where $f''(x)>0\ \forall\ x\ \in(-3,4)$ then

	Column 1		**Column 2**
(A)	$h(x)$ is increasing	**(p)**	$x\in\left(\frac{3}{2},4\right)$
(B)	$h(x)$ is decreasing	**(q)**	$x\in\left(0,\frac{3}{2}\right)$
(C)	The least and greatest value of x^2+y^2-xy where x, y connected by $x^2+4y^2=4$	**(r)**	$\frac{5-\sqrt{13}}{2},\frac{5+\sqrt{13}}{2}$
(D)	$f(x)=2x^3-9x^2+12x+6$. The global minima of $f(x)$ in $(1,3)$.	**(s)**	$x=2$

66. MATCH THE FOLLOWING:

	Column 1		**Column 2**
(A)	The value of a for which $x^3+3(a-7)x^2+3(a^2-9)x-2$ has a +ve point of maxima	**(p)**	9/8
(B)	The minimum value of μ for which $x^3-\lambda x^2+\mu x-6=0$ has real and positive roots	**(q)**	$(-\infty,-3)\cup\left(3,\frac{29}{7}\right)$
(C)	The maximum value of $f(x)=\sin x+\cos 2x$ for $x\in[0,2\pi]$	**(r)**	$\left(3(6)^{\frac{2}{3}},\infty\right)$
(D)	The set of values of x for which $\log(1+x)\le x$	**(s)**	$(-3, 3)$
		(t)	$(-1, \infty)$

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

67. If the difference between the greatest and least values of the function $f(x)=\left(3-\sqrt{4-x^2}\right)^2+\left(1+\sqrt{4-x^2}\right)^3$ is p, then find $\left[\frac{p}{2}\right]$. (where [.] represents greatest integer function).

68. $f(x)$ is a fifth order polynomial in 'x' with every root of $f(x)=0$ is real and distinct. Find the number of real roots of $f''(x)f(x)-(f'(x))^2=0$.

69. Let $f(x)=30-2x-x^3$, then find the number of positive integral values of x which satisfies $f(f(f(x)))>f(f(-x))$.

70. If $|\ln x| = px$ has exactly three distinct solutions, then find $[p]$ (where [.] denote greater integer function).

71. If $f : R \to R$ is a monotonic, differentiable real valued function, a, b are two real numbers and $\int_a^b (f(x)+f(a))(f(x)-f(a))\,dx = k\int_{f(a)}^{f(b)} x(b-f^{-1}(x))dx$, then the value of k is _________.

72. A line passing through $(21,30)$ and normal to the curve $y = 2\sqrt{x}$. If m is slope of the normal then $m+6=$

73. Let $P = x^3 - \frac{1}{x^3}, Q = x - \frac{1}{x}$ and $'a'$ is the minimum value of $\frac{P}{Q^2}$. Then the value of $[a]$ is _________. (where $[x] =$ the greatest integer $\le x$).

74. If θ is the angle of intersection of curves $y = \left[|sin\ x| + |cos\ x|\right]\ and\ x^2 + y^2 = 5$.
Then the value of $|\tan\theta|$ is _________. (where [.] denotes G.I.F.)

75. Let $f : [0,\infty) \to R$ be a continuous, strictly increasing function such that $f^3(x) = \int_0^x tf^2(t)dt$. If a normal is drawn to the curve. $y = f(x)$ with gradient $\frac{-1}{2}$, then find the intercept made by it on the y-axis.

76. The smallest positive integral value of p for which the function $f(x) = 6px - p\sin 4x - 5x - \sin 3x$ is monotonic increasing and has no critical points on R is :

77. l_1 and l_2 are lengths of side of two variable squares S_1 and S_2 respectively for $l_1 = l_2 + l_2^3 + 6$ at $l_2 = 1$. If rate of change of area of S_2 with respect to area of S_1 is equal to $\frac{1}{8m}$, then $m =$.

78. If the function $\int_0^x f(t)dt \to 5$ as $|x| \to 1$, where f is continuous then the number of integers in the range of p so that the equation $2x + \int_0^x f(t)dt = p$ has roots of opposite sign in $(-1,1)$.

79. Let $f(x)$ be a function such that its derivative $f'(x)$ is continuous in $[a,b]$ and derivable in (a,b). Consider a function $\phi(x) = f(b) - f(x) - (b-x)f'(x) - (b-x)^2 A$. If Rolle's theorem is applicable to $\phi(x)$ on $[a,b]$ and for some $c \in (a,b), \phi'(c) = 0$ and $f(b) = f(a) + (b-a)f'(a) + \lambda f''(c)(b-a)^2$ then 6λ is equals to :

80. The values of parameter a such that the line $\left(\log_2\left(1+5a-a^2\right)\right)x - 5y - \left(a^2-5\right) = 0$ is a normal to the curve $xy = 1$, may lie in the interval (c,d) then $c-d$ is equals to :

81. A polynomial function $P(x)$ of degree 5 with leading coefficient one, increases in the interval $(-\infty,1)$ and $(3,\infty)$ and decreases in the interval $(1,3)$. Given that $P(0) = 4$ and $P'(2) = 0$. Find the value $P'(6)$.

82. The circle $x^2 + y^2 = 1$ cuts the x-axis at P and Q. Another circle with center at Q and variable radius intersect the first circle at R above the x-axis and the line segment PQ at S, the maximum area of the triangle QSR is A, thens $[2A]$ is ________ $([.]\text{is } G.I.F)$

83. A wire $20cm$ long be divided into two parts, if one part is to be bent into a circle, the other part is to be bent into a square and the two plane figures are to have areas the sum of which is maximum, then side length of square is____.

84. If $\int_e^x t\, f(t)\,dt = \sin x - \cos x - \frac{x^2}{2}$ for all $x \in R$ then value of $\left(-2f\left(\frac{\pi}{6}\right)\right)$ is ____.

85. If $f(x+h) = f(x) + hf'(x+\theta h), 0 < \theta < 1,$ the value of $4\theta,$ when $f(x) = Ax^2 + Bx + C$ is______________

86. In the interval (a,b) there exists at least one point $c,$ for any two differentiable function f and g such that $\begin{vmatrix} f(a) & f(b) \\ \phi(a) & \phi(b) \end{vmatrix} = \lambda^2 (b-a) \begin{vmatrix} f(a) & f'(c) \\ \phi(a) & \phi'(c) \end{vmatrix},$ then sum of absolute value of λ is______.

87. $y = \tan^{-1}(\cot x) + \cot^{-1}(\tan x),\ \frac{\pi}{2} < x < \pi,$ then $\left(\frac{-dy}{dx}\right)$ is equal to_______.

88. The third derivative of a function $f(x)$ vanishes for all $x.$ If $f(0) = 1,\ f'(1) = 2$ and $f''(1) = -1,$ then find $f'(x)$ at $x = 3.$

89. A lane of width $27m$ runs at right angle out of a road of $64m.$ The maximum length of a pole which can be carried from the road to the lane keeping it horizontal is $L,$ then $\sqrt[3]{L}$ equals to __________

90. If $a^2 + b^2 = 1$ and u is the minimum value of $\frac{b+1}{a+b-2}$ then find the value of $u^2.$

91. Let f be a differentiable function on R and satisfying $f(x) = -\left(x^2 - x + 1\right)e^x + \int_0^x e^{x-y}.f'(y)\,dy.$

If $f(1) + f'(1) + f''(1) = ke,$ where $k \in N,$ then find $k.$

92. Let $P(x_0, y_0)$ be a point on the curve $C:\left(x^2 - 11\right)(y+1) + 4 = 0$ where $x_0, y_0 \in N.$ If area of the triangle formed by the normal drawn to the curve $'C'$ at P and the co-ordinate axes is $\left(\frac{a}{b}\right), a, b \in N$ then the least value of $(a - 6b).$

93. Let f be a twice differentiable function defined in $[-3,3]$ such that $f(0) = -4, f'(3) = 0, f'(-3) = 12$ and $f''(x) \geq -2 \forall x \in [-3,3].$ If $g(x) = \int_0^x f(t)dt$ then find maximum value of $g(x),$ in [–3, 3]

94. If $9 + f''(x) + f'(x) = x^2 + f^2(x),$ where $f(x)$ is twice differentiable function such that $f''(x) \neq 0 \forall x \in R$ and let P be the point of maxima of $f(x)$ then find the number of tangents which can be drawn from P to the circle $x^2 + y^2 = 9.$

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. If $f(x)=\lim_{n\to\infty} e^{x\tan(1/n)\log(1/n)}$, and $\int \frac{f(x)}{\sqrt[3]{\sin^{11}x\cos x}}dx = g(x)+C$ (C being the constant of integration). Then :

(A) $g\left(\frac{\pi}{4}\right)=\frac{3}{2}$ **(B)** $g(x)$ is continuous for all

(C) $g\left(\frac{\pi}{4}\right)=-\frac{15}{8}$ **(D)** $g\left(\frac{\pi}{4}\right)=-\frac{1}{2}$

2. If $f(x)=\lim_{n\to\infty}\left[2x+4x^3+........+2nx^{2n-1}\right](0<x<1)$ then $\int f(x)dx$ is equal to :

(A) $-\sqrt{1-x^2}+c$ **(B)** $\frac{1}{\sqrt{1-x^2}}+c$ **(C)** $\frac{1}{x^2-1}+c$ **(D)** $\frac{1}{1-x^2}+c$

3. The integral $\int \frac{\sec^{3/2}\theta-\sec^{1/2}\theta}{2+\tan^2\theta}\tan\theta d\theta$ is :

(A) $\sqrt{2}\tan^{-1}\left(\frac{\sec\theta+1}{\sqrt{2\sec\theta}}\right)+C$ **(B)** $\frac{1}{\sqrt{2}}\log_e\left|\frac{\sec\theta-\sqrt{2\sec\theta}+1}{\sec\theta+\sqrt{2\sec\theta}+1}\right|+C$

(C) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sec\theta+1}{\sqrt{2\sec\theta}}\right)+C$ **(D)** $\sqrt{2}\log_e\left|\frac{\sec\theta-\sqrt{2\sec\theta}+1}{\sec\theta+\sqrt{2\sec\theta}+1}\right|+C$

4. If $f(x)=\lim_{n\to\infty}\frac{x^n-x^{-n}}{x^n+x^{-n}}, 0<x<1, n\in N$ then $\int(\sin^{-1}x)f(x)dx$ is equal to :

(A) $-\left[x\sin^{-1}x+\sqrt{1-x^2}\right]+C$ **(B)** $x\sin^{-1}x+\sqrt{1-x^2}+C$

(C) $\frac{x^2}{2}+C$ **(D)** $\frac{1}{2}\left(\sin^{-1}x\right)^2+C$

5. If $I_n=\int\cot^n x dx$, then $I_0+I_1+2(I_2+I_3+..........+I_8)+I_9+I_{10}=$

(A) $-\sum_{k=1}^{9}\frac{\cot^k x}{k}$ **(B)** $\sum_{k=1}^{9}\frac{\cot^k x}{k!}$ **(C)** $\sum_{k=1}^{10}\frac{\cot^k x}{10}$ **(D)** $-\sum_{k=1}^{10}k\cot^k x$

6. If $\int\frac{\left(sin^{3/2}\theta+cos^{3/2}\theta\right)d\theta}{\sqrt{sin^3\theta\, cos^3\theta\, sin(\theta+\alpha)}}=a\sqrt{cos\alpha\, tan\theta+sin\alpha}+b\sqrt{cos\alpha+sin\alpha\, cot\theta}+c$ then :

(A) $a=2\sec\alpha, b=2\operatorname{cosec}\alpha, c\in R$ **(B)** $a=2\sec\alpha, b=-2\operatorname{cosec}\alpha, c\in R$

(C) $a=-2\sec\alpha, b=2\operatorname{cosec}\alpha, c\in R$ **(D)** $a=2\operatorname{cosec}\alpha, b=2\sec\alpha, c\in R$

7. If $\int \frac{\cos^2 x + \sin 2x}{(2\cos x - \sin x)^2}\, dx = \frac{\cos x}{2\cos x - \sin x} + ax + b\ln|2\cos x - \sin x| + c$, then :

(A) $a = \frac{1}{5}, b = \frac{2}{5}$ **(B)** $a = \frac{1}{5}, b = -\frac{2}{5}$ **(C)** $a = -\frac{1}{5}, b = \frac{2}{5}$ **(D)** $a = -\frac{1}{5}, b = -\frac{2}{5}$

Paragraph for Questions 8 - 10

Let us consider the integrals of the following forms $f\left(x, \sqrt{mx^2 + nx + p}\right)^{1/2}$

Case I: If $m > 0$, then put $\sqrt{mx^2 + nx + c} = u \pm x\sqrt{m}$ Case II: If p > 0, then put $\sqrt{mx^2 + nx + c} = ux \pm \sqrt{p}$

Case III: If quadratic equation $mx^2 + nx + p = 0$ has real roots α and β then put $mx^2 + nx + p = (x - \alpha)u \, or \, (x - \beta)u$

8. If $\int \frac{dx}{x - \sqrt{9x^2 + 4x + 6}}$ to evaluate I, one of the most proper substitution could be :

(A) $\sqrt{9x^2 + 4x + 6} = u \pm 3x$ **(B)** $\sqrt{9x^2 + 4x + 6} = 3u \pm x$

(C) $x = \frac{1}{t}$ **(D)** $9x^2 + 4x + 6 = \frac{1}{t}$

9. $\int \frac{\left(x + \sqrt{1 + x^2}\right)^{15}}{\sqrt{1 + x^2}} dx$ is equal to :

(A) $\frac{\left(x + \sqrt{1 + x^2}\right)^{16}}{10} + c$ **(B)** $\frac{1}{15\left(\sqrt{1 + x^2} + x\right)} + c$

(C) $\frac{15}{\left(\sqrt{1 + x^2} - x\right)} + c$ **(D)** $\frac{\left(\sqrt{1 + x^2} + x\right)^{15}}{15} + c$

10. To evaluate $\int \frac{dx}{(x - 1)\sqrt{-x^2 + 3x - 2}}$ one of the most suitable substitution could be :

(A) $\sqrt{-x^2 + 3x - 2} = u$ **(B)** $\sqrt{-x^2 + 3x - 2} = \left(ux\sqrt{2}\right)$

(C) $\sqrt{-x^2 + 3x - 2} = u(1 - u)$ **(D)** $\sqrt{-x^2 + 3x - 2} = u(x - 2)$

11. If $I_n = \int \frac{dx}{\left(x^2 + a^2\right)^n}$, where $n \in N$ and $n > 1$. If I_n and I_{n-1} are related by the relation $PI_n = \frac{x}{\left(x^2 + a^2\right)^{n-1}} + QI_{n-1}$. Then P and Q are respectively given by :

(A) $(2n - 1)a^2, 2n - 3$ **(B)** $2a^2(n - 1), 2n - 1$

(C) $a^2(n + 1), 2n + 3$ **(D)** $a^2, a^2(n + 1)$

12. $\int \frac{(ax^2-b)dx}{x\sqrt{c^2x^2-(ax^2+b)^2}} =$

(A) $\sin^{-1}\left(\frac{ax+bx^2}{c}\right)+k$ **(B)** $\tan^{-1}\left(\frac{a+bx^2}{cx}\right)+k$

(C) $\sin^{-1}\left(\frac{ax^2+b}{cx}\right)+k$ **(D)** $\tan^{-1}\left(ax^2+bx+c\right)+k$

13. Let $f:\left[0,\frac{\pi}{2}\right]\to R$ be such that $f(0)=3$ and $f'(x)=\frac{1}{1+\cos x}$. If $a<f\left(\frac{\pi}{2}\right)<b$, then a and b can be

(A) $\frac{\pi}{2},\pi$ **(B)** 3, 4

(C) $3+\frac{\pi}{4},3+\frac{\pi}{2}$ **(D)** $3+\frac{\pi}{2},3+\frac{3\pi}{4}$

Paragraph for Questions 14 - 18

Evaluation of indefinite integral with the help of specific substitution:

In general if we have an integral of type $\int f\left(g(x)\right)g'(x)dx$, we substitute $g(x)=t \Rightarrow g'(x)dx=dt$ and the integral becomes $\int f(t)dt$. Some of the substitution can be guessed by keen observation of the nature of given integrand. For example, we have $\frac{d}{dx}\left(x+\frac{1}{x}\right)=1-\frac{1}{x^2}$. So if the integrand is of the type $f\left(x+\frac{1}{x}\right)\left(1-\frac{1}{x^2}\right)$, we can substitute $x+\frac{1}{x}=t$.

Some more similar forms are given below

For integral $\int f\left(x-\frac{a}{x}\right)\left(1+\frac{a}{x^2}\right)dx$, put $x-\frac{a}{x}=t$ For integral $\int f\left(x+\frac{a}{x}\right)\left(1-\frac{a}{x^2}\right)dx, put\ x+\frac{a}{x}=t$

For integral $\int f\left(x^2-\frac{a}{x^2}\right)\left(x+\frac{a}{x^3}\right)dx, put\ x^2-\frac{a}{x^2}=t$ For integral $\int f\left(x^2+\frac{a}{x^2}\right)\left(x-\frac{a}{x^3}\right)dx, put\ x^2+\frac{a}{x^2}=t$

Many integrands can be brought into above forms by suitable reductions or transformations

14. $\int \frac{x^2+1}{x^4+1}dx =$

(A) $\frac{1}{\sqrt{2}}\tan^{-1}\frac{x^2-1}{\sqrt{2}x}+C$ **(B)** $\sin^{-1}\frac{x^2+1}{\sqrt{2}x}+C$

(C) $\frac{1}{2}\log\frac{\sqrt{2}x+1}{\sqrt{2}x-1}+C$ **(D)** $x^2+\frac{1}{x^2}+C$

15. $\int \frac{x^2-1}{\left(x^4+3x^2+1\right)\tan^{-1}\left(x+\frac{1}{x}\right)}dx =$

(A) $\tan^{-1}\left(x+\frac{1}{x}\right)+C$ **(B)** $\left(x+\frac{1}{x}\right)\tan^{-1}\left(x+\frac{1}{x}\right)+C$

(C) $\ln\left|\tan^{-1}\left(x+\frac{1}{x}\right)\right|+C$ **(D)** $\frac{1}{2}\ln\left|x+\frac{1}{x}\right|+C$

16. $\int \frac{x^4-2}{x^2\sqrt{x^4+x^2+2}}dx=$

(A) $\sqrt{x^2+1+\frac{1}{x^2}}+C$ **(B)** $\sqrt{x^2+1+\frac{2}{x^2}}+C$

(C) $\sqrt{x^2+\frac{1}{x^2}}+C$ **(D)** $\sqrt{x^2+\frac{2}{x^2}}+C$

17. Anti-derivative of $\frac{x-1}{(x+1)\sqrt{x^3+x^2+x}}$ is :

(A) $tan^{-1}\left(x+\frac{1}{x}+1\right)$ **(B)** $tan^{-1}\sqrt{x+\frac{1}{x}+1}$ **(C)** $2\,tan^{-1}\sqrt{x+\frac{1}{x}+1}$ **(D)** $\sqrt{x+\frac{1}{x}+1}$

18. The derivative of $x^{-4}+x^{-5}$ is $-\left(4x^{-5}+5x^{-6}\right)$. So, $\int \frac{5x^4+4x^5}{\left(x^5+x+1\right)^2}dx=$

(A) $x^5+x+1+C$ **(B)** $\frac{1}{x^5+x+1}+C$ **(C)** $x^{-4}+x^{-5}+C$ **(D)** $\frac{x^5}{x^5+x+1}+C$

19. Let $f(xy)=f(x).f(y), \forall x>0, y>0$ and $f(1+x)=1+x\{1+g(x)\}$, where $\lim_{x\to 0} g(x)=0$, then $\int \frac{f(x)}{f'(x)}dx$ is :

(A) $\frac{x^2}{2}+c$ **(B)** $\frac{x^3}{3}+c$ **(C)** $\frac{x^2}{3}+c$ **(D)** None of these

20. If $\int \frac{dx}{x^2\left(x^n+1\right)^{(n-1)/n}}=-\left[f(x)\right]^{1/n}+C,$ then $f(x)$ is

(A) $\left(1+x^n\right)$ **(B)** $1+x^{-n}$ **(C)** x^n+x^{-n} **(D)** None of these

21. If $I_{m,n}=\int \cos^m x \sin nx dx,$ then $7I_{4,3}-4I_{3,2}=$

(A) Constant **(B)** $-cos^2 x+C$ **(C)** $-cos^4 x\,cos\,3x+C$ **(D)** $cos\,7x-cos\,4x+C$

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

22. The value of the integral $\int e^{\sin^2 x}\left(\cos x+\cos^3 x\right)\sin x dx$ is :

(A) $\frac{1}{2}e^{\sin^2 x}\left(3-\sin^2 x\right)+c$ **(B)** $e^{\sin^2 x}\left(1+\frac{1}{2}\cos^2 x\right)+c$

(C) $e^{\sin^2 x}\left(3\cos^2 x+2\sin^2 x\right)+c$ **(D)** $e^{\sin^2 x}\left(2\cos^2 x+3\sin^2 x\right)+c$

23. $I=\int \frac{dx}{(\sin x-2\cos x)(2\cos x+\sin x)}$ is equal to

(A) $\log_e \sqrt[4]{\frac{\tan x-2}{\tan x+2}}+c$ **(B)** $\frac{1}{4}\log_e\left|\frac{\sin x-2\cos x}{\sin x+2\cos x}\right|+c$

(C) $-\frac{1}{4}\log_e\left|\frac{2\sin x+\cos x}{\sin x+2\cos x}\right|+c$ **(D)** None of these

24. If $\int \sqrt{\operatorname{cosec} x+1}\, dx = k fog(x)+c$, where k is a real constant, then :

(A) $k=-2, f(x)=cot^{-1}x, g(x)=\sqrt{cosec x-1}$ **(B)** $k=-2, f(x)=tan^{-1}x, g(x)=\sqrt{cosec x-1}$

(C) $k=2, f(x)=tan^{-1}x, g(x)=\frac{cot\, x}{\sqrt{cosec x-1}}$ **(D)** $k=2, f(x)=cot^{-1}x, g(x)=\frac{cot\, x}{\sqrt{cosec x+1}}$

25. If $f'(x)=\frac{1}{-x+\sqrt{x^2+1}}$ and $f(0)=\frac{-(1+\sqrt{2})}{2}$, then the value of $f(5)$ will be :

(A) More than 12 **(B)** Loss than 5 **(C)** More than 5 **(D)** None of these

26. Let $f(x)=\frac{1}{4-3\cos^2 x+5\sin^2 x}$ and its anti-derivative $F(x)=\frac{1}{3}\tan^{-1}(g(x))+c$, then :

(A) $g(x)$ is equal to 3tanx **(B)** $g\left(\frac{\pi}{4}\right)$ is equal to 3

(C) $g'\left(\frac{\pi}{3}\right)$ is equal to 6 **(D)** $g'\left(\frac{\pi}{3}\right)$ is equal to 12

27. If $\int \frac{\sin x-\cos x}{(\sin x+\cos x)\sqrt{\sin x\cos x+\sin^2 x\cos^2 x}}dx=\operatorname{cosec}^{-1}(g(x))+c\,\forall x\in R$, then

(A) $g(x)=1+\sin 2x$ **(B)** $g(x)=1-\sin 2x$

(C) $g(x)\geq 0$ **(D)** $-1\leq g(x)\leq 1$

28. If $x^2\neq(n\pi-1)\forall n\in N$, then $I=\int \frac{x\sqrt{2\,sin(x^2+1)-sin\,2(x^2+1)}}{2\,sin(x^2+1)+sin\,2(x^2+1)}dx$ is equal to :

(A) $log\left|sec\frac{(x^2+1)}{2}\right|+c$ **(B)** $-log\left|cos\frac{(x^2+1)}{2}\right|+c$

(C) $log\left|tan\frac{(x^2+1)}{2}\right|+c$ **(D)** $-log\left|cot\frac{(x^2+1)}{2}\right|+c$

29. Let $f(x)=\{b^2+(a-1)b+2\}x-\int(\sin^2 x+\cos^4 x)dx$ be an increasing function of $x\in R$ and $b\in R$, then a can take value(s) :

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4

30. If $\int \cos ec^2 x\left(\cos x+\sqrt{\cos 2x}\right)dx = \cot x \log\left(\cos x+\sqrt{\cos 2x}\right)+P\left(\cos ec^2 x-2\right)^t+Q\left(x+\cot x\right)+c$, , then

(A) $t = P+\frac{Q}{2}$ **(B)** $P+Q=0$ **(C)** $P \neq Q \neq t$ **(D)** $P-\frac{Q}{2}=t$

31. $\int \frac{\sin 2x}{8\sin^2 x+17\cos^2 x}dx$ is equal to :

(A) $-\frac{1}{9}\log_e\left(8\sin^2 x+17\cos^2 x\right)+c$ **(B)** $\log_e \frac{1}{\sqrt[9]{8+9\cos^2 x}}+c$

(C) $\log_e \frac{1}{\sqrt[9]{17-9\sin^2 x}}+c$ **(D)** None of these

32. $I=\int \sec^3\left(2\theta\right)d\theta$ is equal to :

(A) $\frac{1}{2}(\sec\theta\tan\theta)+\log_e\sqrt{\sec\theta+\tan\theta}+c$ **(B)** $\frac{1}{4}(\sec 2\theta\tan 2\theta)+\frac{1}{2}\log_e\sqrt{\sec 2\theta+\tan 2\theta}+c$

(C) $\frac{1}{4}(\sec 2\theta\tan 2\theta)+\log_e\sqrt[4]{\sec 2\theta+\tan 2\theta}+c$ **(D)** None of these

33. $I_1=\int 2^x dx = p(x)+c_1$ and $I_2=\int\left(\frac{1}{2}\right)^x dx = m(x)+c_1$ then $p(x)-m(x)$ is equal to

(A) $\{\log_e(2)\}\left(2^x-2^{-x}\right)$ **(B)** $\{\log_e 2\}\left(2^x+2^{-x}\right)$

(C) $\frac{1}{\log_e 2}\left(2^x+2^{-x}\right)$ **(D)** $\log_2 e\left(2^x+2^{-x}\right)$

34. If $\int \frac{dx}{p^2\sin^2 x+r^2\cos^2 x}=\frac{1}{12}\tan^{-1}(3\tan x)+c$, then the value of psinx + rcosx can be :

(A) $\frac{6}{\sqrt{5}}$ **(B)** $\sqrt{5}$ **(C)** $6\sqrt{3}$ **(D)** –4

35. $I=\int\frac{dx}{\left(1+\sqrt{x}\right)^8}$ is equal to:

(A) $\frac{-1}{21\left(1+\sqrt{x}\right)^6}\left(\frac{6\sqrt{x}}{1+\sqrt{x}}+1\right)+c$ **(B)** $\frac{-1}{21\left(1+\sqrt{x}\right)^7}\left(7\sqrt{x}+1\right)+c$

(C) Either (A) and (B) **(D)** None of these

36. If $\int\frac{\sin x}{\sin(x-\alpha)}dx = Ax+B\log\sin(x-\alpha)+c$, then

(A) $A=\sin\alpha$ **(B)** $B=\sin\alpha$ **(C)** $A=\cos\alpha$ **(D)** $B=\cos\alpha$

37. If $\int(\sin 3\theta+\sin\theta)e^{\sin\theta}\cos\theta d\theta=\left(A\sin^3\theta+B\cos^2\theta+C\sin\theta+D\cos\theta+E\right)e^{\sin\theta}+F$ then :

(A) B = 12 **(B)** D = 0 **(C)** B = –12 **(D)** None of these

38. $\int \frac{x-1}{(x+1)\sqrt{x^3+x^2+x}}dx \ \forall\ x>0$ is equal to :

(A) $2\tan^{-1}\sqrt{x^2+\frac{1}{x^2}+1}+c$ **(B)** $\tan^{-1}\sqrt{x+\frac{1}{x}+1}+c$

(C) $2\tan^{-1}\sqrt{x+\frac{1}{x}+1}+c$ **(D)** $2\sec^{-1}\sqrt{x+\frac{1}{x}+2}+c$

39. If $\int \frac{x+\left(cos^{-1}3x\right)^2}{\sqrt{1-9x^2}}dx = P\sqrt{1-9x^2}+Q\left(cos^{-1}3x\right)^3+c$ then :

(A) $P=-\frac{1}{9}$ **(B)** $Q=-\frac{3}{8}$ **(C)** $P=\frac{3}{8}$ **(D)** $Q=-\frac{1}{9}$

40. If $\int \frac{\sec x(2+\sec x)}{(1+2\sec x)^2}$ is $\left(\frac{\cos x+2}{\lambda\sin x}\right)^p$

(A) $\lambda=1$ **(B)** $\lambda=-1$ **(C)** $p=1$ **(D)** $p=-1$

41. A function $f(x)$ continuous on R and periodic with 2π satisfies $f(x)+(\sin x)f(x+\pi)=\sin^2 x$ then,

(A) $f(x)=\frac{\sin^2 x\left(1+\sin^2 x\right)}{(1-\sin x)}$

(B) $f(x)=\frac{\sin^2 x(1-\sin x)}{\left(1+\sin^2 x\right)}$

(C) $\int f(x)dx = x+\cos x-\frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\tan x\right)+\frac{1}{2\sqrt{2}}\ln\frac{\left(\sqrt{2}-\cos x\right)}{\left(\sqrt{2}+\cos x\right)}+c$

(D) None of these

42. $\int \frac{dx}{\left(\prod_{r=0}^{n}(x+r)\right)}$ is equal to :

(A) $\frac{1}{n!}\left[\sum_{r=0}^{n}(-1)^r.{}^nC_r\ln(x+r)\right]+c$ **(B)** $\frac{1}{n!}\left[\sum_{r=0}^{n-1}(-1)^{r-1}.\ln(x+r-1)\right]+c$

(C) $\frac{1}{n!}\ln\left(\prod_{r=0}^{n}(x+r)^{(-1)^r.{}^nC_r}\right)+c$ **(D)** $\frac{1}{n!}\left[\sum_{r=0}^{n-1}\ln(x+r-1)^{(-1)^{r-1}}\right]$

43. In a certain problem the differentiation of product $\left(f(x).g(x)\right)$ appears. One student commits mistake and differentiates as $\left(\frac{d\left(f(x)\right)}{dx}.\frac{d\left(g(x)\right)}{dx}\right)$ but he gets correct result if $f(x)=x^3$ & $g(4)=9, g(2)=-9$ & $g(0)=\frac{-1}{3}$ then :

(A) $g(x)=\frac{9}{(x-3)^3}$

(B) $\left(\frac{d}{dx}\left(f(x-3).g(x)\right)\right)_{at\ x=100}=0$

(C) $\lim\limits_{x\to 0}\frac{f(x).g(x)}{x\left(1+g(x)\right)}=0$

(D) None of these

44. If 'A' is a square matrix and e^A is defined as $e^A=I+A+\frac{A^2}{2!}+\frac{A^3}{3!}+........=\frac{1}{2}\begin{bmatrix} f(x) & g(x) \\ g(x) & f(x) \end{bmatrix}$ where $A=\begin{bmatrix} x & x \\ x & x \end{bmatrix}$ and $0<x<1$, I is an identity matrix, then:

(A) $\int\frac{g(x)}{f(x)}dx=\ln\left(e^x+e^{-x}\right)+c.$

(B) $f(x)\in R^+$ & $g(x)\in R^+$

(C) $f(x)\in R^+$ & $g(x)\in R$

(D) $\int\left(g(x)+1\right)\sin x\,dx=\frac{e^{2x}}{5}(2\sin x-\cos x)+c$

45. $I=\int\frac{\left(x^2-1\right)\sqrt{x^4+2x^3-x^2+2x+1}}{x^2(x+1)^2}dx$ is equal to (where $t=x+\frac{1}{x}$)

(A) $\sqrt{t^2+2t-3}-\ln\left(t+1+\sqrt{t^2+2t-3}\right)-\sqrt{3}\sin^{-1}\left(\frac{t+5}{2t+4}\right)+c$

(B) $\sqrt{t^2+t-3}-\ln\left(t+1+\sqrt{t^2+t-3}\right)-\sqrt{3}\sin^{-1}\left(\frac{t+5}{2t+4}\right)+c$

(C) $\sqrt{t^2+2t-3}-\ln\left(t+1+\sqrt{t^2+2t-3}\right)+\sqrt{3}\cos^{-1}\left(\frac{t+5}{2t+4}\right)+c$

(D) None of these

46. Let $f:R\to R$ be a function as $f(x)=(x-1)(x+2)(x-3)(x-6)-100$ If $g(x)$ is a polynomial of degree ≤ 3 such that $\int\frac{g(x)}{f(x)}dx$ does not contain any logarithm function and $g(-2)=10$, then:

(A) $f(x)=0$ has two real & two imaginary roots

(B) $\left(f(x)\right)_{\min}=-84$

(C) $\int\frac{g(x)}{f(x)}dx=\tan^{-1}\left(\frac{x-2}{2}\right)+c$

(D) $g(2)=-42$

47. $I_1 = \int f(x)dx$ and $I_2 = \int_0^1 f(x)dx$ where $f(x) = x^2 \ln(1-x^2)$, then:

(A) $I_1 = -\left\{\frac{x^3}{1.3} + \frac{x^5}{3.5} + \frac{x^7}{5.7} + ...\right\}$ **(B)** $I_2 = \frac{2}{3}\ln 2 - \frac{8}{9}$

(C) $I_1 = -\left\{\frac{x^5}{1.5} + \frac{x^7}{2.7} + \frac{x^9}{3.9} + ...\right\}$ **(D)** $I_2 = \frac{2}{3}\ln 2 - \frac{5}{9}$

48. Let $f: R \to R$ be a function satisfying

$f(x+2y) = f(x)e^{2y} + f(2y)e^x + x^2(1-e^{2y}) + 4y^2(1-e^x) + 4xy \forall x, y \in R$ and $f'(0) = 1$, then:

(A) $f(x) = xe^x + x^2$. **(B)** $f(x) = xe^x - x^2$.

(C) $\int f(x)dx = e^x(x-1) + \frac{x^3}{3} + c$ **(D)** $\int f(x)dx = e^x(x-1) - \frac{x^3}{3} + c$

49. If $\int\left(\frac{1}{1-x^8}\right)\left[\cos^{-1}\left(\frac{2x}{1+x^2}\right) + \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]dx$. has $p\tan^{-1} f(x)$ & $q\tan^{-1} g(x)$ terms and $x \in (-1, 1)$, then: (where p and q are constant)

(A) $f(x).g(x) = \frac{x^2-1}{\sqrt{2}}$ **(B)** $p.q = \frac{\pi^2}{64\sqrt{2}}$

(C) $p.q = \frac{\pi^2}{\sqrt{2}}$ **(D)** None of these

50. If $A = \int e^{ax}\cos bx\, dx$ and $B = \int e^{ax}\sin bx\, dx$, then which of the following may be correct?

(A) $(A^2 + B^2)(a^2 + b^2) = e^{2ax}$ **(B)** $\tan^{-1}\frac{B}{A} + \tan^{-1}\frac{b}{a} = bx$

(C) (A) and (B) both **(D)** None of these

51. If $\int \sqrt{\frac{\cos x - \cos^3 x}{(1-\cos^3 x)}}dx = f(x) + c$, then $f(x)$ is equal to:

(A) $\frac{2}{3}\sin^{-1}\left(\cos^{\frac{3}{2}} x\right)$ **(B)** $\frac{3}{2}\sin^{-1}\left(\cos^{\frac{3}{2}} x\right)$ **(C)** $\frac{2}{3}\cos^{-1}\left(\cos^{\frac{3}{2}} x\right)$ **(D)** $-\frac{2}{3}\sin^{-1}\left(\cos^{\frac{3}{2}} x\right)$

52. If $\int \frac{dx}{x^{22}(x^7-6)} = A\left\{\ln(p)^6 + 9p^2 - 2p^3 - 18p\right\} + c$, then:

(A) $A = \frac{1}{9072}$, **(B)** $p = \left(\frac{x^7-6}{x^7}\right)$ **(C)** $A = \frac{1}{54432}$, **(D)** $p = \left(\frac{x^7-6}{x^7}\right)^{-1}$

53. $f(x) = \int e^{\tan^{-1}x}(1+x+x^2)d(\cot^{-1}x)$ is equal to:

(A) $-e^{\tan^{-1}x} + C$ **(B)** $f(x)$ is decreasing **(C)** $-xe^{\tan^{-1}x} + C$ **(D)** $f(x)$ is increasing

54. If the anti-derivative of $\dfrac{x^3}{\sqrt{1+2x^2}}$ which passes through (1, 2) is $\dfrac{1}{m}\left(1+2x^2\right)^{1/2}\left(x^2-1\right)+c.$ Then :

(A) $c=6$ **(B)** $m=6$ **(C)** $m+c=12$ **(D)** $m+c=8$

55. If $I=\displaystyle\int \frac{\sin x+\sin^3 x}{\cos 2x}dx = P\cos x+Q\ln\ |\text{f}\left(x\right)|+R$, then :

(A) $P=1/2,\ Q=-\dfrac{3}{4\sqrt{2}}$ $P=1/2,\ Q=\dfrac{3}{4\sqrt{2}},\ f(x)=\dfrac{\sqrt{2}\cos x+1}{\sqrt{2}\cos x-1}$

(B) $P=1/4, Q=-\dfrac{1}{\sqrt{2}}, f\left(x\right)=\dfrac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}$

(C) $P=1/2, Q=-\dfrac{3}{4\sqrt{2}}, f\left(x\right)=\dfrac{\sqrt{2}\cos x-1}{\sqrt{2}\cos x+1}$

(D) $P=-1/2, Q=-\dfrac{3}{4\sqrt{2}}, f\left(x\right)=\dfrac{\sqrt{2}\cos x+1}{\sqrt{2}\cos x-1}$

56. If $f\left(x\right), g\left(x\right)$ and $h\left(x\right)$ are continuous and positive functions such that:

$f\left(x\right)+g\left(x\right)+h\left(x\right)=\sqrt{f\left(x\right)g\left(x\right)}+\sqrt{g\left(x\right)h\left(x\right)}+\sqrt{h\left(x\right)f\left(x\right)}$, then $\int\left(f\left(x\right)+g\left(x\right)-2h\left(x\right)\right)dx$ is/are

(A) Independent of $f\left(x\right)$ **(B)** Independent of $g\left(x\right)$

(C) Independent of $h\left(x\right)$ **(D)** only A and B

57. Let f is a differentiable function such that $f\left(x\right)=x^2+\int_0^x e^{-t}f\left(x-t\right)dt$, then:

(A) $\int e^x f\left(x\right)dx=\dfrac{e^x x^3}{3}+c$ **(B)** $f\left(x\right)$ is neither even nor odd

(C) $\int e^x f\left(x\right)dx=\dfrac{e^x x^2}{2}+c$ **(D)** $f\left(3\right)=18$

58. Let f is a differentiable function such that $f'\left(x\right)=f\left(x\right)+\int_0^2 f\left(x\right)dx, f\left(0\right)=\dfrac{4-e^2}{3}$, then:

(A) $f\left(x\right)=e^{-x}-\dfrac{\left(e^2+1\right)}{3}$ **(B)** $f\left(x\right)=e^{x}-\dfrac{\left(e^2-1\right)}{3}$

(C) $f\left(x\right)$ is increasing **(D)** $f\left(x\right)$ is decreasing

59. Let f be a function such that $f\left(x\right).f\left(y\right)+2=f\left(x\right)+f\left(y\right)+f\left(xy\right), \forall x,y\in R-\{0\}$ and $f\left(0\right)=1, f'\left(1\right)=2$, then:

(A) $3\left(\int f\left(x\right)dx\right)-x\left(f\left(x\right)+2\right)$ is constant **(B)** $3\left(\int f\left(x\right)dx\right)-x\left(f\left(x\right)-2\right)$ is constant

(C) $\int f\left(x\right)dx-x\left(f\left(x\right)-2\right)$ is constant **(D)** the only possible value of $f\left(1\right)$ is 2

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column 1 are labelled as (A), (B), (C) & (D) whereas statements in Column 2 are labeled as p, q, r, s & t. More than one choice from Column 2 can be matched with Column 1.

60. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	If $\int \frac{2^x}{\sqrt{1-4^x}} dx = k \sin^{-1}(f(x)) + C$, then k is greater than	**(p)**	0
(B)	If $\int \frac{(\sqrt{x})^5}{(\sqrt{x})^7 + x^6} dx = a \ln \frac{x^k}{x^k+1} + c$, then ak is less than	**(q)**	1
(C)	$\int \frac{x^4+1}{x\left(x^2+1\right)^2} dx = k \ln \lvert x \rvert + \frac{m}{1+x^2} + n$, where n is the constant of integration, then mk is greater than	**(r)**	3
(D)	$\int \frac{dx}{5+4\cos x} = k \tan^{-1}\left(m \tan \frac{x}{2}\right) + C$, then k/m is greater than	**(s)**	4

61. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	$\int \frac{e^{2x}-1}{e^{2x}+1} dx$ is equal to	**(p)**	$x - \log\left[1+\sqrt{1-e^{2x}}\right] + c$
(B)	$\int \frac{1}{(e^x+e^{-x})^2} dx$ is equal to	**(q)**	$\log(e^x+1) - 1 - e^{-x} + c,$
(C)	$\int \frac{e^{-x}}{1+e^x} dx$ is equal to	**(r)**	$\log(e^{2x}+1) - x + c$
(D)	$\int \frac{1}{\sqrt{1-e^{2x}}} dx$ is equal to	**(s)**	$-\frac{1}{2\left(e^{2x}+1\right)} + c$

62. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	$\int \frac{dx}{\sqrt{x}(x+9)} =$	**(p)**	$\log\left\lvert 1 - \tan\left(\frac{x}{2}\right)\right\rvert + c$
(B)	$\int e^x\left(1 - \cot x + \cot^2 x\right) dx =$	**(q)**	$\log\left\lvert 1 - \cot\left(\frac{x}{2}\right)\right\rvert + c$
(C)	$\int \frac{\sin^3 x + \cos^3 x}{\cos^2 x \sin^2 x} dx =$	**(r)**	$\sec x - \operatorname{cosec} x + c$
(D)	$\int \frac{dx}{1 - \cos x - \sin x} =$	**(s)**	$\frac{2}{3} \tan^{-1} \frac{\sqrt{x}}{3} + c$
		(t)	$-e^x . \cot x + c$

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

63. Let $f(x)=\int x^{\sin x}(1+x\cos x\cdot\ln x+\sin x)dx$ and $f\left(\frac{\pi}{2}\right)=\frac{\pi^2}{4}$, then the value of $|\cos(f(\pi))|$ is

64. Let $g(x)=\int\frac{1+2\cos x}{(\cos x+2)^2}dx$ and $g(0)=0$, then the value of $8g(\pi/2)$ is

65. Let $k(x)=\int\frac{(x^2+1)dx}{\sqrt[3]{x^3+3x+6}}$ and $k(-1)=\frac{1}{\sqrt[3]{2}}$, then the value of $k(-2)$ is

66. If $\int\frac{2\cos x-\sin x+\lambda}{\cos x+\sin x-2}dx=A\ln\left|\cos x+\sin x-2\right|+Bx+C$. Then the value of $A+B+|\lambda|$ is

67. If $\int\left[\left(\frac{x}{e}\right)^x+\left(\frac{e}{x}\right)^x\right]\ln x\,dx=A\left(\frac{x}{e}\right)^x+B\left(\frac{e}{x}\right)^x+C$, then the value of A + B is

68. If $\int\left(x^9+x^6+x^3\right)\left(2x^6+3x^3+6\right)^{1/3}dx=a\left(2x^9+3x^6+6x^3\right)^{4/3}+c$, then the value of 48a must be………

69. If $\int\frac{dx}{1+\sin x}=\tan\left(\frac{x}{2}+a\right)+b$, then the value of $-\frac{4a}{\pi}$ must be…………

70. If $\int\frac{dx}{\sqrt{x+7}-\sqrt[4]{x+7}}=P\sqrt{x+7}+Q\sqrt[4]{x+7}+R\,ln\left|(x+7)^{1/4}-1\right|+c$, Then find the value of P + Q + R.

71. If $\int\frac{\left(x^2+2\right)dx}{\left(x^2+1\right)\left(x^2+4\right)}=k\,tan^{-1}\left(\frac{mx}{c-x^2}\right)$, then find the value of 3k + m + c.

72. If $\int\frac{dx}{\sqrt[4]{(x-1)^3(x+2)^5}}=k\sqrt[4]{\frac{x-1}{x+2}}+c$, then $3k$ is equal to____________.

73. If the primitive of the function $f(x)=\frac{x^{2009}}{\left(1+x^2\right)^{1006}}$ w.r.t x is equal to $\frac{1}{n}\left(\frac{x^2}{1+x^2}\right)^m+C$ then find the value of $(m+n)$ (where $m,n\in N$)

74. Let $f(x)$ is a quadratic function such that $f(0)=1$ & $f(-1)=4$. If $\int\frac{f(x)dx}{x^2(x+1)^2}$ is a rational function, then $f(10)=$.

75. Let $\int\frac{\left(f'(x)g(x)-g'(x)f(x)\right)dx}{\left(f(x)+g(x)\right)\sqrt{f(x)g(x)-g^2(x)}}=\sqrt{m}\tan^{-1}\left(\frac{f(x)-g(x)}{ng(x)}\right)+c$ where $m,n\in N$ and C is constant of integration $\left(g(x)>0\right)$ then the value of m^2+n^2 is

76. Let a matrix $'A'$ be denoted as $A = diag.\left(5^x, 5^{5^x}, 5^{5^{5^x}}\right)$ If the value of $\int\left(\det\left(A\right)\right)dx = \frac{5^{5^{5^x}}}{\left(\ln 5\right)^k} + c$ then k is

77. If $\int\left\{\ln\left(\frac{\cos 2\theta}{1+\sin 2\theta}\right)+\ln\left(\frac{1+\sin 2\theta}{1-\sin 2\theta}\right)^{\cos^2\theta}\right\}d\theta$ is equal to $\frac{1}{a}\ \sin 2\theta \ln.\left|\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right| + b\ln\left|\cos 2\theta\right| + c$ where $a, b \in R - \{0\}$ & c is integration constant such that $\cos\theta > \sin\theta > 0$ then $(a+b)$ is

78. Let f & g be differentiable function for all $x \in R$ & have the following properties

(i) $f'(x) = f(x) - g(x)$ (ii) $g'(x) = g(x) - f(x)$

(iii) $f(0) = 5$ (iv) $g(0) = 1$

Then the value of $\left|f(\ln 2) + g(\ln 3)\right|$ is equal to

79. If $\int\frac{x^3+x+1}{x^4+x^2+1}dx = A_1\ln\left(x^2+x+1\right) + A_2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + A_3\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + A_4\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right) + c$ then the value of $(A_1 + A_2 + A_3 + A_4)$ is

80. Let $\int\frac{\ln\left(x+\sqrt{1+x^2}\right)dx}{\sqrt{1+x^2}} = fog(x) + c,$ where $f(x) = \frac{x^2}{2}$ and g are some functions and c is an arbitrary constant. If $\int f(x).g(x)dx = ax^3 g(x) + b\left(1+x^2\right)^{3/2} + c\left(1+x^2\right)^{1/2} + d.$ then $\left(\frac{1}{a+b+c}\right)$ is equal to

81. $\int\sqrt{x}\tan\left\{2\tan^{-1}\left(\frac{\sqrt{\sqrt{1+\sqrt{x}}+1}-\sqrt{\sqrt{1+\sqrt{x}}-1}}{\sqrt{\sqrt{1+\sqrt{x}}+1}+\sqrt{\sqrt{1+\sqrt{x}}-1}}\right)\right\}dx$ is equal to $ax^b + K\tan^{-1}\left(\frac{\sqrt{x}}{2}\right) + \frac{\alpha}{\sqrt{1+\sqrt{x}}} + c$ then $a+b$ is equal to (where $a, b, k, \alpha \in R$)

82. If $\int\frac{(\cos x - \sin x + 1 - x)}{e^x + \sin x + x}dx = \ln\left(f(x)\right) + g(x) + c$ where c is the constant of integration & $f(x)$ is positive, , then $\frac{f(x)+g(x)}{e^x+\sin x}$ is

83. If $\int\left(x^{2010}+x^{804}+x^{402}\right)\left(2x^{1608}+5x^{402}+10\right)^{\frac{1}{402}}dx = \frac{1}{10a}\left(2x^{2010}+5x^{804}+10x^{402}\right)^{b/402} + c,$ where c is constant Then a is equal to

84. Let $u(x)$ & $v(x)$ are differentiable functions such that $\frac{u(x)}{v(x)} = 7.$ If $\frac{u'(x)}{v'(x)} = P$ & $\left(\frac{u(x)}{v(x)}\right)' = q$

then $\frac{p+q}{p-q}$ has the value equal to

85. If $\int \frac{\sqrt{2-x-x^2}}{x^2}dx = \frac{A\sqrt{2-x-x^2}}{x} + \frac{B}{4\sqrt{2}}\ln\left|\frac{4-x+4\sqrt{2-x-x^2}}{x}\right| - \sin^{-1}\left(\frac{2x+1}{3}\right)+c$ then $|A+B|$ is equal to_____.

86. If $\int\left\{\left(\frac{x^{-6}-64}{4+2x^{-1}+x^{-2}}\right).\left(\frac{x^2}{4-4x^{-1}+x^{-2}}\right)-\frac{4x^2(2x+1)}{(1-2x)}\right\}dx$ is equal to $f(x)$ where $f(1)=2$ then $f(3)=$

87. Let $f(x)=x+\sin x$. Suppose g denotes the inverse function of f. If the value of $g'\left(\frac{\pi}{4}+\frac{1}{\sqrt{2}}\right)$ is l then $2l=$

88. If $\int(\sin(2020x))(\sin^{2018}x)dx$ is equal to $\frac{(\sin(ax)).(\sin x)^b}{c}+k$ (where k is integration constant) then $\frac{a+b+c}{3}=$

89. If $I=\int\frac{dx}{(x-2)\left(1+\sqrt{7x-10-x^2}\right)}=f(t)+c$ (Where $t=\sqrt{\frac{5-x}{x-2}}$) and $f(0)=k\ln\frac{3-\sqrt{5}}{3+\sqrt{5}}$, ($k>0$) then k^2 is equal to _____.

90. $I=\int e^{x\sin x+\cos x}.\left(\frac{x^4\cos^3 x - x\sin x+\cos x}{x^2\cos^2 x}\right)dx = f(x)+c$, where c is constant and $f(\pi)=e^a\left(b+\frac{1}{b}\right)$ then $a+b$ equal to _____.

91. Suppose $\begin{vmatrix} f'(x) & f(x) \\ f''(x) & f'(x)\end{vmatrix}=0$ where $f(x)$ is differentiable function with $f'(x)\neq 0$ & satisfies $f(0)=1, f'(0)=2$ If $f(x)=e^{\lambda x}+\mu$ then $\lambda+\mu$ is _____.

92. If $\int\sqrt{\frac{1+x^{2n}}{x^{2n}}}\frac{\ln(1+x^{2n})-2n\ln x}{x^{2n+1}}dx=\frac{\alpha p^3}{\beta n}[1-3\ln p]+c$ (where p = $\sqrt{1+\frac{1}{x^{2n}}}$, $\alpha,\beta\in N$) then $\alpha+\beta=$

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SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. If $p,\ q,\ r,\ s$ are in arithmetic progression and $f(x)=\begin{vmatrix} p+\sin x & q+\sin x & p-r+\sin x \\ q+\sin x & r+\sin x & -1+\sin x \\ r+\sin x & s+\sin x & s-q+\sin x \end{vmatrix}$ such that $\int_0^2 f(x)\,dx=-4,$ then the common difference of the progression is :

(A) ± 1 **(B)** $\dfrac{1}{2}$ **(C)** ± 2 **(D)** None of these

2. If $A=\int_1^{\sin\theta}\frac{t\,dt}{1+t^2}$ and $B=\int_1^{\operatorname{cosec}\theta}\frac{dt}{t(1+t^2)}$, then the value of $\begin{vmatrix} A & A^2 & B \\ e^A e^B & B^2 & -1 \\ 1 & A^2+B^2 & -1 \end{vmatrix}$ is :

(A) $\sin\theta$ **(B)** $\operatorname{cosec}\theta$ **(C)** 0 **(D)** 1

3. Area bounded by the curves $y=\left[\frac{x^2}{64}+2\right]$ ([.] denotes the greatest integer function), $y=x-1$ and $x=0$ above the x-axis is :

(A) 2 **(B)** 3 **(C)** 4 **(D)** $2\sqrt{3}$

Paragraph for Questions 4 - 7

Evaluating Integrals Dependent on a parameter

Differentiate I with respect to the parameter within the sign of integrals taking variable of the integrand as constant. Now, evaluate the integral so obtained as a function of the parameter and then integrate the result to get I. Constant of integration can be computed by giving some arbitrary values to the parameter and the corresponding value of I.

4. The value of $\int_0^1 \frac{x^a-1}{\log x}dx$ is :

(A) $\log(a-1)$ **(B)** $\log(a+1)$ **(C)** $a\log(a+1)$ **(D)** None of these

5. The value of $\int_0^{\pi/2}\log\left(\sin^2\theta+k^2\cos^2\theta\right)d\theta$, where $k\ge 0$, is :

(A) $\pi\log(1+k)+\pi\log 2$ **(B)** $\pi\log(1+k)$

(C) $\pi\log(1+k)-\pi\log 2$ **(D)** $\log(1+k)-\log 2$

6. The value of $\frac{dI}{da}$ when $I=\int_0^{\pi/2}\log\left(\frac{1+a\sin x}{1-a\sin x}\right)\frac{dx}{\sin x}$ (where $|a|<1$) is :

(A) $\dfrac{\pi}{\sqrt{1-a^2}}$ **(B)** $-\pi\sqrt{1-a^2}$ **(C)** $\sqrt{1-a^2}$ **(D)** $\dfrac{\sqrt{1-a^2}}{\pi}$

7. If $\int_0^{\pi} \frac{dx}{(a-\cos x)} = \frac{\pi}{\sqrt{a^2-1}}$, then the value of $\int_0^{\pi} \frac{dx}{\left(\sqrt{10}-\cos x\right)^3}$ is :

(A) $\frac{\pi}{81}$ **(B)** $\frac{7\pi}{162}$ **(C)** $\frac{7\pi}{81}$ **(D)** None of these

8. The smaller area enclosed by $y = f(x)$, when $f(x)$ is a polynomial of least degree satisfying $\lim_{x \to 0}\left[1+\frac{f(x)}{x^3}\right]^{1/x} = e$ and the circle $x^2 + y^2 = 2$ above the x-axis is :

(A) $\frac{\pi}{2}$ **(B)** $\frac{3}{5}$ **(C)** $\frac{\pi}{2}-\frac{3}{5}$ **(D)** $\frac{\pi}{2}+\frac{3}{5}$

9. The area bounded by the curves $y = \ln x, y = \ln|x|, y = |\ln x|$ and $y = |\ln|x||$ is :

(A) 5 sq. units **(B)** 2 sq. units **(C)** 4 sq. units **(D)** None of these

10. If the line $y = mx + 2$ cuts the parabola $2y = x^2$ at points (x_1, y_1) and $(x_2, y_2)(x_1 < x_2)$, then value of m for which $\int_{x_1}^{x_2}\left(mx+2-\frac{x^2}{2}\right)dx$ is minimum is :

(A) $\sqrt{2}$ **(B)** $\frac{8}{3}$ **(C)** $\frac{1}{\sqrt{3}}$ **(D)** 0

11. Maximum area of rectangle whose two sides are $x = x_0, x = \pi - x_0$ and which is inscribed in a region bounded by $y = \sin x$ and x-axis is obtained when $x_0 \in$

(A) $\left(\frac{\pi}{4}, \frac{\pi}{3}\right)$ **(B)** $\left(\frac{\pi}{6}, \frac{\pi}{4}\right)$ **(C)** $\left(0, \frac{\pi}{6}\right)$ **(D)** None of these

12. If $f(x) = a + bx + cx^2$, where $c > 0$ and $b^2 - 4ac < 0$, then the area enclosed by the co-ordinate axes, the line $x = 2$ and the curve $y = f(x)$ is given by :

(A) $\frac{1}{3}[4f(1) + f(2)]$ **(B)** $\frac{1}{2}[f(0) + 4f(1)]$

(C) $\frac{1}{2}[f(0) + 4f(1) + f(2)]$ **(D)** $\frac{1}{3}[f(0) + 4f(1) + f(2)]$

Paragraph for Questions 13 - 15

$f(x)$ satisfies the relation $f(x) - \lambda \int_0^{\pi/2} \sin x \cos t f(t) dt = \sin x$.

13. If $\lambda > 2$, then $f(x)$ decreases in which of the following interval?

(A) $(0, \pi)$ **(B)** $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ **(C)** $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ **(D)** None of these

14. If $f(x) = 2$ has at least one real root, then :

(A) $\lambda \in [1,4]$ **(B)** $\lambda \in [-1,2]$ **(C)** $\lambda \in [0,1]$ **(D)** $\lambda \in [1,3]$

15. If $\int_0^{\pi/2} f(x) dx = 3$, then the value of λ is :

(A) 1 **(B)** 3/2 **(C)** 4/3 **(D)** None of these

16. If a is a positive integer, then the number of values of a satisfying

$$\int_0^{\pi/2}\left\{a^2\left(\frac{\cos 3x}{4}+\frac{3}{4}\cos x\right)+a\sin x-20\cos x\right\}dx \le -\frac{a^2}{3}$$ is :

(A) Only one **(B)** Two **(C)** Three **(D)** Four

17. If I is the greatest of the definite integrals $I_1=\int_0^1 e^{-x}\cos^2 x\,dx, I_2=\int_0^1 e^{-x^2}\cos^2 x\,dx$

$I_3=\int_0^1 e^{-x^2}dx,\ I_4=\int_0^1 e^{-x^2/2}dx$ then :

(A) $I=I_1$ **(B)** $I=I_2$ **(C)** $I=I_3$ **(D)** $I=I_4$

18. The value of the integral $\int_0^{\pi}\frac{\sin(n+1/2)x}{\sin x/2}dx\,(n\in N)$ is :

(A) π **(B)** 2π **(C)** 3π **(D)** none of these

19. The value of $\int_0^1 \lim_{n\to\infty}\sum_{k=0}^{n}\frac{x^{k+2}2^k}{k!}dx$ is :

(A) e^2-1 **(B)** 2 **(C)** $\frac{e^2-1}{2}$ **(D)** $\frac{e^2-1}{4}$

20. Let $P(x)$ be a polynomial of least degree whose graph has three points of inflection $(-1,-1)$, $(1, 1)$ and a point with abscissa 0 at which the curve is inclined to the axis of abscissa at an angle of 60°. Then $\int_0^1 P(x)dx$ equals to :

(A) $\frac{3\sqrt{3}+4}{14}$ **(B)** $\frac{3\sqrt{3}}{7}$ **(C)** $\frac{\sqrt{3}+\sqrt{7}}{14}$ **(D)** $\frac{\sqrt{3}+2}{7}$

21. The value of a for which the equation $\int_0^x \sin^2(\frac{t}{2})dt=a^2x^2-\frac{1}{2}(3x-1)+\frac{1}{a^2}$ possess a solution are :

(A) $\pm\frac{1}{\sqrt{2n\pi}}, n\in N$ **(B)** $\pm\frac{1}{\sqrt{2n\pi-\frac{\pi}{2}}}, n\in N$ **(C)** $\pm\frac{1}{\sqrt{n\pi+\frac{\pi}{2}}}, n\in N$ **(D)** None of these

22. Let f, g and h be continuous functions on $[0, a]$ such that $f(x)=f(a-x), g(x)=-g(a-x)$ and $3h(x)-4h(a-x)=5.$ Then $\int_0^a f(x)g(x)h(x)\,dx=$

(A) 5/4 **(B)** 3/4 **(C)** 1 **(D)** 0

Paragraph for Questions 23 - 26

Let $f(x)$ and $\phi(x)$ are two continuous functions on R satisfying $\phi(x)=\int_a^x f(t)dt,\ a\ne 0$ and another continuous function $g(x)$ satisfying $g(x+\alpha)+g(x)=0\ \forall\ x\in R, \alpha>0$ and $\int_b^{2k} g(t)dt$ is independent of b.

23. If $f(x)$ is an odd function, then :

(A) $\phi(x)$ is also an odd function **(B)** $\phi(x)$ is an even function

(C) $\phi(x)$ is neither an even nor an odd function

(D) For $\phi(x)$ to be an even function, it must satisfy $\int_0^a f(x)\,dx=0$

24. If $f(x)$ is an even function, then :

(A) $\phi(x)$ is also an even function **(B)** $\phi(x)$ is an odd function

(C) If $f(a-x)=-f(x)$, then $\phi(x)$ is an even function

(D) If $f(a-x)=-f(x)$, then $\phi(x)$ is an odd function

25. Least positive value of c if c, k , b are in A.P. is :

(A) 0 **(B)** 1 **(C)** α **(D)** 2α

26. If m, n are even integers and p, q $\in R$, then $\int_{p+m\alpha}^{q+n\alpha} g(t)dt$ is equal to :

(A) $\int_p^q g(x)dx$ **(B)** $(n-m)\int_0^\alpha g(x)dx$

(C) $\int_p^q g(x)dx+(n-m)\int_0^\alpha g(2x)dx$ **(D)** $\int_p^q g(x)dx+(n-m)\int_0^\alpha g(x)dx$

27. If $G(x,t)=\begin{cases} x(t-1), & \text{when } x\le t \\ t(x-1), & \text{when } t<x \end{cases}$ and if t is continuous function of x in [0, 1]

Let $g(x)=\int_0^1 f(t)G(x,t)dt.$ Then which is incorrect :

(A) $g(0)+g(1)=0$ **(B)** $g(0)=0$ **(C)** $g(1)=1$ **(D)** $g''(x)=f(x)$

28. If $\int_0^1 \frac{\sin t}{1+t}dt=\alpha,$ then the value of $\int_{4\pi-2}^{4\pi}\frac{\sin\frac{t}{2}}{4\pi+2-t}dt$ is :

(A) α **(B)** $-\alpha$ **(C)** $\pi\alpha$ **(D)** 2α

29. The value of $\int_{-\pi}^{\pi}|x\sin[x^2-\pi]|\,dx,$ where [.] denotes the greatest integer function is :

(A) $\sum_{r=1}^{6} r\sin r$ **(B)** $\sum_{r=1}^{6}(-1)^r r\sin r$ **(C)** $\sum_{r=1}^{6} r^2\sin r$ **(D)** None of these

30. If $a\le\int_0^1\frac{dx}{\sqrt{4-x^2-x^3}}\le b,$ then $(a,b)=$

(A) $\left(\frac{\pi}{6},\frac{\pi}{4}\right)$ **(B)** $\left(\frac{\pi}{6},\frac{\pi}{4\sqrt2}\right)$ **(C)** $\left(\frac{\pi}{4\sqrt2},\frac{\pi}{2\sqrt2}\right)$ **(D)** None of these

31. Let $I=\int_{\pi/4}^{\pi/3}\frac{\sin x}{x}dx,$ then I belongs to :

(A) $\left(\frac{\sqrt3}{8},\frac{\sqrt2}{6}\right)$ **(B)** $\left(\frac{\sqrt2}{2},\frac{\sqrt3}{2}\right)$ **(C)** $\left(\frac{1}{2},\frac{\sqrt2}{2}\right)$ **(D)** None of these

32. The sum of the series as $n\to\infty$ $\frac{\sqrt n}{(3+4\sqrt n)^2}+\frac{\sqrt n}{\sqrt2(3\sqrt2+4\sqrt n)^2}+\frac{\sqrt n}{\sqrt3(3\sqrt3+4\sqrt n)^2}+....\frac{1}{49n}$ is :

(A) 1/14 **(B)** 3/28 **(C)** 2/3 **(D)** $\pi/4$

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

33. A function $f(x)$ which satisfies the relation $f(x)=e^x+\int_0^1 e^x f(t)dt$, then :

(A) $f(0)<0$ **(B)** $f(x)$ is a decreased function

(C) $f(x)$ is an increasing function **(D)** $\int_0^1 f(x)dx>0$

34. Let $f(x)=\int_1^x \frac{3^t}{1+t^2}dt, \text{where } x>0$, then :

(A) For $0<\alpha<\beta,\ f(\alpha)<f(\beta)$ **(B)** For $0<\alpha<\beta,\ f(\alpha)>f(\beta)$

(C) $f(x)+\frac{\pi}{4}<\tan^{-1}x, \forall x\ge 1$ **(D)** $f(x)+\frac{\pi}{4}>\tan^{-1}x, \forall x\ge 1$

35. The values of a for which the integral $\int_0^2 |x-a|dx\ge 1$ is satisfied are :

(A) $[2,\infty)$ **(B)** $(-\infty,0]$ **(C)** (0, 2) **(D)** None of these

36. If $\int_a^b |\sin x|dx=8$ and $\int_0^{a+b} |\cos x|dx=9$, then :

(A) $a+b=\frac{9\pi}{2}$ **(B)** $|a-b|=4\pi$ **(C)** $\frac{a}{b}=15$ **(D)** $\int_a^b \sec^2 x dx=0$

37. If $g(x)=\int_0^x 2|t|dt$, , then :

(A) $g(x)=x|x|$ **(B)** g(x) is monotonic

(C) g(x) is differentiable at x = 0 **(D)** $g'(x)$ is differentiable at x = 0

38. Let $f:[1,\infty)\to R$ and $f(x)=x\int_1^x \frac{e^t}{t}dt-e^x$, then

(A) $f(x)$ is an increasing function **(B)** $\lim_{x\to\infty} f(x)\to\infty$

(C) $f'(x)$ has a maxima at x = e **(D)** $f(x)$ is a decreasing function

39. The value of $\int_0^1 \frac{2x^2+3x+3}{(x+1)(x^2+2x+2)}dx$ is :

(A) $\frac{\pi}{4}+2\log 2-\tan^{-1}2$ **(B)** $\frac{\pi}{4}+2\log 2-\tan^{-1}\frac{1}{3}$

(C) $2\log 2-\cot^{-1}3$ **(D)** $-\frac{\pi}{4}+\log 4+\cot^{-1}2$

40. If $A_n = \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x} dx$; $B_n = \int_0^{\pi/2} \left(\frac{\sin nx}{\sin x}\right)^2 dx$, for $n \in N$, then :

(A) $A_{n+1} = A_n$ **(B)** $B_{n+1} = B_n$ **(C)** $A_{n+1} - A_n = B_{n+1}$ **(D)** $B_{n+1} - B_n = A_{n+1}$

41. If $f(x) = \int_0^x [f(x)]^{-1} dx$ and $\int_0^1 [f(x)]^{-1} dx = \sqrt{2}$, then :

(A) $f(2) = 2$ **(B)** $f'(2) = 1/2$ **(C)** $f^{-1}(2) = 2$ **(D)** $\int_0^1 f(x) dx = \sqrt{2}$

42. The value of $\int_0^\infty \frac{dx}{1+x^4}$ is :

(A) Same as that of $\int_0^\infty \frac{x^2+1}{1+x^4} dx$ **(B)** $\frac{\pi}{2\sqrt{2}}$

(C) Same as that of $\int_0^\infty \frac{x^2 dx}{1+x^4}$ **(D)** $\frac{\pi}{\sqrt{2}}$

43. If $f(x) = \int_0^x |t-1| dt$, where $0 \le x \le 2$, then :

(A) Range of $f(x)$ is [0, 1] **(B)** $f(x)$ is differentiable at $x = 1$

(C) $f(x) = \cos^{-1} x$ has two real roots **(D)** $f'\left(\frac{1}{2}\right) = \frac{1}{2}$

44. If $\int_a^b \frac{f(x)}{f(a)+f(a+b-x)} dx = 10$, then :

(A) $b = 22, a = 2$ **(B)** $b = 15, a = -5$ **(C)** $b = 10, a = -10$ **(D)** $b = 10, a = -2$

45. If $I_n = \int_0^1 \frac{dx}{(1+x^2)^n}$, where $n \in N$, which of the following statements hold good?

(A) $2nI_{n+1} = 2^{-n} + (2n-1)I_n$ **(B)** $I_2 = \frac{\pi}{8} + \frac{1}{4}$

(C) $I_2 = \frac{\pi}{8} - \frac{1}{4}$ **(D)** $I_3 = \frac{3\pi}{32} + \frac{1}{4}$

46. If $f(x)$ is integrable over [1, 2], then $\int_1^2 f(x) dx$ is equal to :

(A) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r}{n}\right)$ **(B)** $\lim_{n\to\infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$

(C) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r+n}{n}\right)$ **(D)** $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

47. If $f(2-x)=f(2+x)$ and $f(4-x)=f(4+x)$ for all x and $f(x)$ is a function for which $\int_0^2 f(x)dx=5$, then $\int_0^{50} f(x)dx$ is equal to :

(A) 125 **(B)** $\int_{-4}^{46} f(x)dx$ **(C)** $\int_1^{51} f(x)dx$ **(D)** $\int_2^{52} f(x)dx$

48. $\int_0^x \left\{\int_0^u f(t)dt\right\}du$ is equal to :

(A) $\int_0^x (x-u)f(u)du$ **(B)** $\int_0^x uf(x-u)du$ **(C)** $x\int_0^x f(u)du$ **(D)** $x\int_0^x uf(u-x)du$

49. Which of the following statements(s) is(are) true?

(A) If function $y=f(x)$ is continuous at $x=c$ such that $f(c)\neq 0$, then $f(x)f(c)>0\ \forall x\in(c-h,c+h)$ where h is sufficiently small positive quantity

(B) $\lim_{n\to\infty}\ln\left(\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right).....\left(1+\frac{n}{n}\right)\right)=1+2\ln 2.$

(C) Let f be a continuous and non-negative function defined on [a, b]. If $\int_a^b f(x)dx=0$, then $f(x)=0\ \forall\ x\in[a,b]$.

(D) Let f be a continuous function defined on [a, b]. If $\int_a^b f(x)dx=0$, then there exists at least one $c\in(a,b)$ for which $f(c)=0$.

50. The value of $\int_0^1 e^{x^2-x}dx$ is :

(A) <1 **(B)** >1 **(C)** $>e^{-1/4}$ **(D)** $<e^{-1/4}$

51. If $f(x)=\int_0^x(\cos(\sin t)+\cos(\cos t))dt$, then $f(x+\pi)$ is :

(A) $f(x)+f(\pi)$ **(B)** $f(x)+2f(\pi)$ **(C)** $f(x)+f\left(\frac{\pi}{2}\right)$ **(D)** $f(x)+2f\left(\frac{\pi}{2}\right)$

52. If x satisfies the equation $x^2\left(\int_0^1\frac{dt}{t^2+2t\cos\alpha+1}\right)-x\left(\int_{-3}^3\frac{t^2\sin 2t}{t^2+1}dt\right)-2=0 \quad (0<\alpha<\pi)$, then the value of x is :

(A) $2\sqrt{\left(\frac{\sin\alpha}{\alpha}\right)}$ **(B)** $-2\sqrt{\left(\frac{\sin\alpha}{\alpha}\right)}$ **(C)** $4\sqrt{\left(\frac{\sin\alpha}{\alpha}\right)}$ **(D)** $-4\sqrt{\left(\frac{\sin\alpha}{\alpha}\right)}$

53. If $G(x,t)=\begin{cases}x(t-1), & where\ x\le t\\ t(x-1), & where\ t<x\end{cases}$ and if t is continuous function of x in [0, 1]. Let $g(x)=\int_0^1 f(t)G(x,t)dt$, then :

(A) $g(0)=1$ **(B)** $g(0)=0$ **(C)** $g(1)=1$ **(D)** $g^{11}(x)=f(x)$

54. $\int_{-1/2}^{1/2}\sqrt{\left\{\left(\frac{x+1}{x-1}\right)^2+\left(\frac{x-1}{x+1}\right)^2-2\right\}}dx$ is :

(A) $4ln\left(\frac{4}{3}\right)$ **(B)** $4ln\left(\frac{3}{4}\right)$ **(C)** $-ln\left(\frac{81}{256}\right)$ **(D)** $ln\left(\frac{256}{81}\right)$

55. Let T > 0 be a fixed real number. Suppose $f\ (x)$ is a continuous function for all $x \in R, f\left(x+T\right)=f\left(x\right)$.

If $I=\int_0^T f\left(x\right)dx$ then :

(A) $\int_5^{5+5T} f\left(x\right)dx=5I$ **(B)** $\int_5^{5+5T} f(2x)dx=10I$

(C) $\int_5^{5+5T} f(3x)dx=5I$ **(D)** $\int_5^{5+5T} f(3x)dx=15I$

56. Let $f(x)=\int_{\pi^2/4}^{x^2}\frac{\sin x}{1+\cos^2\sqrt{t}}dt$ then

(A) $f'\left(\frac{\pi}{2}\right)=\pi$ **(B)** $f'\left(-\frac{\pi}{2}\right)=\pi$ **(C)** $f'\left(\frac{3\pi}{2}\right)=-3\pi$ **(D)** $f'\left(\pi\right)=\int_{\pi^2}^{\pi^2/4}\frac{dx}{1+cos^2\sqrt{x}}$

57. Let $I_n=\int_{-\pi}^{\pi}\frac{sin\,nx}{sin\,x}dx; n\in N$ then :

(A) $I_{n+2}=I_n$ **(B)** $\sum_{m=1}^{20} I_{2m+1}=20\pi$

(C) $I_{2m}=0$ where $m=1,2,3,.....$ **(D)** $I_{n+1}=I_n$

58. If y is a function of x, satisfying $x.\int_0^x y(t)\,dt=(x+1)\int_0^x t\cdot y(t)\,dt$, where $x>0$, given $y\,(1)=1$. Then:

(A) $y=x^3.e^{-\frac{1}{x}}$ **(B)** $y=y=\frac{e}{x^3}e^{-1/x}$ **(C)** $y\left(2\right)=\frac{8}{\sqrt{e}}$ **(D)** $y\left(2\right)=\frac{\sqrt{e}}{8}$

59. If $P=\int_0^{\infty}\frac{x^2}{1+x^4}dx;\ Q=\int_0^{\infty}\frac{x\,dx}{1+x^4}$ and $R=\int_0^{\infty}\frac{dx}{1+x^4}$, then:

(A) $Q=\frac{\pi}{4}$, **(B)** $P=R$ **(C)** $P-\sqrt{2}\ Q+R=\frac{\pi}{2\sqrt{2}}$ **(D)** $P-2\sqrt{2}\quad Q+R=\frac{\pi}{\sqrt{2}}$

60. Let $u=\int_0^{\pi/4}\left(\frac{\cos x}{\sin x+\cos x}\right)^2dx$ and $v=\int_0^{\pi/4}\left(\frac{\sin x+\cos x}{\cos x}\right)^2dx$, then:

(A) $v=1+\ln 2$ **(B)** $u=\frac{1+\ln 2}{4}$ **(C)** $\frac{v}{u}=6$ **(D)** $\frac{v}{u}=\frac{1}{6}$

61. If $\int_0^2 \frac{\ln(1+2x)}{1+x^2}dx = (\tan^{-1} a)(\ln\sqrt{b})$ where $a, b \in N$, then:

(A) $a = 2$ **(B)** $b = 5$ **(C)** $a^2 + b^2 = 29$ **(D)** $b - a = 4$

62. Comment upon the nature of roots of the quadratic equation $x^2 + 2x = k + \int_0^1 |t+k|\, dt$ depending on the value of $k \in R$.

(A) real and distinct if $-1 < k < 0$ **(B)** real and distinct if $k < -1$

(C) imaginary if $-1 < k < 0$ **(D)** real and distinct $k > 0$

63. If $\lim_{n\to\infty} \frac{1}{n^2}\sum_{k=0}^{n-1}\left[k\int_k^{k+1}\sqrt{(x-k)(k+1-x)}\,dx\right] = \frac{\pi}{m^n}$, then:

(A) $m = 2, n = 4$ **(B)** $m = 4, n = 2$ **(C)** $m = \sqrt{2}, n = 4$ **(D)** $m = 2^{1/4}, n = 8$

64. Let $g(x) = x^c.e^{2x}$ & let $f(x) = \int_0^x e^{2t}.(3t^2+1)^{1/2}\,dt$. For a certain value of $'c'$, the limit of $\frac{f'(x)}{g'(x)}$ as $x \to \infty$ is finite and non-zero, then:

(A) $c = 1$ **(B)** $\lim_{x\to\infty}\frac{f'(x)}{g'(x)} = \frac{\sqrt{3}}{2}$ **(C)** $\lim_{x\to\infty}\frac{f'(x)}{g'(x)} = \frac{2}{\sqrt{3}}$ **(D)** None of these

65. If $I = \int_3^4 \frac{1}{\sqrt[3]{\ln x}}dx$, then:

(A) $I > 0.92$ **(B)** $I < 1$ **(C)** $I > .8$ **(D)** (B) and (C) only

66. Consider the integrals $I_1 = \int_0^1 e^{-x}\cos^2 x\, dx, I_2 = \int_0^1 e^{-x^2}\cos^2 x\, dx$, $I_3 = \int_0^1 e^{-x^2}dx$ and $I_4 = \int_0^1 e^{-x^2/2}dx$, Then:

(A) $I_4 > I_2$ **(B)** $I_2 > I_1$ **(C)** $I_3 < I_4$ **(D)** None of these

67. Which of the following is/are true?

(A) $\frac{\pi}{3\sqrt{3}} \le \int_0^1 \frac{dx}{1+x^2+2x^5}$ **(B)** $\int_0^1 \frac{dx}{1+x^2+2x^5} \le \frac{\pi}{4}$

(C) $\int_0^{\pi/2}\sqrt{1-\sin^3 x}dx \le \frac{1}{2}\left(\sqrt{2}+\ln\left(1+\sqrt{2}\right)\right)$. **(D)** $1 \le \int_0^{\pi/2}\sqrt{1-\sin^3 x}dx$

68. Let $f(x)$ be a continuous function and $'c'$ is a constant satisfying $\int_0^x f(t)dt = e^x - ce^{2x}\int_0^1 f(t)e^{-t}dt$, then:

(A) $f(x) = e^{2x} - 2e^x$ **(B)** $f(x) = e^x - 2e^{2x}$ **(C)** $c = \frac{1}{3-2e}$ **(D)** $c = \frac{1}{3+2e}$

69. If $f(x) = x + \int_0^1 (xy^2 + x^2y)(f(y))dy$, then:

(A) $f(x) = \frac{260}{119}$ **(B)** $f(-1) = \frac{-100}{119}$

(C) $f(x)$ have positive point of local minimum **(D)** $f(x)$ have negative point of local minimum

70. If $I=\int\limits_{1/2}^{2}\frac{\ln t}{1+t^{n}}dt,\ (t\neq 1)$

(A) $I>0$ if $n=1$ **(B)** $I<0\ \forall n\geq 3$ **(C)** $I=0$ if $n=2$ **(D)** $I<0$ if $n=1$

71. If $\lim\limits_{x\to 0}\frac{\int_0^x \frac{t^2dt}{\left(a+t^r\right)^{1/p}}}{bx-\sin x}=l,$ (where $p\in N, p\geq 2, a>0, r>0$ and $b\neq 0$), then:

(A) If l exists and is non-zero, then $b=1$

(B) If $p=3$ and $l=1$, then $a=8$

(C) If $p=2$ and $a=9$ and l exists and non-zero, then $l=\frac{2}{3}$

(D) If $P=2$ & $a=9$ & l exists, then $l=\frac{1}{3}$

72. Suppose $f(x)$ and $g(x)$ are two continuous functions defined for $0\leq x\leq 1$. Given, $f(x)=\int_0^1 e^{x+t}.f(t)dt$ and $g(x)=\int_0^1 e^{x+t}.g(t)dt+x.$ Then:

(A) $f(1)=0$

(B) $g(0)-f(0)=\frac{2}{3-e^2}$

(C) $\frac{g(0)}{g(2)}=\frac{1}{3}$

(D) $g(0)-f(0)=\frac{2}{3+e^2}$

73. We are given the curves $y=\int_{-\infty}^{x}f(t)dt$ through the point $\left(0,\frac{1}{2}\right)$ and $y=f(x)$ where $f(x)>0$ and $f(x)$ is differentiable, $\forall x\in R$ passes through $(0,1)$. Tangents drawn to both the curves at the points with equal abscissae intersect on the same point on the X-axis. Then:

(A) The number of solutions $f(x)=2ex$ is 2

(B) $\lim\limits_{x\to\infty}\left(f(x)\right)^{f(-x)}=e$

(C) $\lim\limits_{x\to\infty}\left(f(x)\right)^{f(-x)}=1$

(D) The number of solutions $f(x)=2ex$ is 1

74. If $f(x)=\int_0^x\left(4t^4-at^3\right)dt$ and $g(x)$ is quadratic satisfying $g(0)+6=g'(0)-c=g''(0)+2b=0.$ $y=h(x)$ and $y=g(x)$ intersect in 4 distinct points with abscissae $x_i; i=1,2,3,4$ such that $\sum\frac{i}{x_i}=8, a,b,c\in R^{+}$ and $h(x)=f'(x)$. Then :

(A) Abscissae of point of intersection are in AP

(B) $a=20$

(C) $c=25$

(D) $c-a=6$

75. Let $f(x)$ and $g(x)$ be differentiable functions such that $f(x)+\int_0^x g(t)\,dt = \sin x(\cos x - \sin x)$ and $(f'(x))^2 + (g(x))^2 = 1$, then $f(x)$ and $g(x)$ respectively, can be:

(A) $\frac{1}{2}\sin 2x, \sin 2x$ **(B)** $\frac{\cos 2x}{2}, \cos 2x$ **(C)** $\frac{1}{2}\sin 2x, -\sin 2x$ **(D)** $\cos^2 x, \cos 2x$

76. The function $f:[0, 1]\to[0, 1]$ is continuous and has the property $f(f(x)) = 1 - x$ for all $x \in [0,1]$ and $\alpha = \int_0^1 f(x)\,dx$, then:

(A) $f\left(\frac{1}{4}\right)+f\left(\frac{3}{4}\right)=1$

(B) the value of α equals to $1/2$

(C) $f\left(\frac{1}{3}\right).f\left(\frac{2}{3}\right)=1$

(D) $\int_0^{\pi/2} \frac{\sin x\,dx}{(\sin x + \cos x)^3}$ has the same value as α

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column 1 are labelled as (A), (B), (C) & (D) whereas statements in Column 2 are labeled as p, q, r, s & t. More than one choice from Column 2 can be matched with Column 1.

77. If $[.]$ denotes the greatest integer function, then match the following columns:

	Column 1		Column 2
(A)	$\int_{-1}^{1}\left[x+[x+[x]]\right]dx$	**(p)**	3
(B)	$\int_{2}^{5}([x]+[-x])dx$	**(q)**	5
(C)	$\int_{-1}^{3}\operatorname{sgn}(x-[x])dx$	**(r)**	4
(D)	$25\int_{0}^{\frac{\pi}{4}}\left(\tan^6(x-[x])+\tan^4(x-[x])\right)dx$	**(s)**	–3

78. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	$\lim\limits_{n\to\infty}\left[\int_0^2 \frac{\left(1+\frac{t}{n+1}\right)^n}{n+1}dt\right]$	**(p)**	$e-\frac{1}{2}e^2-\frac{3}{2}$
(B)	Let $f(x)$ be a function satisfying $f'(x)=f(x)$ with $f(0)=1$ and g be the function satisfying $f(x)+g(x)=x^2$, then the value of the integral $\int_0^1 f(x)g(x)dx$	**(q)**	e^2
(C)	$\int_0^1 e^{e^x}(1+xe^x)dx$ is equal to	**(r)**	e^2-1
(D)	$\lim\limits_{k\to 0}\frac{1}{k}\int_0^k (1+\sin 2x)^{\frac{1}{x}}\,dx$ is equal to	**(s)**	e^e

79. MATCH THE FOLLOWING:

	Column 1		Column 2
(A)	If $f(x)$ is an integrable function for $x\in\left[\frac{\pi}{6},\frac{\pi}{3}\right]$ and $I_1=\int_{\pi/6}^{\pi/3}\sec^2\theta f(2\sin 2\theta)d\theta$ and $I_2=\int_{\pi/6}^{\pi/3}\operatorname{cosec}^2\theta f(2\sin 2\theta)d\theta$, then I_1/I_2	**(p)**	3
(B)	If $f(x+1)=f(3+x)$ for $\forall x$, and the value of $\int_a^{a+b} f(x)dx$ is independent of a then the value of b can be	**(q)**	1
(C)	The value of $\int_1^4 \frac{\tan^{-1}[x^2]}{\tan^{-1}[x^2]+\tan^{-1}[25+x^2-10x]}$ Where [.] denotes G.I.F.	**(r)**	2
(D)	If $I=\int_0^2\sqrt{x+\sqrt{x+\sqrt{x+\dots\dots\infty}}}\,dx$ (where x >0), then [I] is equal to (where [.] denotes the greatest integer function	**(s)**	4

80. **MATCH THE FOLLOWING :**

	Column 1		Column 2
(A)	If $I=\int_{-2}^{2}\left(\alpha x^3+\beta x+\gamma\right)dx$, then I is	**(p)**	Independent of α
(B)	Let α,β be the distinct positive roots of the equation $\tan x=2x$, then $\gamma\int_0^1(\sin\alpha x.\sin\beta x)dx$ where $\gamma\neq 0$) is	**(q)**	Independent of β
(C)	If $f(x+\alpha)+f(x)=0$, where $\alpha>0$, then $\int_{\beta}^{\beta+2\gamma\alpha}f(x)dx$, where $\gamma\in N$ is	**(r)**	Independent of γ
(D)	$\gamma\int_0^{\alpha}[\sin x]dx$ is, where $\gamma\neq 0$, $\alpha\in\left[(2\beta+1)\pi,(2\beta+2)\pi\right]n\in N$, and where [.] denotes the greatest integer function	**(s)**	Depends on α

81. Let $\mathrm{I(n)}=\int_{\mathrm{i}}^{\mathrm{e}}\mathrm{x^3(\log x)^n\,dx}$, where n is a whole number.

	Column 1		Column 2
(A)	$\frac{64}{5e^4-1}I(2)=$	**(p)**	1
(B)	$\frac{4I(n)+nI(n-1)}{e^4}, for\, n\geq 1=$	**(q)**	3
(C)	The least value of 'n', for which $I(n)<\frac{e^4-4}{4}$, is	**(r)**	2
(D)	$\lim\limits_{n\to\infty}\mathrm{I(n)}=$	**(s)**	0

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

82. Consider a real valued continuous function f such that $f(x)=\sin x+\int_{-\pi/2}^{\pi/2}\left(\sin x+tf(t)\right)dt$. If M and m are maximum and minimum value of the function f, then the value of M/m is _______.

83. Let $f(x)=x^3-\frac{3x^2}{2}+x+\frac{1}{4}$. Then the value of $\left(\int_{1/4}^{3/4}f(f(x))dx\right)^{-1}$ is _______.

84. Let $f:[0,\infty)\to R$ be a continuous strictly increasing function, such that $f^3(x)=\int_0^x t.f^2(t)dt$ for every $x\geq 0$, then value of $f(6)$ is _______.

85. If the value of the definite integral $\int_0^1 {}^{207}C_7 x^{200}.(1-x)^7\,dx$ is equal to 1/k where $k \in N$, then the value of k/26 is____.

86. If the value of $\lim_{n\to\infty}\left(n^{-3/2}\right)\sum_{j=1}^{6n}\sqrt{j}$ is equal to $\sqrt{N}$, then the value of N/12 is ______.

87. The value of $2^{2010}\dfrac{\int_0^1 x^{1004}(1-x)^{1004}dx}{\int_0^1 x^{1004}\left(1-x^{2010}\right)^{1004}dx}$ is _______.

88. Let $J=\int_{-5}^{-4}(3-x^2)\tan(3-x^2)dx$ and $K=\int_{-2}^{-1}(6-6x+x^2)\tan(6x-x^2-6)dx$ Then J + K is _______.

89. Let $I_n=\int_0^1 x^n\sqrt{1-x^2}\,dx$ then find the value of $\lim_{n\to\infty}\dfrac{I_n}{I_{n-2}}$.

90. For differentiable function $f(x)$, if $\int_0^n f'(x)\left([x]-x+\frac{1}{2}\right)dx = A_1\int_0^n f(x)dx + A_2 f(0)+A_3 f(n)+A_4\sum_{r=0}^{n} f(r)$,

(where [.] Denotes the G.I.F and A_1, A_2, A_3, A_4 are constant $n \in N$) then $A_1+A_2+A_3+A_4$ is equal to_____.

91. For positive integers $k=1,2,3,......,n,$ let S_k denotes the area of ΔAOB_k (where 'O' is origin) such that $\angle AOB_k=\dfrac{k\pi}{2n}, OA=1$ and $OB_k=k.$ If the value of $\lim_{n\to\infty}\dfrac{1}{n^2}\sum_{k=1}^{n} S_k=\dfrac{\alpha}{\pi^2},$ then 'α' is equal to]

92. If $\lim_{n\to\infty}\sum_{k=0}^{n}\dfrac{{}^nC_k}{n^k(k+3)}=p$ then p is_____.

93. Let $f(x)$ be a continuous function with continuous first derivative on (a,b), where $b>a$, and let $\lim_{x\to a^+} f(x)=\infty$, $\lim_{x\to b^-} f(x)=-\infty$ and $f'(x)+f^2(x)\ge -1$, for all x in (a,b), if the minimum value of $(b-a)$ equals to k then k is_____.

94. Let $f(x)$ be a continuous function such that $f(x)>0$ for all $x\ge 0$ and $(f(x))^{101}=1+\int_0^x f(t)dt.$ then $(f(101))^{100}$ is equal to_____.

95. If $\int_x^{xy} f(t)dt$ is independent of x and $f(2)=2,$ if the value of $\int_1^x f(t)dt=k.\ln x$ then k is_____.

96. Let $y=f(x)$ be a quadratic function with $f'(2)=1.$ Then the value of the integral $\int_{2-\pi}^{2+\pi} f(x)\cdot\sin\left(\dfrac{x-2}{2}\right)dx$ is__.

97. If a_1, a_2 and a_3 are the three values of a which satisfy the equation $\int_0^{\pi/2}(\sin x + a\cos x)^3 dx$

$-\dfrac{4a}{\pi-2}\int_0^{\pi/2} x\cos x\,dx=2$ then $\left(a_1^2+a_2^2+a_3^2\right)$ is equal to_____.

98. If $\int_0^{\infty} \frac{\ell n\, t}{x^2+t^2}\,dt = \frac{\pi\, \ell n\, 2}{4}$ $(x>0)$ then the number of integral values of $'x'$ satisfying this equation is_____.

99. Let $F(x)=\int_{-1}^{x}\sqrt{4+t^2}\,dt$ and $G(x)=\int_{x}^{1}\sqrt{4+t^2}\,dt$ then the value of $(FG)'(0)$ is___. (where dash denotes the derivative).

100. Let $'x'$ be a real valued differentiable function satisfying $f\left(\frac{x}{y}\right)=f(x)-f(y)$ and $\lim_{x\to 0}\frac{f(1+x)}{x}=3.$ If the area bounded by the curve $y=f(x)$, the Y-axis and the line $y=3$, where $x, y\in R^+$ is K. then K is_____.

101. Let $S=\left\{(x,y):\frac{y(3x-1)}{x(3x-2)}<0\right\}$, $S'=\{(x,y)\in A\times B: -1\le A\le 1, -1\le B\le 1\}$, then the area of the region enclosed by all points in $S\cap S'$ is_____.

102. A positive real valued continuously differentiable functions *f* on the real line such that for all x

$f^2(x)=\int_0^x\left((f(t))^2+(f'(t))^2\right)dt+e^2$ then $f(-1)=$

103. If $\int_1^2 \frac{(x^2-1)\,dx}{x^3\cdot\sqrt{2x^4-2x^2+1}}=\frac{1}{k}$ then k is_____.

104. Let $h(x)=(fog)(x)+K$ where K is any constant. If $\frac{d}{dx}(h(x))=-\frac{\sin x}{\cos^2(\cos x)}$ if $j(x)=\int_{f(x)}^{g(x)}\frac{f(t)}{g(t)}dt,$ where f and g are trigonometric functions then the value of $j(0)$ is equal to $(\cos(1)=.54)$

105. For a $\ge$ 2, if the value of the definite integral $\int_0^{\infty}\frac{dx}{a^2+\left(x-(1/x)\right)^2}$ equals to $\frac{\pi}{5050}$ then $\frac{a}{25}$ is_____.

106. If $\int_0^{\pi}\sqrt{(\cos x+\cos 2x+\cos 3x)^2+(\sin x+\sin 2x+\sin 3x)^2}\,dx$ has the value equal to $\left(\frac{\pi}{k}+\sqrt{w}\right)$ where k and w are positive integers then $k^2+w^2=$

107. If the absolute value of the integral $I=\int_{\pi/4}^{\pi/2}\frac{x\cdot\cos 2x\cdot\cos x}{\sin^7 x}dx$ in the lowest form is $\frac{-a}{b}$ where $a,b\in N,$ then $(a+b)=$

108. If $\int_0^{\pi}\frac{x\sin^3 x}{4-\cos^2 x}dx=\pi\left(1-\frac{a\ln b}{c}\right)$ where a and b are prime and $c\in N,$ then $a+b+c=$

109. Consider a real valued continuous function f such that $f(x)=\sin x+\int_{-\pi/2}^{\pi/2}(\sin x+tf(t))dt.$ then minimum value of $f(x)$ is_____.

JEE Advanced Revision Booklet	Differential Equations

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. The solution of $\frac{xdy}{x^2+y^2} = \left(\frac{y}{x^2+y^2} - 1\right)dx$ is :

(A) $y = x\cot(c-x)$ **(B)** $cos^{-1}\left(\frac{y}{x}\right) = -x+c$

(C) $y = x\tan(c-x)$ **(D)** $\frac{y^2}{x^2} = x\tan(c-x)$

2. The solution of $(y(1+x^{-1})+\sin y)dx + (x+\log_e x + x\cos y)dy = 0$ is :

(A) $(1+y^{-1}\sin y)+x^{-1}\log_e x = C$ **(B)** $(y+\sin y)+xy\log_e x = C$

(C) $xy + y\log_e x + x\sin y = C$ **(D)** None of these

3. The solution of $x^3\frac{dy}{dx} + 4x^2\tan y = e^x\sec y$ satisfying $y(1) = 0$ is :

(A) $\tan y = (x-2)e^x\log x$ **(B)** $\sin y = e^x(x-1)x^{-4}$

(C) $\tan y = (x-1)e^x x^{-3}$ **(D)** $\sin y = e^x(x-1)x^{-3}$

4. The solution of $\frac{xdx+ydy}{xdy-ydx} = \sqrt{\frac{a^2-x^2-y^2}{x^2+y^2}}$ is :

(A) $\sqrt{x^2+y^2} = a\left\{\sin\left(\tan^{-1}\frac{y}{x}+C\right)\right\}$ **(B)** $\sqrt{x^2+y^2} = a\cos\left\{\tan^{-1}\frac{y}{x}+C\right\}$

(C) $\sqrt{x^2+y^2} = a\{\tan(\sin^{-1}y/x)+C\}$ **(D)** None of these

5. The curve $y = f(x)$ is such that the area of the trapezium formed by the coordinate axes ordinate of an arbitrary point and the tangent at this point equals half the square of its abscissa. The curve is

(A) $y = cx^2 \pm x$ **(B)** $y = cx^2 \pm 1$ **(C)** $y = cx \pm x^2$ **(D)** $y = cx^2 \pm x \pm 1$

6. The real value of m for which the substitution $y = u^m$ will transform the differential equation $2x^4y\frac{dy}{dx} + y^4 = 4x^6$ into a homogeneous equation is

(A) $m = 0$ **(B)** $m = 1$ **(C)** $m = \frac{3}{2}$ **(D)** $m = \frac{2}{3}$

7. Solution of the differential equation $x = 1 + xy\frac{dy}{dx} + \frac{x^2y^2}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{x^3y^3}{3!}\left(\frac{dy}{dx}\right)^3 + ...$ is :

(A) $y = ln(x)+c$ **(B)** $y = (ln\,x)^2 + c$

(C) $y = \pm\sqrt{\{(ln\,x)^2+c}$ **(D)** $xy = x^y + c$

8. The solution of the differential equation $\frac{dy}{dx}=\frac{1}{xy[x^2\sin y^2+1]}$ is :

(A) $x^2(\cos y^2-\sin y^2-2Ce^{-y^2})=2$ **(B)** $y^2(\cos x^2-\sin y^2-2Ce^{-y^2})=2$

(C) $x^2(\cos y^2-\sin y^2-e^{-y^2})=4C$ **(D)** None of these

Paragraph for Questions 9 - 13

Let us represent the derivative *dy*/*dx* by p. An equation of the form $y=px+f(p)$ **. . . .(i)**

Is known as Clairut's equation where *f* (*p*) is a function of p. To solve equation (1), we differentiate the equation with respect to x, we get $p=p+x\frac{dp}{dx}+f'(p)\frac{dp}{dx}$ $\Rightarrow\left[x+f'(p)\right]\frac{dp}{dx}=0\Rightarrow\frac{dp}{dx}=0$ **. . . .(ii)** Or, $x+f'(p)=0$ **. . . .(iii)**

Now, (ii) gives p = constant = c, say

Then eliminating p from (i) we get y = cx + *f* (c) **. . . .(iv)**

Which is a solution of equation (i). If we eliminate p between (i) and (iii) we will obtain another solution not contained in the general solution (iv). This solution is known as the singular solution.

9. The general equation of the equation y = px + log p which does not contain the singular solution, is

(A) y = cx + logc **(B)** $y=cx+\frac{1}{c}$ **(C)** y = logx + c **(D)** y = –logx + c

10. The singular solution of the differential equation given in previous problem is :

(A) y = –x + 1 **(B)** y = x + 1 **(C)** y + 1 = logx **(D)** y + 1 = –log(–x)

11. Non-singular solution of the differential equation $x\frac{dy}{dx}=y-\left(\frac{dy}{dx}\right)^2$ is :

(A) $y^2 = cx + c$ **(B)** $y = cx + c^2$ **(C)** $cy = x^2 + c$ **(D)** $y = cx^2 + c$

12. Singular solution of the differential equation given in the previous question is :

(A) $y=\frac{x}{4}$ **(B)** $y=\frac{x^2}{4}$ **(C)** $y=-\frac{x^2}{4}$ **(D)** y = x

13. Solution of the differential equation $x^2\left(y-x\frac{dy}{dx}\right)=y\left(\frac{dy}{dx}\right)^2$ which does not contain singular solution is :

(A) $x^2(y-xc)=yc^2$ **(B)** $y=cx+c^2$ **(C)** $y^2=cx^2+c^2$ **(D)** $xy = cx^2 + c$

14. The solution of $y=2x\left(\frac{dy}{dx}\right)+x^2\left(\frac{dy}{dx}\right)^4$ is :

(A) $y=2c^{1/2}x^{1/4}+c$ **(B)** $y=2\sqrt{c}\,x^2+c^2$ **(C)** $y=2\sqrt{c}\,(x+1)$ **(D)** $y=2\sqrt{cx}+c^2$

15. Solution of the differential equation $x\cos\left(\frac{y}{x}\right)(ydx+xdy)=y\sin\left(\frac{y}{x}\right)(xdy-ydx)$ is :

(A) $y=cx\cos\left(\frac{x}{y}\right)$ **(B)** $\sec\left(\frac{y}{x}\right)=cxy$ **(C)** $\left(\frac{y}{x}\right)\sec\left(\frac{y}{x}\right)=c$ **(D)** none of these

16. Solution of the differential equation $\left\{\frac{1}{x}-\frac{y^2}{(x-y)^2}\right\}dx+\left\{\frac{x^2}{(x-y)^2}-\frac{1}{y}\right\}dy=0$ is :

(A) $ln\left|\frac{x}{y}\right|+\frac{xy}{x-y}=c$ **(B)** $\frac{xy}{x-y}=ce^{x/y}$ **(C)** $ln|xy|=c+\frac{xy}{x-y}$ **(D)** None of these

17. If the solution of the differential equation $\dfrac{xdx-ydy}{xdy-ydx}=\sqrt{\dfrac{1+x^2-y^2}{x^2-y^2}}$ be $\sqrt{f(x,y)}+\sqrt{1+f(x,y)}=c\left(\dfrac{x+y}{\sqrt{f(x,y)}}\right)$, then $f(x,y)$ is :

(A) x^2+y^2 **(B)** $1+x^2-y^2$ **(C)** x^2-y^2 **(D)** $\dfrac{x^2-y^2}{x^2+y^2}$

18. The solution of the equation $x\int_0^x y(t)dt=(x+1)\int_0^x ty(t)dt, x>0$ as $y=f(x)$ is :

(A) $y=ce^{-1/x}$ **(B)** $y=\dfrac{c}{x^3}$ **(C)** $y=-\dfrac{1}{2}-clnx$ **(D)** $y=\dfrac{ce^{-1/x}}{x^3}$

19. Solution of the equation $\dfrac{xdx+ydy}{xdy-ydx}=\sqrt{\dfrac{a^2-x^2-y^2}{x^2+y^2}}$ is :

(A) $\sqrt{x^2-y^2}=a\tan\left\{\tan^{-1}\left(\dfrac{y}{x}\right)+c\right\}$ **(B)** $\sqrt{x^2+y^2}=a\sin\left\{\tan^{-1}\left(\dfrac{y}{x}\right)+c\right\}$

(C) $\sqrt{x^2+y^2}=a\tan\left\{\tan^{-1}\left(\dfrac{y}{x}\right)+c\right\}$ **(D)** $\sqrt{x^2-y^2}=a\cos\left\{\tan^{-1}\left(\dfrac{y}{x}\right)+c\right\}$

20. Through any point (x, y) of a curve which passes through the origin, lines are drawn parallel to the co-ordinate axes. The curve, given that it divides the rectangle formed by the two lines and the axes into two areas, one of which is twice the other, represents a family of :

(A) circles **(B)** pair of straight lines

(C) parabolas **(D)** rectangular hyperbolas

21. The orthogonal trajectories of the family of coaxial circles $x^2+y^2+2gx+C=0$, where g is a parameter are

(A) family of circles with centre on y-axis **(B)** system of coaxial parabolas

(C) $x^2+y^2-C'x-Cy=0$, where C' is an arbitrary constant **(D)** system of circles with centre on x-axis

22. A curve $f(x)$ passes through the point $P(1,1)$. The normal to the curve at point P is $a(y-1)+(x-1)=0$. If the slope of the tangent at any point on the curve is proportional to the ordinate at that point, then the equation of the curve is

(A) $y=e^{ax}-1$ **(B)** $y-1=e^{ax}$ **(C)** $y=e^{a(x-1)}$ **(D)** $y-a=e^{ax}$

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

23. The solution of $\left(\dfrac{dy}{dx}\right)^2+2y\cot x\dfrac{dy}{dx}=y^2$ is :

(A) $y-\dfrac{c}{1+\cos x}=0$ **(B)** $y=\dfrac{c}{1-\cos x}$ **(C)** $x=2\sin^{-1}\sqrt{\dfrac{c}{2y}}$ **(D)** $x=2\cos^{-1}\sqrt{\dfrac{c}{2y}}$

24. The solution of $\dfrac{dy}{dx}+x=xe^{(n-1)y}$ is :

(A) $\dfrac{1}{n-1}\log\left(\dfrac{e^{(n-1)y}-1}{e^{(n-1)y}}\right)=\dfrac{x^2}{2}+C$ **(B)** $e^{(n-1)y}=Ce^{(n-1)y+(n-1)x^2/2}+1$

(C) $\log\left(\dfrac{e^{(n-1)y}-1}{(n-1)e^{(n-1)y}}\right)=x^2+C$ **(D)** $e^{(n-1)y}=ce^{(n-1)x^2/2+x}+1$

25. A tangent drawn to the curve $y = f(x)$ at P(x, y) cuts the x-axis and y-axis at A and B respectively such that BP: AP = 3:1, given that f(1) = 1, then :

(A) Equation of curve is $x\frac{dy}{dx} - 3y = 0$ **(B)** Normal at (1,1) is $x + 3y = 4$

(C) Curve passes through (2, 1/8) **(D)** Equation of curve is $x\frac{dy}{dx} + 3y = 0$

26. If $f(x), g(x)$ be twice differentiable functions on [0, 2] satisfying $f''(x) = g''(x)$, $f'(1) = 2g'(1) = 4$ and $f(2) = 3g(2) = 9$, then :

(A) $f(4) - g(4) = 10$ **(B)** $|f(x) - g(x)| < 2 \Rightarrow -2 < x < 0$

(C) $f(2) = g(2) \Rightarrow x = -1$ **(D)** $f(x) - g(x) = 2x$ has real root

27. Given the differential equation $\frac{dy}{dx} = \frac{6x^2}{2y + \cos y}$; $y(1) = \pi$ and the following statements

(A) Solution is $y^2 - \sin y = -2x^3 + c$ **(B)** Solution is $y^2 + \sin y = 2x^3 + c$

(C) $c = \pi^2 - 2$ **(D)** $c = \pi^2 + 2$

28. The equation of the curve satisfying the differential equation $y_2\left(x^2 + 1\right) = 2xy_1$ passing through the point (0, 1) and having slope of tangent at x = 0 as 3(where y_2 and y_1 represents 2nd and 1st order derivative), then :

(A) $y = f(x)$ is strictly increasing function **(B)** $y = f(x)$ is non-monotonic function

(C) $y = f(x)$ has three distinct real roots **(D)** $y = f(x)$ has only one negative root

29. For the central conics having their axes along the coordinates :

(A) Differential equation is $y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2 + xy\frac{d^2y}{dx^2}$ **(B)** Order is 2 and degree is1

(C) Differential equation is $xy\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2 + y\left(\frac{d^2y}{dx^2}\right)^3$ **(D)** Order is 2 and degree is 3

30. For the differential equation $\left(3x + 2y^2\right)ydx + 2x\left(2x + 3y^2\right)dy = 0$

(A) on simplification it reduces to $2\left(xy^3\right)d\left(xy^3\right) + d\left(x^3y^4\right) = 0$ **(B)** solution is $x^2y^6 + x^3y^4 = c$

(C) on simplification it reduces to $2\left(xy^2\right)d\left(xy^2\right) + d\left(x^2y^3\right) = 0$ **(D)** solution is $x^2y^4 + x^2y^3 = 0$

31. If y_1, y_2 are the solution of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$, then :

(A) $y = y_1 + c(y_1 - y_2)$ is the general solution of equation

(B) $y = y_1 + c(y_1 + y_2)$ is the general solution of equation

(C) $\alpha y_1 + \beta y_2$ is a solution of $\alpha + \beta = 1$ **(D)** $\alpha y_1 + \beta y_2$ is a solution of $\alpha - \beta = 1$

32. If the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of air is 30°C and the substance cools from 37°C to 34°C in 15 min then:

(A) Temperature of substance will be 32°C at t = $15\log_{7/4}\frac{7}{2}$ min

(B) Temperature of substance will be 31°C at t = $15\log_{7/4} 7$ min

(C) Proportional constant $k = \frac{1}{15}\log\frac{7}{4}$

(D) All of these

33. A right circular cylinder with radius R and height H contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant = K > 0). If T is time after which cylinder will be empty, then

(A) T is dependent on R
(B) T is independent of R
(C) T is dependent on H
(D) T is dependent on K

34. Let $S_1 = x^2 + y^2 - kx = 0$ and $S_2 = x^2 - y^2 - cx = 0$, then

(A) S_1 and S_2 intersect at an angle of $\pi/4$
(B) S_1 and S_2 intersect orthogonally
(C) abscissa of the point of intersection of S_1 and S_2 is A.M. of c and k.
(D) point of intersection of S_1 and S_2 is origin.

35. If a curve $y = f(x)$, passing through the point (2, 1) satisfies the condition that length of subtangent is equal to slope of tangent in 1st quadrant given that $\frac{dy}{dx} > 0$, then :

(A) Curve $y = f(x)$ is a parabola
(B) $y = f(x)$ is $y^2 = \frac{x}{2}$
(C) $y = f(x)$ is $y^2 = x - 1$
(D) Area bounded by $y = f(x)$, y-axis and y = 0 , y = 3 is 18 sq. units

36. Consider the differential equation $\cos^2 x\frac{dy}{dx} - (\tan 2x)y = \cos^4 x;\ |x| < \frac{\pi}{4}$ and $y\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{8}$ then :

(A) Integrating factor of the differential equation is $\frac{\cos 2x}{1+\cos 2x}$
(B) Solution is $y = \frac{1}{2}\tan 2x \cos^2 x$
(C) Solution is $y = \tan 2x \cos^2 x - \frac{3\sqrt{3}}{8}$
(D) $y\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{8}$

37. If the length of perpendicular from origin to any normal to the curve y = $f(x)$ is equal to its y intercept, then

(A) Curve is $x^2 = 4y + c$
(B) Curve is y = c
(C) Curve is x = c
(D) Equation of normal to the curve y = k

38. If $f(x) = \int_1^x \frac{\log t}{1+t+t^2}dt,\ x \ge 1$, then :

(A) $f(x) = \int_1^x \frac{t\log t}{1+t+t^2}dt,\ x \ge 1$
(B) $f(x) = \int_1^{1/x} \frac{\log t}{1+t+t^2}dt,\ x \ge 1$
(C) $f(x) = f\left(\frac{1}{x}\right)$
(D) All of these

39. If $f(x)$ is a function such that $x\int_0^x (1-t)f(t)dt = \int_0^x tf(t)dt;\ f(1) = 1$, then :

(A) $f(x) = -\frac{1}{x} - 3\ln x + 1$
(B) $x^2 + y^2 + 2ax + 2by + c = 0$
(C) Degree of the differential equation is 1
(D) All of these

40. The solutions of $y = x\left(\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3\right)$ are given by (where $p = \frac{dy}{dx}$ and k is constant)

(A) The constant function y = 0 **(B)** $y = kp^{-3}e^{1/2p^2}(p+p^3)$

(C) $y = kp^3e^{-1/2p^2}(p+p^3)$ **(D)** $ye^{-1/2p^2} = p^{-2}+1$

41. If $|y| = f(x)$ is solution of $\frac{d^2y}{dx^2} = \frac{x^3}{y^3}\frac{d^2x}{dy^2}$ such that $f(0) = 2$ and $y = g(x)$ is solution of $\frac{d^2y}{dx^2} + \frac{8y^3}{x^3} \cdot \frac{d^2x}{dy^2} = 0$ such that $g(1) = \frac{1}{3}$, then:

(A) Domain of region $f(x) \cap g(x)$ is $[-2, 2]$ **(B)** Domain of region $f(x) \cap g(x)$ is $\left[-\sqrt{3}, \sqrt{3}\right]$

(C) Range of region $f(x) \cap g(x)$ is $[0, 2]$ **(D)** Range of region $f(x) \cap g(x)$ is $\left[0, \sqrt{3}\right]$

42. If $y = e^{-x}\sin x$ and $y_n + a_n y = 0$ where a_n is constant for $n \in N$ & $y_n = \frac{d^n y}{dx^n}$ (nth derivative of y), then:

(A) $a_4 = 4$ **(B)** $a_8 = -16$ **(C)** $a_{12} = 64$ **(D)** $a_{16} = 256$

43. If $y = f(x); f(x) \geq 0$ & $f(0) = 0$ bounding a curvilinear trapezoid with base $[0, x]$ whose area is proportional to 3^{rd} power of $f(x)$. If $f(1) = 3$, then:

(A) Range of $f\left(\sin^2 x\right)$ is $[-3, 3]$

(B) Domain of $f\left(\ln\left(2^x - 3\right)\right)$ is $[2, \infty)$

(C) Range of $f\left(\sec^2 x\right)$ is $[3, \infty)$

(D) Area bounded by $y = f(x)$ line $x = 0, y = 1$ & $y = 2$ is $\frac{7}{9}$

44. If differential equation of the curve $y = ae^{3x} + be^{2x} + ce^x$ is $\frac{d^3y}{dx^3} + m\frac{d^2y}{dx^2} + n\frac{dy}{dx} + py = 0$, then:

(A) $(m+n)$ is a prime number **(B)** $(m-n)$ is always divisible by any odd number

(C) $m+n+p$ is a negative integer **(D)** $|m|$ is divisible by two prime numbers

45. If right circular cone with radius 18 & height 27 contains a liquid which evaporates at a rate proportional to its surface area in contact with air (proportionality constant $= k > 0$). If volume of liquid is V & r is radius of surface of liquid left, then:

(A) $\frac{1}{r^2}\frac{dV}{dr} = \frac{3\pi}{2}$ **(B)** $\frac{1}{r^2}\frac{dr}{dt} = -k\pi$

(C) Radius as function of time $r(t) = \frac{-2k}{3}t + c$ **(D)** Total time taken to empty the cone is $\frac{3}{2}$ unit

46. If A tangent drawn to the curve $y = f(x)$ at (x, y) cuts the $x-$ axis and $y-$ axis at A and B respectively such that $\frac{BP}{AP} = \frac{3}{1}$ given $f(1) = 1$, then:

(A) differential equation of curve may be $x\frac{dy}{dx} + 3y = 0$

(B) differential equation of curve may be $x\frac{dy}{dx} = 3y$

(C) If tangent at $R(\alpha, \beta)$ intersect again at $S(m, n)$ then $m + 2\alpha = 0$

(D) equation of normal at $(1,1)$ is $3y = x + 2$

47. If $\frac{dy}{dx} = \frac{x^2 - y}{x + y}$ such that $y = f(x)$ is a solution of differential equation & $f(0) = 0$, then:

(A) $(f(1)+1)^2 = \frac{5}{3}$ **(B)** $(f(-1)-1)^2 = \frac{1}{3}$ **(C)** $(f(3)+3)^{2/3} = 3$ **(D)** $|f(-3)-3| = 3$

48. Let C be a curve such that the normal at any point P on it meets x-axis and y-axis at A and Y respectively. If $BP : PA = 1:2$ (internally) and the curve passes through the point $(0,4)$ then which of the following alternative(s) is/are correct?

(A) The curves passes through $(\sqrt{10}, -6)$

(B) The equation of tangent at $(4, 4\sqrt{3})$ is $2x + \sqrt{3}y = 20$

(C) The differential equation for the curve is $yy' + 2x = 0$

(D) The curve represent a hyperbola

49. A differentiable function satisfies $f(x) = \int_0^x \{f(t)\cos t - \cos(t - x)\}dt$. which is of the following hold good?

(A) $f(x)$ has a minimum value $1 - e$ **(B)** $f(x)$ has a maximum value $1 - e^{-1}$

(C) $f''\left(\frac{\pi}{2}\right) = e$ **(D)** $f'(0) = 1$

50. Let $\frac{dy}{dx} + y = f(x)$ where y is a continuous function of x with $y(0) = 1$ and $f(x) = \begin{cases} e^{-x}, & \text{if } x \le 2 \\ e^{-2}, & \text{if } x > 2 \end{cases}$. Which is of the following hold(s) good?

(A) $y(1) = 2e^{-1}$ **(B)** $y'(1) = -e^{-1}$ **(C)** $y(3) = -2e^{-3}$ **(D)** $y'(3) = -2e^{-3}$

51. The function $f(x)$ satisfying the equation $f^2(x) + 4f'(x).f(x) + [f'(x)]^2 = 0$

(A) $f(x) = C.e^{(2-\sqrt{3})x}$ **(B)** $f(x) = C.e^{(2+\sqrt{3})x}$

(C) $f(x) = C.e^{(\sqrt{3}-2)x}$ **(D)** $f(x) = C.e^{-(2+\sqrt{3})x}$

52. Which of the following pair(s) is/are orthogonal?

(A) $16x^2 + y^2 = C$ and $y^{16} = kx$ **(B)** $y = x + Ce^{-x}$ and $x + 2 = y + ke^{-y}$

(C) $y = Cx^2$ and $x^2 + 2y^2 = k$ **(D)** $x^2 - y^2 = C$ and $xy = k$

53. The general solution of the differential equation , $x\left(\frac{dy}{dx}\right)=y.\log\left(\frac{y}{x}\right)$ is:

(A) $y=xe^{1-Cx}$ **(B)** $y=xe^{1+Cx}$ **(C)** $y=ex.e^{Cx}$ **(D)** $y=xe^{Cx}$

54. Identify the statement(s) which is/are true?

(A) $f(x,y)=e^{y/x}+\tan\frac{y}{x}$ is homogeneous of degree zero.

(B) $x.\log\frac{y}{x}dx+\frac{y^2}{x}\ \sin^{-1}\frac{y}{x}dy=0$ is homogeneous

(C) $f(x,y)=x^2+\sin x.\cos y$ is not homogeneous.

(D) $\left(x^2+y^2\right)dx-\left(xy^2-y^3\right)dy=0$ is a homogeneous differential equation.

55. A function $y=f(x)$ satisfying the differential equation $\frac{dy}{dx}.\sin x-y\cos x+\frac{\sin^2 x}{x^2}=0$ is such that, $y\left(\frac{\pi}{2}\right)=\frac{2}{\pi}$, then the statement which is correct?

(A) $\lim\limits_{x\to 0} f(x)=1$ **(B)** $\int_0^{\pi/2} f(x)dx$ is less then $\frac{\pi}{2}$

(C) $\int_0^{\pi/2} f(x)dx$ is greater than unity **(D)** $f(x)$ is an odd function

56. Identify the statement(s) which is/are true?

(A) The order of differential equation $\sqrt{1+\frac{d^2y}{dx^2}}=x$ is 1.

(B) solution of the differential equation

$xdy-ydx=\sqrt{x^2+y^2}dx$ **is** $y+\sqrt{x^2+y^2}=Cx^2$**.**

(C) $\frac{d^2y}{dx^2}=2\left(\frac{dy}{dx}-y\right)$ is differential equation of family of curves $y=e^x\left(A\cos x+B\sin x\right)$.

(D) The solution of differential equation

$\left(1+y^2\right)+\left(x-2e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$ is $xe^{\tan^{-1}y}=e^{3\tan^{-1}y}+k$.

57. Let $y=f(x)$ be a curve in the first quadrant such that the triangle formed by the co-ordinate axis and the tangent at any point on the curve has area 2. If $y(1)=1$, then $y(2)=$

(A) 0 **(B)** 1 **(C)** 2 **(D)** $\frac{1}{2}$

58. If $f(x)=x+\int_1^x \frac{f(t)}{t}dt$, then $\int_0^{\pi}\frac{\left(f(\sin\theta)-\sin\theta\right)}{\sin\theta}d\theta$ is equal to:

(A) $-\frac{\pi}{2}\ln 2$ **(B)** $\frac{\pi}{2}\ln 2$ **(C)** $-\pi\ln 2$ **(D)** $\pi\ln 2$

59. If $\frac{dy}{dx}+y\frac{dx}{dy}=x, y(-2)=1$, then :

(A) $x+3y^2=1$ **(B)** $2x+y+3=0$ **(C)** $x+y+1=0$ **(D)** $x^2-4y=0$

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column 1 are labelled as (A), (B), (C) & (D) whereas statements in Column 2 are labeled as p, q, r, s & t. More than one choice from Column 2 can be matched with Column 1.

60. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	The value of $\int_{-2}^{2}\left(ax^3+bx+c\right)$ dx depends on	**(p)**	$\frac{1}{n-1}$
(B)	$\int_{-1}^{1}\left(ax^3+bx\right)dx=0$ is true for all real values of	**(q)**	0
(C)	$\left(\sum_{n=1}^{10}\int_{-2n-1}^{-2n}\sin^{27}(x)dx+\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}(x)dx\right)$	**(r)**	$\frac{1}{n+1}$
(D)	$\int_{0}^{\pi/4}\left(\tan^n(x)+\tan^{n-2}(x)\right)d\left(x-[x]\right)$ (where [.] is G.I.F.)	**(s)**	a, b
		(t)	a, c
		(u)	c

61. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	If the curve satisfy the equation $\left(e^x+1\right)ydy=(y+1)e^x dx$ passes through (0, 0) and (k, 1) then k is	**(p)**	1
(B)	If the curve satisfy the equation $x\frac{dy}{dx}+y=xy^3$ passes, through (1,1) and $\left(\frac{3}{2}, p\right)$ then p is	**(q)**	Not defined
(C)	If $\frac{dy}{dx}=\frac{xy+y}{xy+x}$, then the solution of the differential equation always passes through the point origin and (k, 1) then k is	**(r)**	$\ln(e-1)$
(D)	The solution of the equation $\frac{dy}{dx}=\frac{3x-4y-2}{3x-4y-3}$ passes through origin the distance of it from (–1, 1) is	**(s)**	ϕ

62. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	Order of differential equation whose general solution is given by $y=(c_1+c_2)\cos(x+c_3)-c_4e^{x+c_5}+c_5\sin x$, where c_1, c_2, c_3, c_4, c_5 are arbitrary constants, is	**(p)**	1
(B)	Order of differential equation formed by eliminating the constants from $y=a\sin^2 x+b\cos^2 x+c\sin 2x+d\cos 2x+e\sin x,$, where a, b, c , d are arbitrary constants, is	**(q)**	2
(C)	The degree of equation $\frac{d^2y}{dx^2}-\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}=0$	**(r)**	3
(D)	Order of differential equation whose solution is $y=cx+c^2-3c^{3/2}+2$, where c is a parameter is	**(s)**	4

63. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	Order 1	**(p)**	Of all parabolas whose axis is the x-axis
(B)	Order 2	**(q)**	Of family of curve $y=a(x+a)^2$, where a is an arbitrary constant
(C)	Degree 1	**(r)**	$\left(1+3\frac{dy}{dx}\right)^{2/3}=\frac{4d^3y}{dx^3}$
(D)	Degree 3	**(s)**	Of family of curve $y^2=2c\left(x+\sqrt{c}\right)$, where c > 0

64. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	If the function $y=e^{4x}+2e^{-x}$ is a solution of the differential equation $\frac{\frac{d^3y}{dx^3}-13\frac{dy}{dx}}{y}=K$, then the value of K/3 is	**(p)**	3
(B)	Number of solutions which satisfy the differential equation $\frac{dy}{dx}+x\left(\frac{dy}{dx}\right)^2-y=0$ is	**(q)**	4
(C)	If real value of m for which the substitution, $y=u^m$ will transform the differential equation, $2x^4y\frac{dy}{dx}+y^4=4x^6$ in to a homogenous equation, then the value of 2m is	**(r)**	2
(D)	If the solution of differential equation $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=12y$ is $y=Ax^m+Bx^{-n}$, then $\lvert m-n\rvert$ is	**(s)**	1

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

65. Let $\frac{d}{dx}F(x)=\frac{e^{\sin x}}{x}; x>0$. If $\int_1^4 \frac{2e^{\sin x^2}}{x}dx = F(k)-F(1)$, then find one of the possible value of k/4.

66. If the dependent variable y is changed to 'z' by the substitution $y=\tan z$ and the differential equation $\frac{d^2y}{dx^2}=1+\frac{2(1+y)}{1+y^2}\left(\frac{dy}{dx}\right)^2$ is changed to $\frac{d^2z}{dx^2}=\cos^2 z+k\left(\frac{dz}{dx}\right)^2$, then the value of k equals ____________.

67. Let $y=y(t)$ be a solution to the differential equation $y'+2ty=t^2$, then $16\lim\limits_{t\to\infty}\frac{y}{t}$ is ____________.

68. If the solution of the differential equation $\frac{dy}{dx}=\frac{1}{x\cos y+\sin 2y}$ is $x=ce^{\sin y}-k(1+\sin y)$, then the value of k is

69. If the independent variable x is changed to y, then the differential equation $x\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3-\frac{dy}{dx}=0$ is changed to $x\frac{d^2x}{dy^2}+\left(\frac{dx}{dy}\right)^2=k$ where k equals ____________.

70. The curve passing through the point (1, 1) satisfies the differential equation $\frac{dy}{dx}+\frac{\sqrt{(x^2-1)(y^2-1)}}{xy}=0$. If the curves passes through the point $(\sqrt{2},k)$ then the value of [k] is (where [.] represents greatest integer function.

71. Tangent is drawn at the point (x_i, y_i) on the curve y = $f(x)$, which intersects the x-axis at $(x_{i+1},0)$. Now, again a tangent is drawn at (x_{i+1}, y_{i+1}) on the curve which intersects the x-axis at $(x_{i+2},0)$ and the process is repeated n times, i.e., $i=1,2,3,........,n$. If $x_1,x_2,x_3,........,x_n$ form an arithmetic progression with common difference equal to $\log_2 e$ and curve passes through (0, 2). Now if curve passes through the point (–2, k), then the value of k is ____________.

72. The perpendicular from the origin to the tangent at any point on a curve is equal to the abscissa of the point of contact. Also curve passes through the point (1, 1). Then the length of intercept of the curve on the x-axis is ____________.

73. If the solution of the differential equation $\frac{dy}{dx}-y=1-e^{-x}$ and $y(0)=y_0$ has a finite value, when $x\to\infty$, then the value of $|2/y_0|$ is ____________.

74. Find the constant of integration by the general solution of the differential equation $(2x^2y-2y^4)dx+(2x^3+3xy^3)dy=0$ if curve passes through (1, 1).

75. A tank initially contains 50 gallons of fresh water. Brine contains 2 pounds per gallon of salt, flows into the tank at the rate of 2 gallons per minutes and the mixture kept uniform by stirring, runs out at the same rate. If it will take for the quantity of slat in the tank to increase from 40 to 80 pounds (in seconds) is 206λ, then find λ. (given $\ln 3 = 1.0986$)

76. If $f : R-\{-1\} \to R$ and f is differentiable function which satisfies:

$f\left(x+f(y)+xf(y)\right) = y+f(x)+yf(x) \forall x, y \in R-\{-1\}$, $f(1) \neq 1$ then find the value of $2019\ \left[1+f\left(2018\right)\right]$.

77. If $\phi(x)$ is a differential real-valued function satisfying $\phi'(x)+2\phi(x) \le 1$, then the maximum value of $2\phi\ (x)$, equal to ____.

78. The degree of the differential equation satisfied by the curves $\sqrt{1+x} - a\sqrt{1+y} = 1$, is ____.

79. Let $f(x)$ be a twice differentiable bounded function satisfy $2f^5(x).f'(x)+2\left(f'(x)\right)^3.f^5(x) = f''(x)$. If $f(x)$ is bounded in between $y = k_1$, and $y = k_2$, Then the number of integers between k_1 and k_2 is/are (where $f(0) = f'(0) = 0$)

80. Let $y(x)$ be a function satisfying $d^2y/dx^2 - dy/dx + e^{2x} = 0, y(0) = 2$ and $y'(0) = 1$. If maximum value of $y(x)$ is $y(\alpha)$, Then integral part of (2α) is____.

81. $y = f(x)$ is a particular solution of differential equation $\left(\left(\dfrac{dy}{dx}\right)^2 + y\dfrac{d^2y}{dx^2}\right) = e^x$ such that $f(0) = 1 = f'(0)$, then the value of $f^2(\ln 2)$ is ____.

82. The differential equation $\dfrac{dy}{dx} = \dfrac{(x+y+2)^2 - (y-1)^2}{(x+3)^2}$ determines a curve $y = f(x)$ & $f(0) = 1$ then $f(3)$ is equal to ____.

83. If $y = f(x)$ is solution of differential equation $\dfrac{dy}{2x\,dx} + \dfrac{y}{x^2} = \dfrac{\sin^{-1}x}{2x^2}$, if $f(0) = 0$, then, $\dfrac{\pi}{f(1)}$ is equal to ____.

84. If $f'(x) < 2f(x)$ where $f : \left[\dfrac{1}{2}, 1\right] \to R$ such that $f\left(\dfrac{1}{2}\right) = 2e$ then maximum value of $f(\ln 2)$ is____.

85. If differential equation of the curve $x^2 - y^2 = c\left(x^2+y^2\right)^2$ is $\dfrac{dy}{dx} = \dfrac{x\left(my^2 - x^2\right)}{y\left(3x^2+ny^2\right)}$, then $(m+n)$ is ____.

86. If $x\displaystyle\int_0^x f(t)\,dt = (x+1)\int_0^x t\,f(t)\,dt$ for $x \in R^+$. $f(1) = \dfrac{1}{e}$ and $g(x) = e^{\frac{1}{x}}.f(x)$ then, $|g'(1)|$ (where g' denotes $\dfrac{dg}{dx}$) is ____.

87. If differential equation of first degree of a curve is given by $x^2\left(\dfrac{dy}{dx}\right)^2 - x(2y-1)\dfrac{dy}{dx} + \left(y^2-y-2\right) = 0$, then $(y - 2015\,.\,x)$ is a positive prime number $'P'$ then the value of P is ____.

88. The order of differential equation of family of circles in a plane is m and highest power of second differential $\left(\frac{d^2y}{dx^2}\right)$ is n then $(m+n)$ ____.

89. Let $y = f(x)$ be a curve C_1 passing through $(2,2)$ and $\left(8,\frac{1}{2}\right)$ and satisfying a differential equation $y\left(\frac{d^2y}{dx^2}\right) = 2\left(\frac{dy}{dx}\right)^2$. Curve C_2 is the director circle of the circle $x^2 + y^2 = 2$. If the shortest distance between the curves C_1 and C_2 is $\left(\sqrt{p} - q\right)$ where $p, q \in N$, then find the value of $\left(p^2 - q\right)$.

90. A function $y = f(x)$ satisfies $xf'(x) - 2f(x) = x^4 f^2(x), \forall x > 0$ and $f(1) = -6$. Find the value of $f'\left(3^{1/5}\right)$.

91. For $y < 0$, if y is a differentiable function of x such that $y(x+y) = x$ and $\int \frac{dx}{x+2y} = -\ln(k-y) + c$ where $k \in N$ then $k =$

92. Let C be the curve passing through the point (1, 1) has the property that the perpendicular distance of the origin from the normal at any point P of the curve is equal to the distance of P from the x-axis, If the area bounded by the curve C and $x-$axis in the first quadrant is $\frac{k\pi}{2}$ square units, then the value of 'k' is____.

93. Let $y = f(x)$ be a curve passing through $\left(e, e^e\right)$ which satisfy the differential equation $\left(2ny + xy\log_e x\right)dx - x\log_e x\,dy = 0,\ x > 0,\ y > 0$. If $g(x) = \lim_{n\to\infty} f(x)$, then $\int_{1/e}^{e} g(x)\,dx =$

JEE Advanced Revision Booklet	Vector

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

★1. If $\hat{a}, \hat{b}, \hat{c}$ are three non-coplanar, mutually perpendicular unit vectors, then $[\hat{a}\ \vec{p}\ \vec{q}]\hat{a}+[\hat{b}\ \vec{p}\ \vec{q}]\hat{b}+[\hat{c}\ \vec{p}\ \vec{q}]\hat{c}$ is equal to :

(A) $(\hat{a}+\hat{b}+\hat{c})\times(\vec{p}\times\vec{q})$ **(B)** $\hat{a}+\hat{b}+\hat{c}+\vec{p}+\vec{q}$

(C) $\vec{p}+\vec{q}$ **(D)** $\vec{p}\times\vec{q}$

2. If vectors $\vec{a}=\dfrac{\vec{i}+\vec{j}}{\sqrt{2}}$, $\vec{b}=\dfrac{-\vec{i}+\vec{j}}{\sqrt{2}}$ and $\vec{c}=\vec{k}$ then the value of $(\vec{r}\cdot\vec{a})^2+(\vec{r}\cdot\vec{b})^2+(\vec{r}\cdot\vec{c})^2$ is equal to :

(A) $|\vec{r}|^2$ **(B)** $2\vec{r}$ **(C)** 0 **(D)** None of these

★3. If $\overline{a}$ is a unit vector and projection of $\overline{x}$ along $\overline{a}$ is 2 units and $(\overline{a}\times\overline{x})+\overline{b}=\overline{x}$, then $\overline{x}$ is given by :

(A) $\frac{1}{2}[\overline{a}-\overline{b}+(\overline{a}\times\overline{b})]$ **(B)** $\frac{1}{2}[2\overline{a}+\overline{b}+(\overline{a}\times\overline{b})]$

(C) $[\overline{a}+(\overline{a}\times\overline{b})]$ **(D)** None of these

★4. A vector $\vec{\alpha}=a\hat{i}+b\hat{j}+c\hat{k}$ is said to be rational vector, if a, b, c are all rational. If a rational vector with magnitude as positive integer makes an angle $\pi/4$ with vector $\overline{\beta}=\sqrt{2}\hat{i}+3\sqrt{2}\hat{j}+4\hat{k}$, then $\overline{\alpha}$:

(A) Surely lies in xy plane **(B)** Surely lies in xz plane

(C) Surely lies in yz plane **(D)** Nothing can be said

★5. Let $\overline{a},\overline{b}$ and $\overline{c}$ be three non-coplanar vectors and $\overline{d}$ be a non-zero vector, which is perpendicular to ($\overline{a}+\overline{b}+\overline{c}$). Now if $\overline{d}=\sin x(\overline{a}\times\overline{b})+\cos y(\overline{b}\times\overline{c})+2(\overline{c}\times\overline{a})$, then minimum value of x^2+y^2 is equal to :

(A) π^2 **(B)** 0 **(C)** $\dfrac{\pi^2}{4}$ **(D)** $\dfrac{5\pi^2}{4}$

★6. Let $ABCD$ be a tetrahedron in which position vectors of A, B, C and D are $\vec{i}+\vec{j}+\vec{k}$, $2\vec{i}+\vec{j}+2\vec{k}$, $3\vec{i}+2\vec{j}+\vec{k}$ and $2\vec{i}+3\vec{j}+2\vec{k}$. If ABC be the base of tetrahedron then height of tetrahedron is :

(A) $\sqrt{\dfrac{3}{2}}$ **(B)** $\sqrt{\dfrac{3}{5}}$ **(C)** $\dfrac{1}{3}\sqrt{\dfrac{2}{3}}$ **(D)** None of these

7. The position vectors of the vertices A, B and C of a triangle are three unit vectors $\hat{a}$, $\hat{b}$ and $\hat{c}$. A vector $\overline{d}$ is such that $\overline{d}\cdot\hat{a}=\overline{d}\cdot\hat{b}=\overline{d}\cdot\hat{c}$ and $\overline{d}=\lambda(\hat{b}+\hat{c})$, then triangle ABC is :

(A) Acute angled **(B)** Obtuse angled **(C)** Right angled **(D)** None of these

8. If $\overline{a}$, $\overline{b}$, $\overline{c}$ and $\overline{d}$ are four non-coplanar unit vectors, $\overline{a}$, $\overline{b}$, $\overline{c}$ are mutually perpendicular, such that $\overline{d}$ makes equal angles with all the three vectors $\overline{a}$, $\overline{b}$, $\overline{c}$, then :

(A) $\vec{a}+\vec{b}=\vec{b}+\vec{c}=\vec{c}+\vec{a}$ **(B)** $\overline{a}\times\overline{b}=\overline{b}\times\overline{c}=\overline{c}\times\overline{a}$

(C) $[\overline{d}\ \overline{a}\ \overline{b}]=[\overline{d}\ \overline{b}\ \overline{c}]=[\overline{d}\ \overline{a}\ \overline{c}]$ **(D)** $[\overline{d}\ \overline{a}\ \overline{b}]=[\overline{d}\ \overline{b}\ \overline{c}]=[\overline{d}\ \overline{c}\ \overline{a}]$

9. Let the vectors $\overline{a},\overline{b},\overline{c}$ and $\overline{d}$ be such that $\overline{c}-3\overline{d}=-\overline{b}-\overline{a}$. Then the points with position vectors as $\overline{a}$, $\overline{b}$, $\overline{c}$ and $\overline{d}$ are :

(A) coplanar **(B)** collinear **(C)** non-coplanar **(D)** None of these

10. Let $\vec{\alpha} = a\hat{i} + b\hat{j} + c\hat{k}$ and $\vec{\beta} = b\hat{i} + c\hat{j} + a\hat{k}$, where $a, b, c \in R$. If 'θ' be the angle between $\vec{\alpha}$ and $\vec{\beta}$ then :

(A) $\theta \in (0, \pi/2)$ **(B)** $\theta \in [0, 2\pi/3]$ **(C)** $\theta \in (2\pi/3, \pi]$ **(D)** None of these

11. If the lines $\vec{r} = \vec{a} + t\left(\vec{b} \times \vec{c}\right)$, and $\vec{r} = \vec{b} + s\left(\vec{c} \times \vec{a}\right)$ intersect (t and s are scalars) then :

(A) $\vec{a} \cdot \vec{c} = 0$ **(B)** $\vec{a} \cdot \vec{c} = \vec{b} \cdot \vec{c}$ **(C)** $\vec{b} \cdot \vec{c} = 0$ **(D)** None of these

12. If $4\overline{a} + 5\overline{b} + 9\overline{c} = 0$, then $(\overline{a} \times \overline{b}) \times [(\overline{b} \times \overline{c}) \times (\overline{c} \times \overline{a})]$ is equal to :

(A) A vector perpendicular to plane of $\overline{a}, \overline{b}$ and $\overline{c}$ **(B)** A scalar quantity

(C) $\vec{0}$ **(D)** None of these

★13. Given that $\vec{a}$ is perpendicular to $\vec{b}$ and p is a non-zero scalar, if $p\,\vec{r} + \left(\vec{r}\,.\,\vec{b}\right)\vec{a} = \vec{c}$ then $\vec{r} =$:

(A) $\vec{r} = \dfrac{\vec{c}}{p^2} - \dfrac{\vec{c}.\vec{b}}{p}\vec{a}$ **(B)** $\vec{r} = \dfrac{\vec{c}}{p} + \dfrac{\vec{c}.\vec{b}}{p^2}\vec{a}$ **(C)** $\vec{r} = \dfrac{\vec{c}}{p} - \dfrac{\vec{c}.\vec{b}}{p^2}\vec{a}$ **(D)** None of these

★14. If the plane faces of a tetrahedron are represented by the equations $\overline{r} \cdot (l\hat{i} + m\hat{j}) = 0$, $\overline{r} \cdot (n\hat{k} + m\hat{j}) = 0$, $\overline{r} \cdot (n\hat{k} + l\hat{i}) = 0$ and $\overline{r} \cdot (l\hat{i} + m\hat{j} + n\hat{k}) = p$, then the volume of the tetrahedron is :

(A) $\dfrac{lmn}{6p^3}$ **(B)** $\dfrac{3lmn}{2p^3}$ **(C)** $\dfrac{lmn}{p^3}$ **(D)** None of these

15. Let $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$. The point of intersection of the lines $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ is :

(A) $\hat{i} + \hat{j} - 3\hat{k}$ **(B)** $3\hat{i} + \hat{j} + \hat{k}$ **(C)** $3\hat{i} + \hat{j} - \hat{k}$ **(D)** None of these

16. If the non-zero vectors $\vec{a}$ and $\vec{b}$ are perpendiculars to each other then the solution of the equation $\vec{r} \times \vec{a} = \vec{b}$ is given by:

(A) $\vec{r} = x\vec{a} + \dfrac{1}{\vec{a}.\vec{a}}(\vec{a} \times \vec{b})$; $x \in R$ **(B)** $\vec{r} = x\vec{b} + \dfrac{1}{\vec{b}.\vec{b}}(\vec{a} \times \vec{b})$; $x \in R$

(C) $\vec{r} = x\,\vec{a} \times \vec{b}$; $x \in R$ **(D)** None of these

17. Let *OPQR* is a tetrahedron such that *O* is origin and $\vec{p}, \vec{q}, \vec{r}$ are position vectors of *P, Q, R* respectively and α is the angle which *OP* makes with face *PQR* then :

(A) $|\sin\alpha| = \dfrac{\left|[\vec{p}\,\vec{q}\,\vec{r}]\right|}{|\vec{q} \times \vec{r} + \vec{r} \times \vec{p} + \vec{p} \times \vec{q}\,| \cdot |\vec{p}|}$ **(B)** $\sin^2\alpha - \cos^2\alpha = \sqrt{3}$

(C) $\tan\alpha = \dfrac{\left|[\vec{p}\vec{q}\vec{r}]\right|}{|\vec{q} \times \vec{r} + \vec{r} \times \vec{p} + \vec{p} \times \vec{r}| \cdot |\vec{p}|}$ **(D)** None of these

18. If $\overline{r} = l(\overline{b} \times \overline{c}) + m(\overline{c} \times \overline{a}) + n(\overline{a} \times \overline{b})$ and $[\overline{a}\ \overline{b}\ \overline{c}] = 2$, then $l + m + n$ is equal to :

(A) $\overline{r} \cdot (\overline{b} \times \overline{c} + \overline{c} \times \overline{a} + \overline{a} \times \overline{b})$ **(B)** $1/2\ \overline{r} \cdot (\overline{a} + \overline{b} + \overline{c})$

(C) $(\overline{a}\ \overline{b}\ \overline{c})$ **(D)** None of these

19. If vectors $\overline{b} = (\tan\alpha, -1, 2\sqrt{\sin\frac{\alpha}{2}})$ and $\overline{c} = (\tan\alpha, \tan\alpha, -\dfrac{3}{\sqrt{\sin\alpha/2}})$ are orthogonal and vector $\overline{a} = (1, 3, \sin 2\alpha)$ makes an obtuse angle with the *z*-axis, then :

(A) $\alpha = \tan^{-1}(-2)$ **(B)** $\alpha = \tan^{-1}(-3)$ **(C)** $\alpha = \tan^{-1}(2)$ **(D)** None of these

20. Let position vector of point A be $\hat{i}+\hat{j}+\hat{k}$ and that of point B be $-\hat{i}+\hat{k}$, then the position vector of point $R(\vec{r})$ such that AR is perpendicular to BR and $\vec{r}$ is not perpendicular to $\left(\vec{r}-\left(\hat{j}+2\hat{k}\right)\right)$ is :

(A) $\vec{r}=\hat{i}+2\hat{j}$ **(B)** $\vec{r}=2\hat{i}+\hat{j}-\hat{k}$ **(C)** $\vec{r}=\hat{k}+2\hat{i}$ **(D)** None of these

★21. If three coterminous edges of a tetrahedron are $\vec{a}$, $\vec{b}$, $\vec{c}$ such that $|\vec{a}|=2$, $|\vec{b}|=3$, $|\vec{c}|=4$, angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$, $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{4}$ and $\vec{c}$ and $\vec{a}$ is $\frac{\pi}{6}$. The area of the base is 2 sq. units, then the height of the tetrahedron is :

(A) $3\sqrt{\left(\sqrt{3}-2\right)}$ **(B)** $3\sqrt{\left(\sqrt{6}-2\right)}$ **(C)** $\frac{3\sqrt{\left(\sqrt{6}-2\right)}}{2}$ **(D)** None of these

★22. If $\left[(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})\right].(\bar{c}\times\bar{b})=0$ then which of the following is always true :

(A) $\bar{a},\bar{b},\bar{c}$ and $\bar{d}$ are necessary coplanar **(B)** at least one of $\bar{a}$ or $\bar{d}$ must lie in plane of $\bar{b}$ and $\bar{c}$

(C) at least one of $\bar{b}$ or $\bar{c}$ must lie in plane of $\bar{a}$ and $\bar{d}$

(D) at least one of $\bar{a}$ or $\bar{b}$ must lie in plane of $\bar{c}$ and $\bar{d}$

★23. If $\hat{\alpha}$ and $\hat{\beta}$ be two perpendicular unit vectors such that $\vec{x}=\hat{\beta}-(\hat{\alpha}\times\vec{x})$, then $|\vec{x}|$ is equal to :

(A) 1 **(B)** $\sqrt{2}$ **(C)** $\frac{1}{\sqrt{2}}$ **(D)** None of these

24. If $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ are on a circle of radius R whose centre is at origin and $\vec{c}-\vec{a}$ is perpendicular to $\vec{d}-\vec{b}$, then $|\vec{d}-\vec{a}|^2+|\vec{b}-\vec{c}|^2=$ (AC is diameter)

(A) R^2 **(B)** $R^2/2$ **(C)** $2R^2$ **(D)** $4R^2$

25. The vector, directed along the internal bisector of the angle between the vectors $\vec{a}=7\hat{i}-4\hat{j}-4\hat{k}$ & $\vec{b}=-2\hat{i}-\hat{j}+2\hat{k}$ with $|\vec{c}|=5\sqrt{6}$ is:

(A) $\frac{5}{3}\left(\hat{i}-7\hat{j}+2\hat{k}\right)$ **(B)** $\frac{5}{3}\left(\hat{i}+7\hat{j}-2\hat{k}\right)$ **(C)** $\frac{5}{3}\left(-\hat{i}-7\hat{j}+2\hat{k}\right)$ **(D)** None of these

Passage for Questions 26 - 30

Unit vectors $\hat{i}$ and $\hat{j}$ are parallel to the adjacent edges of a large square table. The directions $\hat{i}$ and $\hat{j}$ are referred to as East and North. An ant creeping on the table makes the following movements successively :

(i) 4 *cm* 30° East of North **(ii)** 12 *cm* South West

(ii) 6 *cm* East and **(iv)** 9 *cm* West North

26. The resultant displacement of the ant after first two steps is :

(A) $\left(2+12\sqrt{2}\right)\hat{i}+\sqrt{3}\hat{j}$ **(B)** $\left(2-6\sqrt{2}\right)\hat{i}+\left(2\sqrt{3}+6\sqrt{2}\right)\hat{j}$

(C) $\left(2-6\sqrt{2}\right)\hat{i}+\left(2\sqrt{3}-6\sqrt{2}\right)\hat{j}$ **(D)** None of these

27. The final resultant displacement of the ant is :

(A) $\left(\frac{8\sqrt{2}+21}{\sqrt{2}}\right)\hat{i}+\left(\frac{2\sqrt{6}-3}{\sqrt{2}}\right)\hat{j}$ **(B)** $\left(\frac{8\sqrt{2}+21}{\sqrt{2}}\right)\hat{i}+\left(\frac{2\sqrt{6}+3}{\sqrt{2}}\right)\hat{j}$

(C) $\left(\frac{8\sqrt{2}-21}{\sqrt{2}}\right)\hat{i}+\left(\frac{2\sqrt{6}+3}{\sqrt{2}}\right)\hat{j}$ **(D)** $\left(\frac{8\sqrt{2}-21}{\sqrt{2}}\right)\hat{i}+\left(\frac{2\sqrt{6}-3}{\sqrt{2}}\right)\hat{j}$

28. Magnitude of the resultant displacement is given by :

(A) $\sqrt{301+186\sqrt{2}}$ **(B)** $\sqrt{301-168\sqrt{2}-6\sqrt{6}}$

(C) $\sqrt{301+6\sqrt{6}}$ **(D)** None of these

29. Direction of the ant's resultant displacement is :

(A) $\tan\theta=\frac{2\sqrt{6}+3}{8\sqrt{2}+21}$ **(B)** $\tan\theta=\frac{2\sqrt{6}-3}{8\sqrt{2}-21}$ **(C)** $\tan\theta=\frac{2\sqrt{6}-3}{8\sqrt{2}+21}$ **(D)** None of these

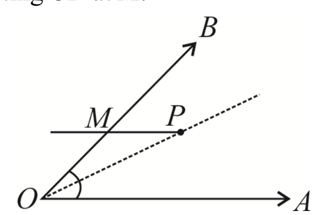
30. Direction of the ant's resultant displacement after first three steps is :

(A) $\tan\theta=\frac{\sqrt{3}-3\sqrt{2}}{4-3\sqrt{2}}$ **(B)** $\tan\theta=\frac{\sqrt{3}+3\sqrt{2}}{4+3\sqrt{2}}$ **(C)** $\tan\theta=\frac{\sqrt{3}-3\sqrt{2}}{4+3\sqrt{2}}$ **(D)** None of these

*Paragraph for Questions 31 - 35

Let two unit vectors along two lines $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be $\hat{a}$ and $\hat{b}$ respectively. Take their point of intersection as the origin and let *P* be any point on the bisector of angle between the lines *OA* and *OB*. Draw *PM* parallel to *AO* cutting *OB* at *M*.

$$\angle AOP=\angle POM=\angle OPM \text{ and hence } OM=PM.$$



But $\overrightarrow{OM}=t\hat{b}$ and $\overrightarrow{MP}=t\hat{a}$

(Since $\overrightarrow{OM}\parallel\hat{b}$ and $\overrightarrow{MP}\parallel\hat{a}$ and their magnitudes are same).

Then $\overrightarrow{OP}=\vec{r}=\overrightarrow{OM}+\overrightarrow{MP}=t(\hat{b}+\hat{a})$ **...(i)**

For external bisector *OP*', the angle between *OB* and *OA* is the same as the internal bisector of the angle between the unit vectors along them being $-\hat{b}$ and $\hat{a}$ and hence the equation of $\overrightarrow{OP}'$ be

$\overrightarrow{OP}'=\vec{r}=t(\hat{a}-\hat{b})$ **...(ii)**

For any two vectors $\vec{a}$ and $\vec{b}$ the equations (i) and (ii) reduce to $\vec{r}=t\left(\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{b}}{|\vec{b}|}\right)$

31. A vector $\vec{c}$, directed along the internal bisector of the angle between the vectors $\vec{a}=7\hat{i}-4\hat{j}-4\hat{k}$ and $\vec{b}=-2\hat{i}-\hat{j}+2\hat{k}$ with $|\vec{c}|=5\sqrt{6}$, is :

(A) $\frac{5}{3}(\hat{i}-7\hat{j}+2\hat{k})$ **(B)** $\frac{5}{3}(5\hat{i}+5\hat{j}+2\hat{k})$ **(C)** $\frac{5}{3}(\hat{i}+7\hat{j}+2\hat{k})$ **(D)** $\frac{5}{3}(-5\hat{i}+5\hat{j}+2\hat{k})$

32. Let *ABC* be a triangle and $\vec{a},\vec{b},\vec{c}$ be the position vectors of the point *A, B, C* respectively. External bisectors of $\angle B$ and $\angle C$ meet at *P* with the sides of the triangle as *a, b, c,* the position vector of *P* becomes :

(A) $\frac{(-b)\vec{b}+(-c)\vec{c}}{(b+c)}$ **(B)** $\frac{a\vec{a}+(-b)\vec{b}+(-c)\vec{c}}{(a-b-c)}$

(C) $\left(\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right)(abc)$ **(D)** $\frac{a\vec{a}+b\vec{b}+c\vec{c}}{(\vec{a}+\vec{b}+\vec{c})}$

33. If the interior and exterior bisectors of the angle *A* of a triangle *ABC* meet the base *BC* at *D* and *E*, then :

(A) $2BC=BD+BE$ **(B)** $BC^2=BD\times BE$

(C) $\frac{2}{BC}=\frac{1}{BD}+\frac{1}{BE}$ **(D)** None of these

34. If ABC be a triangle of sides a, b, c with position vectors of A, B, C as $\vec{a}, \vec{b}$ and $\vec{c}$ respectively, then the position vector of its incentre is :

(A) $\dfrac{(\vec{a}+\vec{b}+\vec{c})}{3}$ **(B)** $\left(\dfrac{\vec{a}\times\vec{b}+\vec{b}+\vec{c}+\vec{c}\times\vec{a}}{a^2+b^2+c^2}\right)$

(C) $\left(\dfrac{a\vec{a}+b\vec{b}+c\vec{c}}{a+b+c}\right)$ **(D)** None of these

35. *ABC* is a triangle. *AD, AD'* are internal and external bisectors of angle *A*, meeting *BC* at *D* and *D'* respectively. *A'* is the mid-point of *DD'* and *B', C'* are similar points on *CA* and *AB*. Then *A', B', C'*

(A) Lie on a plane **(B)** Form an equilateral triangle.

(C) Form an isosceles triangle **(D)** None of these

Paragraph for Questions 36 - 38

If $\vec{a},\vec{b},\vec{c}$ are cyclically permuted, the volume of the scalar triple product remains same. The change in scalar triple product changes the sign of scalar triple product but not the magnitude. In scalar triple product, the position of the dot and cross can be interchanged provided its cyclic nature is preserved. Also, the scalar triple product is ZERO if any two of them are equal.

36. $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$ is $\vec{a}$ and $\vec{c}$ are :

(A) perpendicular **(B)** collinear **(C)** parallel **(D)** None of these

37. The vector $OP = 5i + 12j + 13\hat{k}$ turns through an angle of $\dfrac{\pi}{2}$ about O passing through the positive side of $\hat{j}$ axis an iff way. The vector in the new position is:

(A) $\dfrac{2}{\sqrt{97}}\left(-30\hat{i}+97\hat{j}-78\hat{k}\right)$ **(B)** $\dfrac{2}{\sqrt{98}}\left(-30\hat{i}+97\hat{j}-78\hat{k}\right)$

(C) $\dfrac{3}{\sqrt{97}}\left(-30\hat{i}+97\hat{j}-78\hat{k}\right)$ **(D)** None of these

38. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-parallel unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b}$, the angle between $\vec{b}$ and $\vec{c}$ is:

(A) $\pi/2$ **(B)** $\pi/3$ **(C)** $\pi/4$ **(D)** $\pi/6$

Paragraph for Questions 39 - 40

Let $\vec{r}$ is a position vector of a variable point in Cartesian OXY plane such that $\vec{r}\cdot(10\hat{j}-8\hat{i}-\vec{r})=40$ and

$p_1=\max\left\{|\vec{r}+2\hat{i}-3\hat{j}|^2\right\}$, $p_2=\min\left\{|\vec{r}+2\hat{i}-3\hat{j}|^2\right\}$. A tangent line is drawn to the curve $y=\dfrac{8}{x^2}$ at the point A with abscissa 2. The drawn line cuts x-axis at a point B.

39. p_2 is equal to :

(A) 9 **(B)** $2\sqrt{2}-1$ **(C)** $6\sqrt{2}+3$ **(D)** $9-4\sqrt{2}$

40. p_1+p_2 is equal to :

(A) 2 **(B)** 10 **(C)** 18 **(D)** 5

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

41. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors and $\vec{d}$ is a unit vector which makes equal angles with $\vec{a}$, $\vec{b}$ and $\vec{c}$, then $|\vec{a}+\vec{b}+\vec{c}+\vec{d}|^2$ is equal to :

(A) $4+\sqrt{3}$ **(B)** $4-\sqrt{3}$ **(C)** $4+2\sqrt{3}$ **(D)** $4-2\sqrt{3}$

★42. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-coplanar vectors and $\vec{d}$ be a non-zero vector which is perpendicular to $\left(\vec{a}+\vec{b}+\vec{c}\right)$ and is represented as $\vec{d} = x\left(\vec{a}\times\vec{b}\right)+y\left(\vec{b}\times\vec{c}\right)+z\left(\vec{c}\times\vec{a}\right)$. Then :

(A) $x^3+y^3+z^3=3xyz$ **(B)** $xy+yz+xz\le 0$

(C) $x=y=z$ **(D)** $x^2+y^2+z^2=xy+yz+zx$

43. Let $\vec{a}$ and $\vec{b}$ be two units vectors if $\vec{u}=\vec{a}-\left(\vec{a}\cdot\vec{b}\right)\vec{b}$ and $\vec{v}=\vec{a}\times\vec{b}$ then $|\vec{v}|$ is :

(A) $|\vec{u}|$ **(B)** $|\vec{u}|+|\vec{u}\cdot\vec{a}|$ **(C)** $|\vec{u}|+|\vec{u}\cdot\vec{b}|$ **(D)** $|\vec{u}|+\vec{u}.\left(\vec{a}+\vec{b}\right)$

44. If $\vec{b}$ and $\vec{c}$ are two non-collinear vectors such that $\vec{a}.\left(\vec{b}+\vec{c}\right)=4$ and $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(x^2-2x+6\right)\vec{b}+\left(\sin y\right)\vec{c}$, then :

(A) $x=1$ **(B)** $y=-1$ **(C)** $y=\dfrac{\pi}{2}$ **(D)** $x+y=0$

★45. Let $\vec{\alpha}=a\hat{i}+b\hat{j}+c\hat{k}$, $\vec{\beta}=b\hat{i}+c\hat{j}+a\hat{k}$ and $\vec{\gamma}=c\hat{i}+a\hat{j}+b\hat{k}$ be three coplanar vectors, and $\vec{v}=\hat{i}+\hat{j}+\hat{k}$. Then $\vec{v}$ may be :

(A) $\perp$ to $\vec{\alpha},\vec{\beta},\vec{\gamma}$ **(B)** $\parallel$ to $\vec{\alpha},\vec{\beta},\vec{\gamma}$ **(C)** anti $\parallel$ to $\vec{\alpha},\vec{\beta},\vec{\gamma}$ **(D)** None of these

46. Let $\hat{\alpha}, \hat{\beta}$ and $\hat{\gamma}$ be the unit vectors such that $\hat{\alpha}$ and $\hat{\beta}$ are mutually perpendicular and $\hat{\gamma}$ is equally inclined to $\hat{\alpha}$ and $\hat{\beta}$ at an angle θ. If $\hat{\gamma}=x\hat{\alpha}+y\hat{\beta}+z\left(\hat{\alpha}\times\hat{\beta}\right)$, then :

(A) $z^2=1-2x^2$ **(B)** $z^2=1-2y^2$ **(C)** $z^2=1-x^2-y^2$ **(D)** $x^2=y^2$

47. Unit vectors $\hat{a}$ and $\hat{b}$ are inclined at an angle 2θ and $\left|\hat{a}-\hat{b}\right|<1$. If $0\le\theta<\pi$ then θ may belong to :

(A) $[0, \pi/6)$ **(B)** $(5\pi/6, \pi)$ **(C)** $[\pi/6, \pi/2]$ **(D)** $[\pi/6, 5\pi/6]$

48. The position vectors of the vertices *A, B* and *C* of a triangle are $\hat{i}+\hat{j}$, $\hat{j}+\hat{k}$ and $\hat{i}+\hat{k}$ respectively. A unit vector $\hat{r}$ lying in the plane of ΔABC and perpendicular to *IA*, where *I* is the incentre of the triangle is :

(A) $\dfrac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ **(B)** $\dfrac{\hat{i}-\hat{j}}{\sqrt{2}}$ **(C)** $\dfrac{\hat{j}-\hat{i}}{\sqrt{2}}$ **(D)** $\dfrac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

49. *A* and *B* are two points in space with position vector $\overline{a}$ and $\overline{b}$ respectively. Then the value of λ such that the system of equations $|3\overline{r}-2\overline{a}-\overline{b}|=|\overline{a}-\overline{b}|$ and $[\overline{r}-\lambda\overline{a}-(1-\lambda)\overline{b}]\cdot(\overline{a}-\overline{b})=0$ does not have any solution :

(A) 2 **(B)** 3 **(C)** –2 **(D)** None of these

50. The position vectors of the vertices A, B, C of a triangle are $\bar{a}, \bar{b}$ and $\bar{c}$ respectively, where $\bar{c} = \bar{a} \times \bar{b}$ and $\bar{a}$ and $\bar{b}$ are non–collinear vectors. If $\bar{d}$, the position vector of the centroid of the triangle ABC, makes equal angles 'α' with the vectors $\bar{a}, \bar{b}$ and $\bar{c}$, then :

(A) $|\bar{a}| = |\bar{b}|$. **(B)** $|\bar{a}| \neq |\bar{b}|$.

(C) the value of α is $\cos^{-1}\frac{1}{\sqrt{3}}$ if $\bar{a}.\bar{b} = 0$. **(D)** the value of α is $\cos^{-1}\sqrt{\frac{2}{5}}$ if $\bar{a}.\bar{b} = 0$.

51. The vector sum of $\vec{a}$ and $\vec{b}$ trisects the angle θ between them. If $|\vec{a}| = a$; $|\vec{b}| = b$; $a > b$, then:

(A) $\theta = 3\cos^{-1}\left(\frac{2b}{a}\right)$ **(B)** $\theta = 3\cos^{-1}\left(\frac{a}{2b}\right)$ **(C)** $|\vec{a}+\vec{b}| = \frac{a^2+b^2}{b}$ **(D)** $|\vec{a}+\vec{b}| = \frac{a^2-b^2}{b}$

52. The position vectors of the points A, B, C are respectively (1,1,1), (1,–1,2), (0,2,–1). The unit vector parallel to the plane determined by A, B, C and perpendicular to the vector (1,0,1) is/are :

(A) $\frac{5i+j-\hat{k}}{3\sqrt{3}}$ **(B)** $\frac{i+5j-\hat{k}}{3\sqrt{3}}$ **(C)** $\frac{-5i-j+\hat{k}}{3\sqrt{3}}$ **(D)** $\frac{-i-5j+\hat{k}}{3\sqrt{3}}$

53. A line passes through the points whose position vectors are $\vec{i}+\vec{j}-2\vec{k}$ and $\vec{i}-3\vec{j}+\vec{k}$. The position vector of a point on it at a unit distance from the first point is :

(A) $\frac{1}{5}(5\vec{i}+\vec{j}-7\vec{k})$ **(B)** $\frac{1}{5}(5\vec{i}+9\vec{j}-13\vec{k})$ **(C)** $\vec{i}-4\vec{j}+3\vec{k}$ **(D)** $\frac{1}{\sqrt{29}}(2\vec{i}-4\vec{j}+3\vec{k})$

54. The volume of a right triangular prism $ABC\,A_1B_1C_1$ is equal to 3. If the position vectors of the vertices of the base ABC are $A(1, 0, 1)$; $B(2,0, 0)$ and $C(0, 1, 0)$ the position vectors of the vertex A_1 can be :

(A) (2, 2, 2) **(B)** (0, 2, 0) **(C)** (0, –2, 2) **(D)** (0, –2, 0)

55. For non-zero vectors $\vec{a}, \vec{b}, \vec{c}$, $|(\vec{a}\times\vec{b}).\vec{c}| = |\vec{a}||\vec{b}||\vec{c}|$ holds then :

(A) $\vec{a}.\vec{b}=0, \vec{b}.\vec{c}=0$ **(B)** $\vec{b}.\vec{c}=0, \vec{c}.\vec{a}=0$ **(C)** $\vec{c}.\vec{a}=0, \vec{a}.\vec{b}=0$ **(D)** $\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$

56. Three points having position vectors $\vec{a}, \vec{b}$ and $\vec{c}$ will be collinear if :

(A) $\lambda\vec{a}+\mu\vec{b}=(\lambda+\mu)\vec{c}$ **(B)** $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

(C) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=0$ **(D)** $\vec{a}\times\vec{c}=\vec{b}$

57. If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are any four vectors, then $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})$ is a vector:

(A) Perpendicular to $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$

(B) Along the line of intersection of two planes, one containing $\vec{a}, \vec{b}$ other containing $\vec{c}, \vec{d}$

(C) Equally inclined to both $\vec{a}\times\vec{b}$ and $\vec{c}\times\vec{d}$

(D) None of these

58. $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle $\alpha(\alpha\in[0,\pi])$ to each other and $|\vec{a}+\vec{b}|<1$ then α can lie in :

(A) $\alpha\in\left(\frac{\pi}{3},\frac{2\pi}{3}\right)$ **(B)** $\alpha\in\left(\frac{\pi}{3},\frac{\pi}{2}\right)$ **(C)** $\alpha\in\left(\frac{2\pi}{3},\frac{5\pi}{6}\right)$ **(D)** $\alpha\in\left(\frac{2\pi}{3},\frac{5\pi}{7}\right)$

59. If $\vec{a}=2\hat{i}-\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}-2\hat{k}$ be three vectors. A vector in the plane of the $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{\frac{2}{3}}$ is :

(A) $2\hat{i}+3\hat{j}-3\hat{k}$ **(B)** $2\hat{i}+3\hat{j}+3\hat{k}$ **(C)** $-2\hat{i}-\hat{j}+5\hat{k}$ **(D)** $2\hat{i}+\hat{j}+5\hat{k}$

60. The vector along the bisector of the angle between the two vectors $2\hat{i}-2\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}-2\hat{k}$ having magnitude of 2 units is:

(A) $\frac{2}{\sqrt{10}}\left(3\hat{i}-\hat{k}\right)$ **(B)** $\frac{1}{\sqrt{26}}\left(\hat{i}-4\hat{i}+3\hat{k}\right)$ **(C)** $\frac{2}{\sqrt{26}}\left(\hat{i}-4\hat{i}+3\hat{k}\right)$ **(D)** $\frac{1}{\sqrt{26}}\left(\hat{i}-4\hat{i}-3\hat{k}\right)$

61. Let $\vec{\lambda}=\vec{a}\times(\vec{b}+\vec{c}),\ \vec{\mu}=\vec{b}\times(\vec{c}+\vec{a})$ and $\vec{v}=\vec{c}\times(\vec{a}+\vec{b})$. Then :

(A) $\vec{\lambda}+\vec{\mu}=\vec{v}$

(B) $\vec{\lambda},\ \vec{\mu}$ and $\vec{v}$ are coplanar

(C) $\vec{\lambda}+\vec{\mu}+\vec{v}=\vec{0}$

(D) $\vec{\lambda}+\vec{v}=\vec{\mu}$

62. Let the position vectors of the points A, B, C be $\vec{a}=\hat{i}+2\hat{j}+3\hat{k},\ \vec{b}=-\hat{i}-\hat{j}+8\hat{k}$ and $\vec{c}=-4\hat{i}+4\hat{j}+6\hat{k}$ respectively, then:

(A) ΔABC is equilateral

(B) ΔABC is right angled

(C) $|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=2|\vec{a}-\vec{b}|^2$

(D) $|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=|\vec{a}-\vec{b}|^2$

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column 1 are labelled as (A), (B), (C) & (D) whereas statements in Column 2 are labeled as p, q, r, s & t. More than one choice from Column 2 can be matched with Column 1.

63. If $\vec{a},\vec{b}$ and $\vec{c}$ are the position vectors of the points *A, B* and *C,* where $\vec{c}=x\vec{a}+y\vec{b}$, then match the position of the point *C* according to the given conditions.

	Column 1		Column 2
(A)	$x>0,\ y>0,\ x+y<1$	**(p)**	Outside ΔOAB
(B)	$x>0,\ y>0,\ x+y>1$	**(q)**	Inside $\angle OAB$
(C)	$x>0,\ y<0,\ x+y<1$	**(r)**	Inside $\angle OBA$
(D)	$x<0,\ y>0,\ x+y<1$	**(s)**	Inside $\angle AOB$

64. **MATCH THE FOLLOWING :**

	Column 1		Column 2
(A)	If $\vec{a},\vec{b},\vec{c}$ are three mutually perpendicular vectors where $|\vec{a}|=|\vec{b}|=2,|\vec{c}|=1$, then $1/12[\vec{a}\times\vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}]$ is	**(p)**	–3/4
(B)	If $\vec{a},\vec{b}$ are two unit vectors inclined at $\pi/3$, then $[\vec{a}\ \vec{b}+\vec{a}\times\vec{b}\ \vec{b}]$ is	**(q)**	0
(C)	If $\vec{b},\vec{c}$ are orthogonal unit vectors and $\vec{b}\times\vec{c}=\vec{a}$, then $[\vec{a}+\vec{b}+\vec{c}\ \vec{a}+\vec{b}\ \vec{b}+\vec{c}]$ is	**(r)**	4/3
(D)	If $[\vec{x}\ \vec{y}\ \vec{a}]=[\vec{x}\ \vec{y}\ \vec{b}]=[\vec{a}\ \vec{b}\ \vec{c}]=0$ each vector being a non-zero vector, then $[\vec{x}\ \vec{y}\ \vec{c}]$ is	**(s)**	1

65. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	The possible value of a if $\vec{r}=(\hat{i}+\hat{j})+\lambda(\hat{i}+2\hat{j}-\hat{k})$ and $\vec{r}=(\hat{i}+2\hat{j})+\mu(-\hat{i}+\hat{j}+a\hat{k})$ are two skew lines	**(p)**	–4
(B)	The angle between the vectors $\vec{a}=\lambda\hat{i}-3\hat{j}-\hat{k}$ and $\vec{b}=2\lambda\hat{i}+\lambda\hat{j}-\hat{k}$ is acute, whereas the vector $\vec{b}$ makes with axes of coordinates an obtuse angle, then λ may be	**(q)**	–2
(C)	The possible value of a such that $\left(2\hat{i}-\hat{j}+\hat{k}\right),\left(\hat{i}+2\hat{j}+(1+a)\hat{k}\right)$ and $\left(3\hat{i}+a\hat{j}+5\hat{k}\right)$ are coplanar is	**(r)**	2
(D)	If $\vec{A}=2\hat{i}+\lambda\hat{j}+3\hat{k}$, $\vec{B}=2\hat{i}+\lambda\hat{j}+\hat{k}$, $\vec{C}=3\hat{i}+\hat{j}$ and $\vec{A}+\lambda\vec{B}$ is perpendicular to $\vec{C}$, then $\lvert 2\lambda \rvert$ is	**(s)**	3

66. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	$\vec{a}$ and $\vec{c}$ are unit vectors and $\lvert\vec{b}\rvert=4$ with $\vec{a}\times\vec{b}=2\vec{a}\times\vec{c}$. The angle between $\vec{a}$ and $\vec{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$. If $\vec{b}-2\vec{c}=\lambda\vec{a}$, then λ belongs to	**(p)**	$\left(0, \frac{\pi}{2}\right)$
(B)	If Vector $\vec{a}=(x, y, z)$ makes equal angles with the vectors $\vec{b}=(y, -2z, 3x)$ and $\vec{c}=(2z, 3x, -y)$ and is perpendicular to the vector $\vec{d}=(1, -1, 2)$ with $\lvert\vec{a}\rvert=2\sqrt{3}$ and the angle between $\vec{a}$ and the unit vector $\hat{j}$ is obtuse then $x+y-z$ belongs to	**(q)**	$(0, e)$
(C)	Let ABC be a triangle whose centroid is G orthocentre is H and circumcentre is origin O. If D is any point in the plane of the triangle such that no three of O, A, B, C, D are collinear satisfying the relation $\overrightarrow{AD}+\overrightarrow{BD}+\overrightarrow{CH}+3\overrightarrow{HG}=\lambda\overrightarrow{HD}$, then scalar λ belongs to	**(r)**	$[0, \pi)$
(D)	If $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\lvert\vec{a}\rvert=\lvert\vec{c}\rvert=1, \lvert\vec{b}\rvert=4$ and $\lvert\vec{b}\times\vec{c}\rvert=\sqrt{15}$ if $\vec{b}-2\vec{c}=4\lambda\vec{a}$, then λ belongs to	**(s)**	(e, π)

67. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	If $\vec{a}, \vec{b}, \vec{c}$ represents the sides of the triangle ABC, then	**(p)**	$\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}$
(B)	If $\vec{a}, \vec{b}, \vec{c}$ represents three co-terminus edges of regular tetrahedron, then	**(q)**	$\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$
(C)	If $\vec{a}\times\vec{b}=\vec{c}$, $\vec{b}\times\vec{c}=\vec{a}$, then	**(r)**	$\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$
(D)	$\vec{a}+\vec{b}+\vec{c}=\vec{0}$, where $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, then	**(s)**	$\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-\frac{3}{2}$

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

***68.** Two given points P and Q in the rectangular cartesian coordinates lie on $y = 2^{x+2}$ such that $\vec{OP}.\hat{i} = -1$ and $\vec{OQ}.\hat{i} = +2$ where $\hat{i}$ is a unit vector along the x-axis . The magnitude of $\frac{\vec{OQ} - 4\vec{OP}}{2}$ is _______.

***69.** Line L_1 is parallel to a vector $\vec{\alpha} = 3\hat{i} + 2\hat{j} + 4\hat{k}$ and passes through a point A (7,6,2) and the line L_2 is parallel to a vector $\overline{\beta} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through a point B (5, 3,4). Now a line L_3 parallel to a vector $\vec{r} = 2\hat{i} + 2\hat{j} + \hat{k}$ intersects the lines L_1 and L_2 at points C and D respectively. then $|\overline{CD}|$ is_______.

70. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}.\vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is 30^0, then the value of $|(\vec{a} \times \vec{b}) \times 2\vec{c}|$ is_______.

71. If $\vec{a} \times (\vec{b} \times \vec{c}) + (\vec{a}.\vec{b})\vec{b} = (4 - 2\beta - \sin\alpha)\vec{b} + (\beta^2 - 1)\vec{c}$ and $(\vec{c}.\vec{c})\ \vec{a} = \vec{c}$ where $\vec{b}$ and $\vec{c}$ are non-collinear and α, β are scalars then $\beta =$ _______.

***72.** Let $\vec{u}$ and $\vec{v}$ be unit vectors. if $\vec{\omega}$ is a vector such that $\vec{\omega} + (\vec{\omega} \times \vec{u}) = \vec{v}$, if the maximum volume of the parallelepiped formed by $\vec{u}, \vec{v}$ and $\vec{\omega}$ is p then $12p =$ _______.

73. Let $\overline{u}, \overline{v}, \overline{w}$ be such that $|\overline{u}| = 1,\ |\overline{v}| = 2,\ |\overline{w}| = 3$. If the projection $\overline{v}$ along $\overline{u}$ is equal to that of $\overline{w}$ along $\overline{u}$ and $\overline{v}$, $\overline{w}$ are perpendicular to each other, then $\frac{|\overline{u} - \overline{v} + \overline{w}|^2}{2}$ equals _______.

74. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = x_1\hat{i} + x_2\hat{j} + x_3\hat{k}$, where $x_1, x_2, x_3 \in \{-3, -2, -1, 0, 1, 2\}$. Number of possible vectors $\vec{b}$ such that $\vec{a}$ and $\vec{b}$ are mutually perpendicular, is p then $\frac{p}{5} =$

75. If in a triangle ABC, $\overrightarrow{BC} = \frac{\vec{u}}{|\vec{u}|} - \frac{\vec{v}}{|\vec{v}|}$ and $\overrightarrow{AC} = \frac{2\vec{u}}{|\vec{u}|}$ where $|\vec{u}| \neq |\vec{v}|$, then 1 + cos 2A + cos 2B + cos 2C =

76. Let $\overline{u}$ and $\overline{v}$ are unit vectors and $\overline{w}$ is a vector such that $\overline{u} \times \overline{v} + \overline{u} = \overline{w}$ and $\overline{w} \times \overline{u} = \overline{v}$, Then the value of $[uvw]$ =_______.

77. If $|\vec{a}| = |\vec{b}| = |\vec{c}| = 2$ and $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 1$, then $\sqrt{[\vec{a} \times \vec{b}\ \ \vec{b} \times \vec{c}\ \ \vec{c} \times \vec{a}]}$ is_______.

78. $\vec{a}$ and $\vec{b}$ are two non-collinear vectors then the points with position vectors $l_1\vec{a} + m_1\vec{b}, l_2\vec{a} + m_2\vec{b}, l_3\vec{a} + m_3\vec{b}$ are collinear then find the value of $\begin{vmatrix} 1 & 1 & 1 \\ l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{vmatrix}$.

79. Let $A(2\hat{i} + 3\hat{j} + 5\hat{k}), B(-\hat{i} + 3\hat{j} + 2\hat{k})$ and $C(\lambda\hat{i} + 5\hat{j} + \mu\hat{k})$ are vertices of a triangle and its median through A is equally inclined to the positive directions of the axes. Find the value of $2\lambda - \mu$

80. Let A, B, C be points with position vectors $r_1 = 2\hat{i} - \hat{j} + \hat{k}, r_2 = \hat{i} + 2\hat{j} + 3\hat{k}$ and $r_3 = 3\hat{i} + \hat{j} + 2\hat{k}$ relative to the origin 'O'. Find the shortest distance between point B and plane OAC.

81. Volume of tetrahedron whose vertices are the points with position vectors $\hat{i} - 6\hat{j} + 10\hat{k}, -\hat{i} - 3\hat{j} + 7\hat{k}, 5\hat{i} - \hat{j} + h\hat{k}$ and $7\hat{i} - 4\hat{j} + 7\hat{k}$ is 11 cubic units then the value of h is _______ $(h > 1)$

82. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} \cdot \vec{b} = 0 = \vec{a} \cdot \vec{c}$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. Then the value of $\left|\vec{a} \times \vec{b} - \vec{a} \times \vec{c}\right|$ is ______.

83. Given that the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ (no two of them are collinear). Further if $(\vec{a} + \vec{b})$ is collinear with $\vec{c}, (\vec{b} + \vec{c})$ is collinear with $\vec{a}$ and $|\vec{a}| = |\vec{b}| = |\vec{c}| = \sqrt{2}$. Then the value of $|\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}|$ is _____.

84. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}, \vec{c} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{d} = 2\hat{i} - \hat{j} + \hat{k}$, then the shortest distance between the lines $\vec{r} = \vec{a} + t\vec{b}$ and $\vec{r} = \vec{c} + p\vec{d}$ is k, then the value of $\frac{1}{k^2}$ is _____.

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. The equation of line intersecting and perpendicular to the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and passing through (–2, –5, 7) is :

(A) $\frac{x+2}{14}=\frac{y+5}{123}=\frac{z-7}{104}$ **(B)** $\frac{x+2}{14}=\frac{y+5}{137}=\frac{z-7}{-204}$

(C) $\frac{x+2}{76}=\frac{y+5}{137}=\frac{z-7}{-254}$ **(D)** None of these

***2.** The direction cosines of the projection of the line $\frac{1}{2}(x-1)=-y=z+2$ on the plane $2x+y-3z=4$ are :

(A) $\left(\frac{2}{\sqrt{6}},\frac{-1}{\sqrt{6}},\frac{1}{\sqrt{6}}\right)$ **(B)** $\left(\frac{-2}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}}\right)$ **(C)** $\left(\frac{2}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{-1}{\sqrt{6}}\right)$ **(D)** None of these

3. A mirror and a source of light are situated at the origin O and at a point on the line OX respectively. A ray of light from the source strikes the mirror at O and is reflected. The direction ratios of the normal to the plane of the mirror are (1, –1, 1); then the direction cosines of the reflected ray are :

(A) $\frac{1}{3},-\frac{2}{3},\frac{2}{3}$ **(B)** $-\frac{1}{3},\frac{2}{3},\frac{2}{3}$ **(C)** $-\frac{1}{3},-\frac{2}{3},-\frac{2}{3}$ **(D)** $\frac{-1}{3},\frac{-2}{3},\frac{2}{3}$

4. The planes $x+y-z=0$, $y+z-x=0$, $z+x-y=0$ meet :

(A) in a line **(B)** taken two at a time in parallel lines

(C) in a unique point **(D)** None of these

5. If the line $x=y=z$ intersect the line $\sin A\,x+\sin B\,y+\sin C\,z=2d^2$, $\sin 2A\,x+\sin 2B\,y+\sin 2C\,z=d^2$ then $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ is equal to : (where $A+B+C=\pi$)

(A) 1/16 **(B)** 1/8 **(C)** 1/32 **(D)** 1/12

6. In a three dimensional co-ordinate system P, Q and R are images of a point A (a, b, c) in the x-y, the y-z and the z-x planes respectively. If G is the centroid of triangle PQR then area of triangle AOG is : (O is the origin)

(A) $\frac{3}{2}(a^2+b^2+c^2)$ **(B)** $a^2+b^2+c^2$ **(C)** $\frac{2}{3}(a^2+b^2+c^2)$ **(D)** 0

7. A variable plane passes through a fixed point (a, b, c) and meets the coordinate axes in A, B, C. The locus of the point common to the plane and also planes through A, B, C parallel to coordinate planes is:

(A) $ayz+bzx+cxy=xyz$ **(B)** $axy+byz+czx=xyz$

(C) $axy+byz+czx=abc$ **(D)** $bcx+acy+abz=abc$

8. The equation of the plane bisecting the acute angle between the planes $2x-y+2z+3=0$ and $3x-2y+6z+8=0$ is :

(A) $23x-13y+32z+45=0$ **(B)** $5x-y-4z=3$

(C) $5x-y-4z+45=0$ **(D)** $23x-13y+32z+3=0$

9. Direction cosines of normal to the plane containing lines $x=y=z$ and $x-1=y-1=\frac{z-1}{d}$ (where $d\in R-\{1\}$), are :

(A) $\left\{\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0\right\}$ **(B)** $\left\{\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right\}$ **(C)** $\left\{0,-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right\}$ **(D)** None of these

10. The equation of the straight line through the origin parallel to the line $(b + c)x + (c + a)y + (a + b)z = k = (b - c)x + (c - a)y + (a - b)z$ is :

(A) $\frac{x}{b^2 - c^2} = \frac{y}{c^2 - a^2} = \frac{z}{a^2 - b^2}$ **(B)** $\frac{x}{b} = \frac{y}{c} = \frac{z}{a}$

(C) $\frac{x}{a^2 - bc} = \frac{y}{b^2 - ca} = \frac{z}{c^2 - ab}$ **(D)** None of these

11. A variable plane makes intercepts on the co-ordinate axes the sum of whose squares is constant and equal to k^2. Then the locus of the foot of the perpendicular from the origin to the plane is

(A) $(x^{-2} + y^{-2} + z^{-2})(x^2 + y^2 + z^2) = k^2$ **(B)** $k^2(x^{-2} + y^{-2} + z^{-2})(x^2 + y^2 + z^2) = 1$

(C) $(x^2 + y^2 + z^2)^2 = \frac{1}{k}$ **(D)** None of these

12. P is a point on the plane $lx + my + nz = p$. A point Q is taken on the line OP such that $OP.OQ = p^2$. Then the locus of Q is :

(A) $p(lx + my + nz) = x^2 + y^2 + z^2$ **(B)** $(lx + my + nz)(x^2 + y^2 + z^2) = p^2$

(C) $lx + my + nz = (x^2 + y^2 + z^2)p$ **(D)** None of these

13. The orthogonal projection of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ on the plane $3x + 4y + 5z = 0$ is :

(A) $\frac{x}{7} = \frac{y}{1} = \frac{z}{-5}$ **(B)** $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ **(C)** $\frac{x}{1} = \frac{y}{-2} = \frac{z}{1}$ **(D)** None of these

14. Shortest distance between the lines $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$ and $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{1}$ is equal to :

(A) $\sqrt{14}$ **(B)** $\sqrt{7}$ **(C)** $\sqrt{2}$ **(D)** None of these

***15.** The point of intersection of the line, passing through (0, 0, 1) and intersecting the lines $x + 2y + z = 1, -x + y - 2z = 2$ and $x + y = 2, x + z = 2$ with xy plane is :

(A) $\left(\frac{5}{3}, -\frac{1}{3}, 0\right)$ **(B)** (1, 1, 0) **(C)** $\left(\frac{2}{3}, -\frac{1}{3}, 0\right)$ **(D)** $\left(-\frac{5}{3}, \frac{1}{3}, 0\right)$

16. If $abc \neq 0$ and let (p_1, q_1, r_1) be the image of (p, q, r) in the plane $ax + by + cz + d = 0$, then :

(A) $\frac{p_1 - p}{a} = \frac{q_1 - q}{b} = \frac{r_1 - r}{c}$ **(B)** $a(p + p_1) + b(q + q_1) + c(r + r_1) + 2d = 0$

(C) Both **(A)** and **(B)** **(D)** None of these

17. A perpendicular is drawn from a point (1, 6, 3) to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. What will be coordinates of the foot of perpendicular :

(A) (1, 3, 5) **(B)** (0, 3, –2) **(C)** (2, 4, –5) **(D)** (1, 3, 4)

18. The foot of the perpendicular from the point $O(0, 0, 0)$ to the line of intersection of the planes $x + y + z = 4$ and $2x + y + 3z = 1$ is point A. Then the equation of line OA is :

(A) $\frac{x}{2} = \frac{y}{1} = \frac{z}{3}$ **(B)** $\frac{x}{8} = \frac{y}{29} = \frac{z}{-13}$ **(C)** $\frac{x}{-2} = \frac{y}{1} = \frac{z}{1}$ **(D)** None of these

19. A variable line passing through the point $P(0, 0, 2)$ always makes angle 60^0 with z–axis, intersects the plane $x + y + z = 1$. Then the locus of point of intersection of the line and the plane is :

(A) $x^2 + y^2 + 3(z-2)^2 = 0$ **(B)** $x^2 + y^2 = 3(z+2)^2$

(C) $x^2 + y^2 = 2(z-3)^2$ **(D)** $x^2 + y^2 = 3(z-2)^2$

***20.** A ray of light is coming along the line $\frac{x-2}{3} = \frac{y-1}{4} = \frac{z-6}{5}$ and strikes the plane mirror kept along the plane through the points (2, 1, 0), (5, 0, 1) and (4, 1, 1). Then the equation of reflected ray is :

(A) $\frac{x+10}{4} = \frac{y+15}{5} = \frac{z+14}{3}$ **(B)** $\frac{x-10}{4} = \frac{y-15}{5} = \frac{z-14}{3}$

(C) $\frac{x-15}{-4} = \frac{y-14}{-5} = \frac{z-10}{3}$ **(D)** None of these

***21.** The equation of a line on the plane $x + y + z = 1$ such that the line $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-1}{1}$ and the required line form a plane which is perpendicular to the plane $x + y + z = 1$ is :

(A) $\frac{3x+1}{2} = \frac{3y+1}{-1} = \frac{3z+1}{-1}$ **(B)** $\frac{3x-1}{-2} = \frac{3y-1}{1} = \frac{3z-1}{1}$

(C) $\frac{3x-1}{2} = \frac{3y-1}{-1} = \frac{3z-1}{-1}$ **(D)** None of these

22. The image of the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ in the plane $x + 2y + z = 12$ is :

(A) $\frac{x-6}{2} = \frac{y+\frac{7}{2}}{-2} = \frac{z-13}{2}$ **(B)** $\frac{x-6}{4} = \frac{y+\frac{7}{2}}{-7} = \frac{z-13}{10}$

(C) $\frac{x+6}{2} = \frac{y-\frac{7}{2}}{-3} = \frac{z-13}{6}$ **(D)** None of these

23. If a plane passes through the intersection of $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2 (a_1, a_2 \neq 0)$ and contains the x-axis, then :

(A) $d_1a_2 = a_1d_2$ **(B)** $a_1a_2 = d_1d_2$ **(C)** $d_1a_1 = d_2a_2$ **(D)** None of these

24. The plane $x = 0$ is rotated through an angle α about its line of intersection with the plane $z = 0$. Then equation of the plane in new position is (are) :

(A) $x \pm \sqrt{\sec\alpha - 1}\, z = 0$ **(B)** $x \pm \sqrt{\cos\alpha + 1}\, z = 0$

(C) $x \pm \sqrt{\sec\alpha + 1}\, z = 0$ **(D)** None of these

***<u>Passage for Questions 25 - 29</u>**

A conic section is obtained by the intersection of two inverted cones (having same vertex) with a plane. If a plane passes through the vertex, then its intersection with the cones either represents a pair of straight lines or a point depending upon whether it intersects the cones or not. If plane does not pass through vertex, then section may be hyperbola or circle depending upon whether plane is parallel to axis of the cone or perpendicular to it. If it has any other inclination, then intersection of plane and cone gives an ellipse.

25. If a variable point P moves such that the line passing through *P* and *Q* (0, 0, 2) makes an angle 60º with z-axis, then locus of *P* is :

(A) $x^2 - y^2 - 3(z-2)^2 = 0$ **(B)** $x^2 - y^2 + 3(z-2)^2 = 0$

(C) $x^2 + y^2 - 3(z-2)^2 = 0$ **(D)** $x^2 + y^2 + 3(z-2)^2 = 0$

26. The locus of intersection of locus of *P* with the plane $x + y + z = 1$ is :

(A) a pair of straight lines **(B)** a hyperbola

(C) a circle **(D)** an ellipse

27. The locus of intersection of locus of *P* and the plane $x + y + z = 2$

(A) a pair of straight lines **(B)** a hyperbola

(C) a circle **(D)** a point

28. The locus of intersection of locus of *P* with $x + y = 2$

(A) a straight lines **(B)** a hyperbola **(C)** a circle **(D)** an ellipse

29. The locus of intersection of locus of P with $2x + y + z = 2$ is :

(A) a straight lines **(B)** a hyperbola **(C)** a circle **(D)** a point

***Passage for Questions 30 - 33**

Consider a three dimensional Cartesian system with origin at *O* and three rectangular coordinates axes x, y and *z*-axis. Suppose that the distance between two points *P* and *Q* in the space having their coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively be defined by the following formula $d(P,Q) = |x_2 - x_1| + |y_2 - y_1| + |z_2 - z_1|$.

Although the formula of distance between two points has been defined in a new way, yet the other definition remain same (like section formula, direction cosines etc.). So, in general equations of straight line in space remain unchanged. Now answer the following questions.

30. If l, m, n represent direction cosines (if we can call it) of a vector $\overrightarrow{OP}$, then which of the following relations holds?

(A) $l^2 + m^2 + n^2 = 1$ **(B)** $l + m + n = 1$ **(C)** $|l + m + n| = 1$ **(D)** $|l| + |m| + |n| = 1$

31. Locus of point *P* if $d(O,P) = k$, where *k* is a positive constant number, represents :

(A) a sphere of radius *k* **(B)** a set of eight planes forming an octahedron

(C) a set of eight planes forming hexagonal prism **(D)** an infinite cylinder of radius *k*

32. Let *A* be a point (1, 2, 3) in the given reference system. Then locus of the point *P* in the first octant satisfying the equation $d(O,P) = d(A,P)$ does not contain :

(A) any of the coordinates axes **(B)** any of the coordinates planes

(C) any plane parallel to coordinates axes **(D)** any plane parallel to coordinate planes

33. An equilateral triangle has its vertices on the axes of coordinates and area $\sqrt{3}$ square units. The coordinates of the orthocenter of the triangle are :

(A) (1, 1, 1) **(B)** $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ **(C)** $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ **(D)** $\left(\frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{3}\right)$

Passage for Questions 34 - 36

Two lines whose equations are $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{\lambda}$ and $\frac{x-2}{3}=\frac{y-3}{2}=\frac{z-2}{3}$ lie in the same plane, then:

34. The value of $\sin^{-1}\sin\lambda$ is equal to:

(A) 3 **(B)** $\pi-3$ **(C)** 4 **(D)** $\pi-4$

35. Point of intersection of the line lies on:

(A) $3x+y+z=20$ **(B)** $3x+y+z=25$ **(C)** $3x+2y+z=24$ **(D)** None of these

36. Equation of plane containing both lines is:

(A) $x+5y-3z=10$ **(B)** $x+6y+5z=20$ **(C)** $x+6y-5z=10$ **(D)** None of these

Passage for Questions 37 - 38

Consider the tetrahedron formed by the planes $y+z=0$, $z+x=0$, $x+y=0$, $x+y+z=a$.

37. The direction cosines of the shortest distance lie between the planes $y+z=0$ and $z+x=0$ is:

(A) $-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}$ **(B)** $-\frac{1}{\sqrt{6}},-\frac{1}{\sqrt{6}},-\frac{1}{\sqrt{6}}$

(C) $-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$ **(D)** None of these

38. The shortest distance between any two opposite edges of the tetrahedron is:

(A) $\frac{2}{\sqrt{6}}a$ **(B)** $\frac{1}{\sqrt{6}}a$ **(C)** $\frac{1}{\sqrt{3}}a$ **(D)** None of these

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

★39. The locus of the point equidistant from the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x}{2}=\frac{y}{3}=\frac{z}{1}$ can be:

(A) $x+y+2z=0$ **(B)** $x+y-2z=0$ **(C)** $3x+5y+4z=0$ **(D)** $3x+5y+4z+1=0$

40. The equation of a plane passing through the line $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-2}{-2}$ and making an angle of 30^0 with the plane $x+y+z=5$ is :

(A) $x+(3+2\sqrt{2})y+(2+\sqrt{2})z-11-6\sqrt{2}=0$ **(B)** $x+(3-2\sqrt{2})y+(2-\sqrt{2})z-11+6\sqrt{2}=0$

(C) $x+(3-\sqrt{2})y+(2+\sqrt{2})z-11-6\sqrt{2}=0$ **(D)** $x+(3+\sqrt{2})y+(2-\sqrt{2})z-11+6\sqrt{2}=0$

★41. A plane meets a set of three mutually perpendicular planes in the sides of a triangle whose angles are A, B and C respectively. The squares of cosines of angles which first plane makes with the other planes are:

(A) $\cot B\cot C, \cot C\cot A, \cot A\cot B$

(B) $\tan B\tan C, \tan C\tan A, \tan A\tan B$

(C) $\text{cosec } B\text{cosec } C, \text{cosec } C\text{cosec } A, \text{cosec } A\text{cosec } B$

(D) None of these

★42. The straight lines whose direction cosines are given by the relations $al + bm + cn = 0$ and $fmn + gnl + hlm = 0$ are perpendicular if :

(A) $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$ **(B)** $\frac{a}{f}+\frac{b}{g}+\frac{c}{h}$ **(C)** $\frac{h}{a}+\frac{g}{b}+\frac{f}{c}$ **(D)** None of these

★43. The coordinates of points, whose perpendicular distances from yz, zx and xy-planes are in $A.P.$ and whose distances from x, y and z axes are $\sqrt{13}$, $\sqrt{10}$ and $\sqrt{5}$ respectively is :

(A) (1, 2, 3) **(B)** (–1, 2, 3) **(C)** (1, –2, 3) **(D)** (–1, –2, –3)

44. The plane $3y + 4z = 0$ is rotated about its line of intersection with the plane $x = 0$ through an angle 60^0. The equation of the plane in its new position is :

(A) $3y+4z+5\sqrt{3}x=0$ **(B)** $3y+4z-5\sqrt{3}x=0$

(C) $3y-4z-5\sqrt{3}x=0$ **(D)** $3y-4z+5\sqrt{3}x=0$

45. The line $\frac{x-2}{3}=\frac{y+1}{2}=\frac{z-1}{-1}$ intersects the curve $xy = c^2$, $z = 0$ if c is equal to :

(A) 1/3 **(B)** –1/3 **(C)** $\sqrt{5}$ **(D)** $-\sqrt{5}$

46. Consider the lines $\frac{x}{2}=\frac{y}{3}=\frac{z}{5}$ and $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ the equation of the line which :

(A) Bisects the angle between the lines is $\frac{x}{3}=\frac{y}{3}=\frac{z}{6}$

(B) Bisects the angle between the lines is $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$

(C) Passes through origin and is perpendicular to the given lines is $x = y = -z$

(D) None of these

47. The equations of the planes through the origin which are parallel to the line $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z+1}{-2}$ and at a distance 5/3 from it are :

(A) $2x + 2y + z = 0$ **(B)** $x + 2y + 2z = 0$ **(C)** $2x - 2y + 3z = 0$ **(D)** $x - 2y + 2z = 0$

★48. The line $\frac{x+6}{5}=\frac{y+10}{3}=\frac{z+14}{8}$ is the hypotenuse of an isosceles right angled triangle whose opposite vertex is (7, 2, 4). The equations of the remaining sides are :

(A) $\frac{x-7}{5}=\frac{y-2}{2}=\frac{z-4}{6}$ **(B)** $\frac{x-7}{3}=\frac{y-2}{6}=\frac{z-4}{2}$

(C) $\frac{x-7}{2}=\frac{y-2}{-2}=\frac{z-4}{6}$ **(D)** $\frac{x-7}{2}=\frac{y-2}{-3}=\frac{z-4}{6}$

★49. A ray M is sent along the line $\frac{x-0}{2}=\frac{y-2}{2}=\frac{z-1}{0}$ and is reflected by the plane $x = 0$ at point A. The reflected ray is again reflected by the plane $x+2y=0$ at point B. The initial ray and final reflected ray meets at point J. Then :

(A) The co-ordinates of point B is $(4,-2,1)$ **(B)** The co-ordinates of point J is $(-3,-1,1)$

(C) The centroid of ΔABJ is (0, 0, 0) **(D)** The co-ordinates of point J is $(2,-1,1)$

50. The equation of a line in *xz* plane equally inclined with *x* and *z* axes which is at a unit distance from the line $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z}{1}$ is :

(A) $\frac{x-\sqrt{6}+1}{1}=\frac{y}{0}=\frac{z}{-1}$ **(B)** $\frac{x-1-\sqrt{2}}{1}=\frac{y}{0}=\frac{z}{1}$

(C) $\frac{x-1}{1}=\frac{y}{0}=\frac{z}{1}$ **(D)** None of these

★51. Projection of line $\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z+3}{4}$ on the plane $x + 2y + z = 6$; has equation :

(A) $x + 2y + z - 6 = 0 = 9x - 2y - 5z - 8$ **(B)** $x + 2y + z + 6 = 0,\ 9x - 2y + 5z = 4$

(C) $\frac{x-1}{4}=\frac{y-3}{-7}=\frac{z+1}{10}$ **(D)** $\frac{x+3}{4}=\frac{y-2}{7}=\frac{z-7}{-10}$

52. If three planes $P_1 \equiv 2x+y+z-1=0,\ P_2 \equiv x-y+z-2=0$ and $P_3 = \alpha x-y+3z-5=0$ intersect each other at point P on XOY plane and at point Q on YOZ plane, where O is the origin then identify the correct statement(s)

(A) the value of α is 4

(B) straight line perpendicular to plane P_3 and passing through P is $\frac{x-1}{4}=\frac{y+1}{-1}=\frac{z}{3}$

(C) the length of projection of $\overrightarrow{PQ}$ on *x*-axis is 1

(D) centroid of the triangle OPQ is $\left(\frac{1}{3},-\frac{1}{2},\frac{1}{2}\right)$

53. The coordinate of the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ which are at a distance $3\sqrt{2}$ from the point (1, 2, 3)

(A) (–2, –1, 3) **(B)** (2, 2, 4) **(C)** $\left(\frac{56}{17},\frac{43}{17},\frac{111}{17}\right)$ **(D)** $\left(\frac{47}{11},\frac{42}{11},\frac{56}{11}\right)$

54. The equations of the lines of shortest distance between the lines $\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$ and $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ are:

(A) $3(x-21)=3y+92=3z-32$ **(B)** $\frac{x-62/3}{1/3}=\frac{y+31}{1/3}=\frac{z-31/3}{1/3}$

(C) $\frac{x-21}{1/3}=\frac{y+92/3}{1/3}=\frac{z-32/3}{1/3}$ **(D)** $\frac{x-2}{1/3}=\frac{y+3}{1/3}=\frac{z-1}{1/3}$

55. The equation of planes bisecting the angle between the planes $2x-y+2z+3=0$ and $3x-2y+6z+8=0$ is / are:

(A) $5x-y-4z-45=0$ **(B)** $5x-y-4z-3=0$

(C) $23x-13y+32z+45=0$ **(D)** $23x-13y+32z+5=0$

56. The equation of the plane parallel to plane $x+y+2z=5$ at a distance $\sqrt{6}$ units from this plane is / are:

(A) $x+y+2z+11=0$ **(B)** $x+y+2z=11$

(C) $x+y+2z+1=0$ **(D)** $x+y+2z-1=0$

57. If the line $\frac{x-2}{-1}=\frac{y+2}{1}=\frac{z+k^2-1}{4}$ is one of the angle bisector of the lines $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and $\frac{x}{-2}=\frac{y}{3}=\frac{z}{1}$, then the value of *k* is /are:

(A) – 3 **(B)** 2 **(C)** 3 **(D)** – 2

58. Let PM be perpendicular from the point $P(1, 2, 3)$ to x-y plane. If $\overrightarrow{OP}$ makes an angle θ with the positive direction of z-axis and $\overrightarrow{OM}$ makes an angle ϕ with the positive direction of x-axis, where O is the origin and θ and ϕ are acute angles, then:

(A) $\tan\theta = \frac{\sqrt{5}}{3}$ **(B)** $\sin\theta\sin\phi = \frac{2}{\sqrt{14}}$ **(C)** $\tan\phi = 2$ **(D)** $\cos\theta\cos\theta = \frac{1}{\sqrt{14}}$

59. If lines $x = y = z$, $x = \frac{y}{2} = \frac{z}{3}$ and the third line passing through (1, 1, 1) form a triangle of area $\sqrt{6}$ units, then point of intersection of third line with second line will lie on:

(A) $\frac{x-2}{3} = \frac{y-4}{8} = \frac{z-6}{9}$ **(B)** $x + 2y + z = 16$

(C) $x - 2y - z = 16$ **(D)** $\frac{x-3}{3} = \frac{y-4}{8} = \frac{z-3}{5}$

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column 1 are labelled as (A), (B), (C) & (D) whereas statements in Column 2 are labelled as p, q, r, s & t. More than one choice from Column 2 can be matched with Column 1.

60. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	The point in which the line $\frac{x+1}{-1} = \frac{y-12}{5} = \frac{z-7}{2}$ cuts the surface $11x^2 - 5y^2 + z^2 = 0$ is(are)	**(p)**	$(2, -3, 1)$
(B)	A line with direction cosines proportional to 1, –5, –2 meets each of the lines $x = y + 5 = z + 1$ and $x + 5 = 3y = 2z$. The coordinates of each of the points of intersection are given by	**(q)**	$(1, 2, 3)$
(C)	Let $P = 0$ is the equation of the plane passing through the line of intersection of the planes $2x - y = 0$ and $3z - y = 0$ and perpendicular to the plane $4x + 5y - 3z = 8$. Then the points which lies on the plane $P = 0$ is(are)	**(r)**	$(0, 9, 17)$
(D)	The image of the point (1, 3, 4) in the plane $y + z - 6 = 0$ is(are)	**(s)**	$\left(\frac{1}{2}, 1, \frac{1}{3}\right)$
		(t)	None of these

61. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	The plane $x-2y+7z+21=0$ contains the line	**(p)**	$\frac{x-3}{3}=\frac{y+5}{-4}=\frac{z-7}{5}$
(B)	An equation of the line passing through $3\hat{i}-5\hat{j}+7\hat{k}$ and perpendicular to the plane $3x-4y+5z=8$ is (λ, μ are parameters)	**(q)**	$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$
(C)	Equation of the line of shortest distance between the lines $\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$ and $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ is	**(r)**	$\frac{x-3}{-2}=\frac{y-1}{7}=\frac{z-4}{13}$
(D)	The line of intersection of the planes $\vec{r}.\left(3\hat{i}-\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(\hat{i}+4\hat{j}-2\hat{k}\right)=2$ is parallel to the line given by	**(s)**	$3(x-21)=3y+92=3z-32$
		(t)	None of these

62. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	The coordinates of a point on the line $x=4y+5,\ z=3y-6$ at a distance 3 from the point (5, 0, –6) is(are)	**(p)**	(0, 0, 0)
(B)	The plane containing the lines $\frac{x+1}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-6}{7}$ passes through	**(q)**	(17, 3, 3)
(C)	A line passes through two points A(2, –3, –1) and B(8, –1, 2). The coordinates of a point on this line nearer to the origin at a distance of 14 units from A is(are)	**(r)**	(2, 5, 7)
(D)	The coordinates of the foot of the perpendicular from the point (3, –1, 11) on the line $\frac{x}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ is(are)	**(s)**	(14, 1, 5)
		(t)	None of these

63. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	The D.R.[s] of the line which is perpendicular to lines $\frac{x-7}{2}=\frac{y+17}{-3}=\frac{z-6}{-1}$ and $\frac{x+5}{-1}=\frac{y+3}{2}=\frac{z-4}{-2}$ is	**(p)**	(0, 4, 1)
(B)	Image of (2, 1, 1); in the plane $x+y-z=3$ is	**(q)**	(8, 5, 1)
(C)	Foot of perpendicular from (0, 2, 7) to the line $\frac{x+2}{-1}=\frac{y-1}{3}=\frac{z-3}{-2}$ is	**(r)**	$\left(-\frac{3}{2},-\frac{1}{2},4\right)$
(D)	The point where the line joining (2, 1, 3) and (4, –2, 5) meets the plane $2x+y-z=3$ is	**(s)**	$\left(\frac{8}{3},\frac{5}{3},\frac{1}{3}\right)$
		(t)	None of these

64. MATCH THE FOLLOWING :

	Column 1		Column 2
(A)	Let image of the line $\frac{x-1}{3}=\frac{y-3}{5}=\frac{z-4}{2}$ in the plane $2x-y+z+3=0$ be L. A plane $7x+By+Cz+D=0$ is such that it contains the line L and perpendicular to the plane $2x-y+z+3=0$ then value of $\frac{B+C+D}{10}$ is	**(p)**	4
(B)	ABC is a triangle where $A(2,3,5)$; $B(-1,3,2)$ and $C(\lambda,5,\mu)$. If the median through A is equally inclined to the axes then value of $\mu-\lambda+1$ is	**(q)**	3
(C)	Length of projection of line segment joining $P(-1,2,0)$ and $Q(1,-1,2)$ on $2x-y-2z=4$ is	**(r)**	$\frac{7}{2}$
(D)	If $A(a,b,c)$ is any point on plane $3x+2y+z=7$, then the least value of $a^2+b^2+c^2$ is	**(s)**	$\frac{6}{11}$
		(t)	None of these

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

65. The shortest distance between the lines $2x+y+z=1$, $3x+y+2z=2$ and $x=y=z$ is d then $1/d^2=$ _____.

66. Three lines $y-z-1=0$, $x=0$; $x+z+1=0$, $y=0$; $x-z-1=0$, $y=0$ intersect the xy plane at A, B and C. If the orthocentre of ΔABC is (p, q, r) then $3p+q+r=$ ____________.

67. The minimum distance of the point (1, 1, 1) from the plane $x+y+z=1$ measured perpendicular to the line $\frac{x-x_1}{1}=\frac{y-y_1}{2}=\frac{z-z_1}{3}$ is d then $\frac{3d^2}{7}=$ ____________.

68. The maximum distance between the point $P(0, 0, 3)$ and the circle $x^2 + y^2 - 2\sqrt{5}\,x - 4y + 8 = 0; z = 0$ is ____________.

69. The equation of the plane which is equidistant from lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x-2}{3} = \frac{y-3}{1} = \frac{z-1}{2}$ is $Ax + By + Cz + D = 0$ then $|A + B + C + D| =$ ____________.

70. Plane $2x + 3y + 4z = 5$ is rotated about the line where it cuts the xy-plane by an angle α. In the new position the plane contains the point (1, 1, 1). If the angle $\alpha = \cos^{-1}\sqrt{\frac{p}{q}}$, ($\frac{p}{q}$ is rational number in its simplest form) then $q - 2p =$ ____________.

71. A variable plane is at a constant distance p from the origin and meets the axes in A, B and C. If the locus of the centroid of the tetrahedron $OABC$ is $x^{-2} + y^{-2} + z^{-2} = \lambda p^{-2}$ then the value of $\sqrt{\lambda}$ is______.

72. If the planes $x = cy + bz, y = az + cx, z = bx + ay$ pass through one line then the value of $a^2 + b^2 + c^2 + 2abc$ is________.

73. If the planes $x - y + z + 1 = 0,\ \lambda x + 3y + 2z - 3 = 0,\ 3x + \lambda y + z - 2 = 0$ form a triangular prism then λ is _____.

74. If a, b, c be the lengths of the intercepts of the plane passing through the intersection of the planes $2x + y + 2z = 9,\ 4x - 5y - 4z = 1$ and the point (3, 2, 1) on the coordinate axes, then $(5a + b + c)/2 =$ _____.

75. If Q is the foot of perpendicular from the point $P(4, -5, 3)$ on the line $\frac{x-5}{3} = \frac{y+2}{-4} = \frac{z-6}{5}$, then $[PQ] =$ _____. (where [.] denote greatest integer function)

76. The projection of the line $\frac{x}{2} = \frac{y-1}{2} = \frac{z-1}{1}$ on a plane P is $\frac{x}{1} = \frac{y-1}{1} = \frac{z-1}{-1}$. The plane P passes through $(k, -2, 0)$ then $k =$______.

77. If $10y - 8x - (x^2 + y^2 + z^2) = 40$, $P_1 = \max\left\{\sqrt{(x+2)^2 + (y-3)^2 + z^2}\right\}$, $P_2 = \min\left\{\sqrt{(x+2)^2 + (y-3)^2 + z^2}\right\}$, then $P_1 - P_2$ is ____.

78. Let for $\lambda \in [0, \infty)$ such that $(x, y, z) \neq (0, 0, 0)$ and $(\hat{i} + \hat{j} + 3\hat{k})x + (3\hat{i} - 3\hat{j} + \hat{k})y + (4\hat{i} + 5\hat{j})z = \lambda(x\hat{i} + y\hat{j} + 3\hat{k})$, then the value of $\frac{x - y - z}{x}$ is equal to ______.

79. If the distance between the plane $Ax - 2y + z = d$ and the plane containing the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ is $\sqrt{6}$, then $|d|$ is equal to _____.

80. If the distance of point of intersection of lines $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z+10}{8}$ from $(1, -4, 7)$ is α, then $\frac{\alpha^2}{13}$ is equal to ______.

JEE Advanced Revision Booklet	Probability

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. Entries of a 2×2 determinant are chosen from the set $\{-1, 1\}$. The probability that determinant has zero value is :

(A) 1/4 **(B)** 1/3 **(C)** 1/2 **(D)** None of these

2. A number is chosen at random from the numbers 10 to 99. By seeing the number a man will laugh if product of the digits is 12. If he choose three numbers with replacement then the probability that he will laugh at least once is :

(A) $1-\left(\frac{3}{5}\right)^3$ **(B)** $\left(\frac{43}{45}\right)^3$ **(C)** $1-\left(\frac{4}{25}\right)^3$ **(D)** $1-\left(\frac{43}{45}\right)^3$

★3. 8n players $P_1, P_2, P_3 ....P_{8n}$ play a knock out tournament. It is known that all the players are of equal strength. The tournament is held in 3 rounds where the players are paired at random in each round. If it is given that P_1 wins in the third round. The probability that P_2 looses in the second round is :

(A) $\frac{n}{8n-1}$ **(B)** $\frac{n}{8n+1}$ **(C)** $\frac{2n}{4n-1}$ **(D)** None of these

4. Two subsets A and B of a set containing n elements are chosen at random. The probability that $A \subseteq B$ is :

(A) $\frac{1}{2}$ **(B)** $\frac{2^n}{n!}$ **(C)** $\left(\frac{2}{3}\right)^n$ **(D)** $\left(\frac{3}{4}\right)^n$

5. Three different dice are rolled three times. The Probability that they show different numbers only two times is :

(A) $\frac{1}{3}$ **(B)** $\left(\frac{{}^6P_3}{6^3}\right)^2$ **(C)** $\frac{107}{(54)^3}$ **(D)** $\frac{100}{243}$

★6. 2^n players of equal strength are playing a knock out tournament. If they are paired randomly in all rounds, the probability that out of two particular players S_1 and S_2 exactly one will reach in semi final is $(n \in N, n \geq 2)$:

(A) $\frac{8\times(2^n-4)}{2^n(2^n-1)}$ **(B)** $\frac{(2^n-4)}{2^n(2^n-1)}$ **(C)** $\frac{(2^{n-1}-4)}{2^n(2^n-1)}$ **(D)** None of these

7. A speaks truth in 60% cases and B speaks truth in 70% cases. The probability that they will say the same thing while describing single event is :

(A) 0.56 **(B)** 0.54 **(C)** 0.38 **(D)** 0.94

★8. Three smallest squares are chosen randomly on a chess board the probability that these squares have exactly two corners, but no side common is :

(A) $\frac{80}{{}^{64}C_3}$ **(B)** $\frac{72}{{}^{64}C_3}$ **(C)** $\frac{6^3}{{}^{64}C_3}$ **(D)** None of these

★9. Four die are thrown simultaneously. The probability that 4 and 3 appear on two of the die given that 5 and 6 have appeared on other two die is :

(A) 1/6 **(B)** 1/36 **(C)** 12/151 **(D)** None of these

★10. A fair coin is tossed 5 times then the probability that no two consecutive heads occur, is :

(A) $\frac{11}{32}$ **(B)** $\frac{15}{32}$ **(C)** $\frac{13}{32}$ **(D)** None of these

★**11.** The probability that a randomly chosen 3 digit number has exactly 3 factors :

(A) $\frac{2}{225}$ **(B)** $\frac{7}{900}$ **(C)** $\frac{1}{800}$ **(D)** None of these

12. Out of 6 pairs of distinct gloves 8 gloves are randomly selected, then the probability that there exist exactly 2 pairs in it is :

(A) $\frac{16}{33}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{12}{33}$ **(D)** $\frac{10}{33}$

13. A cube painted red on all sides is cut into 125 equal small cubes. A small cube when picked is found to show red color on one of its faces. The probability that two more faces also show red color is :

(A) $\frac{4}{75}$ **(B)** $\frac{4}{49}$ **(C)** $\frac{1}{8}$ **(D)** $\frac{8}{125}$

★**14.** 2n balls (all distinct in size) are arranged in a row. First few of these balls are black rest all white, both odd in number. The probability that there is exactly, one black ball in one of all possible arrangements is:

(A) $\frac{n}{2^{2n-1}}$ **(B)** $\frac{n}{2^{n-1}}$ **(C)** $\frac{n}{2^{2n-2}}$ **(D)** $\frac{n}{2^{n-2}}$

15. Each of 10 passengers board any of the three buses randomly which had no passenger initially. The probability that each bus has got at least one passenger is :

(A) $1-\frac{2^{10}}{3^{10}}$ **(B)** $1-\frac{{}^{10}C_3\times 3^7}{3^{10}}$ **(C)** $\frac{{}^{10}P_3 3^7}{3^{10}}$ **(D)** $\frac{3^{10}-3.2^{10}+3}{3^{10}}$

★**16.** *A* and *B* play a game of tennis. The situation of the game is as follows; if one scores two consecutive points after a deuce he wins; if loss of a point is followed by win of a point, it is deuce. The chance of a server to win a point is 2/3. The game is at deuce and *A* is serving. Probability that *A* will win the match is : (Serves are changed after each game)

(A) 3/5 **(B)** 2/5 **(C)** 1/2 **(D)** 4/5

17. Six different balls are put in three different boxes, no box being empty. The probability of putting balls in the boxes in equal numbers is :

(A) 3/10 **(B)** 1/6 **(C)** 1/5 **(D)** None of these

★**18.** The probability so that '*r*' 1 × 1 squares which are selected from a m × n chess board such that no two of them share the same row or same column is :

(A) $\frac{{}^mC_r\ {}^nC_r}{{}^{mn}C_r}$ **(B)** $\frac{{}^mC_r\ {}^nC_{r-1}}{{}^{mn}C_r}$ **(C)** $\frac{r\,{}^mC_r\ {}^nC_r}{{}^{mn}C_r}$ **(D)** None of these

19. A business man is expecting two telephone calls. Mr Walia may call any time between 2 p.m and 4 p.m. while Mr Sharma is equally likely to call any time between 2.30 p.m. and 3.15 p.m. The probability that Mr Walia calls before Mr Sharma is :

(A) 1/18 **(B)** 1/6 **(C)** 1/16 **(D)** None of these

20. A student appears for test I, II and III. The student is successful if he passes either in test I, II or I, III. The probability of the student passing in test I, II and III are respectively *p*, *q* and 1/2. If the probability of the student to be successful is 1/2 then :

(A) $p=q=1$ **(B)** $p=q=\frac{1}{2}$ **(C)** $p=1, q=0$ **(D)** $p=1, q=\frac{1}{2}$

21. Three of six vertices of a regular hexagon are chosen at random. The probability that the triangle with three vertices is equilateral equal to :

(A) 1/2 **(B)** 1/5 **(C)** 1/10 **(D)** 1/20

★22. A machine containing n different balls, when switched on, can throw up any number of balls one by one. The probability of throwing r balls is directly proportional to r. Given that a particular ball is the first ball to pop up, the probability that machine has thrown up all the balls is :

(A) $\frac{2}{n+1}$ **(B)** $\frac{1}{n+1}$ **(C)** $\frac{2}{n}$ **(D)** None of these

★23. From $4m + 1$ tickets numbered as 1, 2, … $4m + 1$. Three tickets are chosen at random. The probability that the numbers are in A.P. with even common difference is

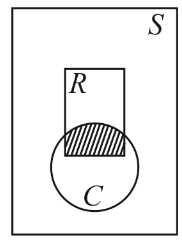
(A) $\frac{2(2m-1)}{3(16m^2-1)}$ **(B)** $\frac{3(2m+1)}{2(16m^2+1)}$ **(C)** $\frac{3(2m-1)}{2(4m^2-1)}$ **(D)** None of these

24. Let A and B be two events such that $P(A \cap B') = 0.20$, $P(A' \cap B) = 0.15$, $P(A' \cap B') = 0.1$, then $p(A/B)$ is equal to,

(A) 11/14 **(B)** 2/11 **(C)** 2/7 **(D)** 1/7

Paragraph for Questions 25 - 28

Consider a random experiment in which the outcomes cannot be identified discretely. Then the sample space of such an experiment will not contain distinguishable elements. An example of such a sample space can be an interval in the set of real numbers. Consider the following experiment:



Let your pen drop, tip downwards, into one of the pages of your notebook and note the point on the paper that the pen first touches. Here the sample space S consists of all the points on the paper. Let R and C be the events that the pencil drops into a rectangle and a circle as illustrated in the adjacent figure.

Clearly the sample space S and event sets R and C contain a uniform distribution of points. We consider only those sample spaces which have some finite geometrical measurement such as length area, and in which a point is selected at random. The probability of an event R, i.e. the selected point belongs to R will be given by $P(R) = \frac{\text{area of } R}{\text{area of } S}$.

Such a probability space is said to be uniform or continuous.

25. A point is selected at random inside a circle. The probability that the point is closer to the centre of the circle than to its circumference :

(A) 1/2 **(B)** 1/3 **(C)** 1/4 **(D)** $1/\sqrt{2}$

26. A point is selected at random inside an equilateral triangle whose side length is 3. The probability its distance to any corner is greater that 1 is

(A) $\frac{2\pi}{9\sqrt{3}}$ **(B)** $1-\frac{2\pi}{9\sqrt{3}}$ **(C)** $\frac{\sqrt{3}\pi}{9}$ **(D)** $1-\frac{\sqrt{3}\pi}{9}$

27. A point X is selected at random from a line segment AB with mid point O. The probability that the line segments AX, XB and AO can form a triangle is :

(A) 1/2 **(B)** 1/3 **(C)** 1/4 **(D)** 3/4

★28. A coin of diameter 1/2 is tossed randomly onto the rectangular cartesian plane. The probability that the coin does not intersect any line whose equation is of the form $x = k$, or $y = k, k$ is integer, is :

(A) 1/2 **(B)** 1/3 **(C)** 1/4 **(D)** 2/3

Paragraph for Questions 29 - 31

A player '*A*' plays a game against a machine. At reach round he deposits one rupee in a slot and then flips a coin which has a probability *p* of showing a head. If a head shows, he gets back the rupee he deposited and one more rupee from the machine. If a tail shows, he loses his rupee. Let *A* starts with 10 rupee and $q = 1 - p$

29. The probability that he will be drained out of his rupee exactly at the eleventh round is

(A) q^{11} **(B)** $1 - p^{11}$ **(C)** $pq^{10} + q^{11}$ **(D)** 0

30. The probability that all his money will be finished exactly at the twelfth round is :

(A) q^{12} **(B)** $1 - p^{12}$ **(C)** ${}^{10}C_1\, pq^{11}$ **(D)** None of these

31. The probability that he is left with no money by the 14th round or earlier is :

(A) $q^{10}(1 + 10pq + 65p^2q^2)$ **(B)** $q^{14}(p^2q + 36pq + 7)$

(C) $q^{12} + 3pq^{13} + 3p^{13}q + p^{12}$ **(D)** $1 - {}^{10}C_1\, pq^{11} - {}^{10}C_2 p^2 q^{12}$

Paragraph for Questions 32 - 34

Let B_n denotes the event that n fair dice are rolled once with $P(B_n) = \frac{1}{2^n}, n \in N$, e.g. $P(B_1) = \frac{1}{2}, ..., P(B_n) = \frac{1}{2^n}$.

Hence $B_1, B_2, B_3, ..., B_n$ are pairwise mutually exclusive events as $n \to \infty$. The event *A* occurs with atleast one of the event $B_1, B_2, B_3, ..., B_n$ and denotes that sum of the numbers appearing on the dice is *S*.

32. If even number of dice has been rolled, the probability that $S = 4$, is :

(A) very closed to $\frac{1}{2}$ **(B)** very closed to $\frac{1}{4}$ **(C)** very closed to $\frac{1}{16}$ **(D)** very closed to $\frac{1}{32}$

33. Probability that greatest number on the dice is 4 if three dice are known to have been rolled, is :

(A) $\frac{37}{216}$ **(B)** $\frac{64}{216}$ **(C)** $\frac{27}{216}$ **(D)** $\frac{31}{216}$

34. If $S = 3$, $P(B_2 / S)$ has the value equal to :

(A) $\frac{8}{169}$ **(B)** $\frac{24}{169}$ **(C)** $\frac{72}{169}$ **(D)** $\frac{16}{169}$

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

35. Suppose *X* is a random variable which takes values 0, 1, 2, 3, …… and $P(X = r) = pq^r$. Where $0 < p < 1, q = 1 - p$ and $r = 0, 1, 2, 3$ ……. Then :

(A) $P(X \geq a) = q^a$ **(B)** $P(X \geq a + b \mid X \geq a) = P(X \geq b)$

(C) $P(X = a + b \mid X \geq a) = P(X = b)$ **(D)** $P(X \geq a + b \mid X \geq b) = P(X \geq a)$

36. Consider the cartesian plane R^2 and let *X* denotes the subset of points for which both coordinates are integers. A coin of diameter 1/2 is tossed randomly into the plane. The probability *p* that the coin covers a point of *X* satisfies:

(A) $p = \frac{\pi}{16}$ **(B)** $p < \frac{\pi}{3}$ **(C)** $p > \frac{\pi}{30}$ **(D)** $p = \frac{1}{4}$

37. A square in inscribed in a circle. If p_1 is the probability that a randomly chosen point of the circle lies within the square and p_2 is the probability that the point lies outside the square, then :

(A) $p_1 = p_2$ **(B)** $p_1 > p_2$ **(C)** $p_1 < p_2$ **(D)** $p_1^2 - p_2^2 < 1/3$

38. If $\frac{1+4p}{4}$, $\frac{1-p}{3}$ and $\frac{1-2p}{2}$ are the probabilities of three mutually exclusive events then p may be :

(A) 1/8 **(B)** 1/2 **(C)** 3/8 **(D)** 5/8

★39. Players $P_1, P_2, P_3, \dots, P_m$, of equal skill, play a game consecutively in pairs as $P_1P_2, P_2P_3, P_3P_4, \dots P_{m-1}P_m, P_mP_1, \dots,$ and any player who wins two consecutive games (i.e k and $(k+1)$th game) wins the match. If the chance that the match is won at the rth game is k then :

(A) $k=\frac{1}{8}, \text{if } r=5$ **(B)** $k=\frac{5}{32}, \text{if } r=5$ **(C)** $k=\frac{3}{32}, \text{if } r=6$ **(D)** $k=\frac{5}{64}, \text{if } r=6$

★40. Two persons A, and B, have respectively $n+1$ and n coins, which they toss simultaneously. Then probability P that A will have more heads then B belongs :

(A) $\frac{1}{4}<P<\frac{3}{4}$ **(B)** $\frac{1}{2}<P<\frac{3}{4}$ **(C)** $\frac{1}{4}<P<\frac{1}{2}$ **(D)** $\frac{1}{3}<P<\frac{3}{4}$

41. If A and B are two events such that $P(A) = 1/2$ and $P(B) = 2/3$, then :

(A) $P(A\cup B) \geq 2/3$ **(B)** $P(A\cap B') \leq 1/3$

(C) $1/6 \leq P(A\cap B) \leq 1/2$ **(D)** $1/6 \leq P(A'\cap B) \leq 1/2$

42. A bag contains n (white and black) balls. It is given that the probability that the bag contains exactly r white balls is directly proportional to r $(0 \leq r \leq n)$. A ball is drawn at random and is found to be white. The probability that there is only one white ball in the bag is p then :

(A) $p=\frac{1}{55}$ if $n=5$ **(B)** $p=\frac{6}{55}$ if $n=5$ **(C)** $p=\frac{1}{91}$ if $n=6$ **(D)** $p=\frac{13}{93}$ if $n=6$

43. Let X be a set containing n elements. If two subsets A and B of X are picked at random, the probability that A and B have the same number of elements is :

(A) $\frac{{}^{2n}C_n}{2^{2n}}$ **(B)** $\frac{1}{{}^{2n}C_n}$ **(C)** $\frac{1.3.5......(2n-1)}{2^n.(n!)}$ **(D)** $\frac{3^n}{4^n}$

44. A parent particle can be divided into 0, 1 or 2 particles with probabilities $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$ respectively, after splitting. Beginning with one particle, the progenitor, let us denote by X_i, the number of particles in the i generation. Then :

(A) $P(X_2>0)=\frac{39}{64}$ **(B)** $P(X_2>0)=\frac{37}{64}$ **(C)** $P\left(\frac{(X_1=2)}{(X_2=1)}\right)=\frac{1}{5}$ **(D)** $P\left(\frac{(X_1=2)}{(X_2=1)}\right)=\frac{3}{64}$

45. If A and B are two events such that $P(A)=\frac{3}{4}$ and $P(B)=\frac{5}{8}$ then:

(A) $P(A\cup B)\geq\frac{3}{4}$ **(B)** $P(A'\cap B)\leq\frac{1}{4}$ **(C)** $\frac{3}{8}\leq P(A\cap B)\leq\frac{5}{8}$ **(D)** $\frac{1}{8}\leq P(A\cap B')\leq\frac{3}{8}$

46. A student has to match historical levents viz., Dandi march, Quit India Movement and Mahatma Gandhi's assassination with the years 1948, 1930 and 1942. The student has no knowledge of the correct answer decides to match the events and years randomly. Let $E_i\ (0\leq i\leq 3)$ denote the event that the student gets exactly i correct answers, then which of the following is/are NOT correct?

(A) $P(E_0)+P(E_3)=P(E_1)$ **(B)** $P(E_0).P(E_1)=P(E_3)$

(C) $P(E_0\cap E_1)=P(E_2)$ **(D)** $P(E_0)+P(E_1)+P(E_3)=1$

47. A bag contains 20 blue marbles, 12 red marbles and some other number of green marbles. If the probability of drawing green marble in one try is $\frac{1}{y}$ then which of the following statements is/are correct?

(A) Number of possible integral values of y is 5 **(B)** Number of possible integral values of y is 6
(C) Sum of possible integral values of y is 69 **(D)** Sum of possible integral values of y is 96

48. A dice is rolled three times. Let E_1 denote the event of getting a number larger than the previous number each time and E_2 denote the event that the numbers (in order) form an increasing AP then:

(A) $P(E_1) \geq P(E_2)$ **(B)** $P(E_2 \cap E_1) = \frac{1}{36}$ **(C)** $P(E_2 / E_1) = \frac{3}{10}$ **(D)** $P(E_1) = \frac{10}{3} P(E_2)$

49. One card is missing from a pack of cards. Let A be the event that missing card is a spade. Then two cards are drawn, and S be the event that they are spades then:

(A) $P(A') = \frac{3}{4}$ **(B)** $P(S/A) = \frac{{}^{13}C_2}{{}^{51}C_2}$ **(C)** $P(A/S) = \frac{11}{50}$ **(D)** $P(A) = P(A/S)$

50. A square is inscribed in a circle. If p_1 is the probability that a randomly chosen point of the circle lies within the square and p_2 is the probability that the point lies outside the square, then:

(A) $p_1 = p_2$ **(B)** $p_1 > p_2$ **(C)** $p_1 < p_2$ **(D)** $p_1^2 - p_2^2 < \frac{1}{3}$

51. A student appears for tests I, II, and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II, III are p, q and $\frac{1}{2}$, respectively. If the probability that the student is successful is $\frac{1}{2}$, then: (Assuming his performance in tests are independent).

(A) $p = 1, q = 0$ **(B)** $p = 2/3, q = 1/2$
(C) $p = 3/5, q = 2/3$ **(D)** There are infinitely many values of p and q

52. The letters of the word PROBABILITY are written down at random in a row. Let E_1 denote the event that two I, s are together and E_2 denote the event that two B's are together, then:

(A) $P(E_1) = P(E_2) = \frac{3}{11}$ **(B)** $P(E_1 \cap E_2) = \frac{2}{55}$ **(C)** $P(E_1 \cup E_2) = \frac{18}{55}$ **(D)** $P(E_1 / E_2) = \frac{1}{5}$

53. Let a, b, c be three integers such that $a^2 + b^2 + c^2 = 2$. Then for the system of simultaneous equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, where $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, which of the following statements is/are true?

(A) The probability that the system of equations has unique solution is 1/2
(B) The number of triplets (a, b, c) for which of the system of equations has infinitely many solutions is 6
(C) If $a = 0$, the number of ordered pairs (b, c) for which the system of equations has no solution is 2
(D) The number of elements in the range of $ab + bc + ca$ is 2

54. Sixteen players S_1, S_2, …., S_{16} play in a tournament. They are divided into eight pairs at random. From each pair, a winner is decided on the basis of a game played between the two players of the pair. Assume that all the players are of equal strength then:

(A) The probability that the player S_1 is among the eight winners is 1/2

(B) The probability that the player S_1 is among the eight winners is 1/4

(C) The probability that exactly one of the two players S_1 and S_2 is among the eight winners is 8/15

(D) The probability that exactly one of the two players S_1 and S_2 is among the eight winners is 7/15

55. *A* and *B* are two independent events. The probability that both *A* and *B* occur is 1/6 and the probability that neither of them occurs is 1/3. Then the probability of the occurrence of *A* may be:

(A) 1/2 **(B)** 1/3 **(C)** 1/4 **(D)** 2/3

56. The probabilities of events $A \cap B$, A, B and $A \cup B$ are respectively in A.P. with second term equal to the common difference. Therefore, *A* and *B* are:

(A) Mutually exclusive **(B)** Independent

(C) Such that one of them must occur **(D)** Such that one is twice as likely as the other

57. 5 players of equal strength play one each with each other. *P*(*A*) = probability that at least one player wins all matches he(they) play. *P*(*B*) = probability that at least one player loses all his (their) matches.

(A) $P(A)=\frac{5}{16}$ **(B)** $P(B)=\frac{7}{16}$ **(C)** $P(A \cap B)=\frac{5}{32}$ **(D)** $P(A \cup B)=\frac{15}{32}$

58. If *A* and *B* are exhaustive events in a sample space such that probabilities of the events $A \cap B$, A, B and $A \cup B$ are in A.P. If $P(A)=K$, where $0<K\le 1$, then:

(A) $P(B)=\frac{K+1}{2}$ **(B)** $P(A\cap B)=\frac{3K-1}{2}$ **(C)** $P(A\cup B)=1$ **(D)** $P(A'\cup B')=\frac{3(1-K)}{2}$

59. A boy has a collection of the blue and green marbles. The number of blue marbles belong to the set {2, 3, 4, …,13}. If two marbles are chosen simultaneously and random from his collection, then the probability that they have different colours is 1/2. Possible number of blue marbles is:

(A) 3 **(B)** 6 **(C)** 10 **(D)** 12

60. A fair coin is tossed *n* times. Let *X* denote the number of times head occurs. If $P(X=4), P(X=5)$ and $P(X=6)$ are in arithmetic progression, then the value of *n* can be:

(A) 14 **(B)** 12 **(C)** 10 **(D)** 7

61. If A_1, A_2, …., A_n be any events of the same sample space then:

(A) $\sum_{i=1}^{n} P(A_i)=1$ **(B)** $\sum P(A_i)\le 1$ if $A_1, A_2, ..., A_n$ are disjoint

(C) $\sum P(A_i)\ge 1$ if $A_1, A_2, ..., A_n$ are exhaustive events

(D) None of these

62. A family has three children. Event '*A*' is that family has at most one boy, Event '*B*' is that family has at least one boy and one girl, Event '*C*' is that the family has at most one girl. Then:

(A) Events '*A*' and '*B*' are independent **(B)** Events '*A*' and '*B*' are not independent

(C) Events *A*, *B*, *C* are not independent **(D)** Events *A*, *B*, *C* are not independent

63. A certain coin is tossed with probability of showing head being '*p*' Let '*q*' denote the probability that when the coin is tossed four times the number of heads obtained is even. Then:

(A) There is no value of *p*, if $q=\frac{1}{4}$ **(B)** There is exactly one value of *p* if $q=\frac{3}{4}$

(C) There are exactly two values of *p* if $q=\frac{3}{5}$ **(D)** There are exactly four values of *p* if $q=\frac{4}{5}$

64. A bag contains four tickets marked with numbers 112, 121, 211, and 222. One ticket is drawn at random from the bag. Let $E_i\left(i=1,2,3\right)$ denote the event that i^{th} digit on the ticket is 2. Then:

(A) E_1 and E_2 are independent **(B)** E_2 and E_3 are independent

(C) E_3 and E_1 are independent **(D)** E_1, E_2, E_3 are independent

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column 1 are labelled as (A), (B), (C) & (D) whereas statements in Column 2 are labeled as p, q, r, s & t. More than one choice from Column 2 can be matched with Column 1.

65. Observe the following columns:

	Column 1		Column 2
(A)	The probability that *A, B, C* solve a problem is $\frac{1}{2},\frac{1}{3}$ and $\frac{1}{4}$. If the probability that the problem will be solved is λ and that the problem is solved by only one of them is μ, then	**(p)**	$\lambda+\mu=\frac{13}{24}$
(B)	The probability of hitting a target by three men is $\frac{1}{2},\frac{1}{3}$ and $\frac{1}{4}$ respectively. If the probability that exactly two of them will hit the target is λ and that at least two of them hit the target is μ, then	**(q)**	$\lambda+\mu=\frac{29}{24}$
(C)	A bag contains 4 white and 2 black balls. Another contains 3 white and 5 black balls. One ball is drawn from each bag. If the probability that both are black is λ and that both are white is μ, then	**(r)**	$\lambda+\mu=\frac{11}{24}$
		(s)	$\lambda-\mu=7/24$
		(t)	$\mu-\lambda=1/24$

66. **MATCH THE FOLLOWING :**

	Column 1		Column 2
(A)	A pack of cards contain 51 cards. Cards are drawn from the pack without replacement. If 1^{st} 13 cards drawn are all red, then the probability that the missing card is black	**(p)**	$\frac{1}{2}$
(B)	A pack of cards is counted with face downwards. It is found that one card is missing. One card is drawn and is found to be red. The probability that the missing card is red.	**(q)**	$\frac{25}{51}$
(C)	A box has 2 white, 4 black and 6 green balls. Person *A*, draws a ball from it. Then from the remaining balls person *B* draws two balls which are found to be green. The probability that *A* has drawn a black ball.	**(r)**	$\frac{8}{9}$
(D)	Let *p, q* be chosen one by one from the set $\{1, \sqrt{2}, \sqrt{3}, 2, e, \pi\}$ with replacement. Now a circle is drawn taking (*p, q*) as its centre. The probability that at the most two rational points exist on the circle. (Rational points are those points whose both the coordinates are rational).	**(s)**	$\frac{2}{5}$
		(t)	None of these

★67. In a tournament there are twelve players $S_1, S_2, ..., S_{12}$ and divided into six pairs at random. From each game a winner is decided on the basis of a game played between the two players of the pair. Assuming all the pairs are of equal strength, then match the following.

	Column 1		Column 2
(A)	Probability that S_2 is among the losers is	**(p)**	5/22
(B)	Probability that exactly one S_3 and S_4 is among the losers, is	**(q)**	10/11
(C)	Probability that both S_2 and S_4 are among the winners is	**(r)**	1/2
(D)	Probability of S_4 and S_5 not playing against each other is	**(s)**	6/11
		(t)	None of these

★68. 'n' whole numbers are randomly chosen and multiplied:

	Column 1		Column 2
(A)	The probability that the last digit is 1, 3, 7 or 9 is	**(p)**	$\frac{8^n - 4^n}{10^n}$
(B)	The probability that the last digit is 2, 4, 6, 8 is	**(q)**	$\frac{5^n - 4^n}{10^n}$
(C)	The probability that last digit is 5 is	**(r)**	$\frac{4^n}{10^n}$
(D)	The probability that the last digit is zero is	**(s)**	$\frac{10^n - 8^n - 5^n + 4^n}{10^n}$
		(t)	None of these

69. A bag contains some white and some black balls, all combinations being equally likely. The total number of balls in the bag is 12. Four balls are drawn at random without replacement. Match the entries in the following two lists.

	Column 1		Column 2
(A)	The probability that all the four balls are black is equal to	**(p)**	$\frac{14}{33}$
(B)	If the bag contains 10 black and 2 white balls, then the probability that all four balls are black is equal to	**(q)**	$\frac{1}{5}$
(C)	If all the four balls are black, then the probability that the bag contains 10 black balls is equal to	**(r)**	$\frac{70}{429}$
(D)	The probability that two balls are black and two are white is equal to	**(s)**	$\frac{13}{165}$
		(t)	None of these

SUBJECTIVE INTEGER TYPE

Each of the following question has an integer answer between 0 and 9.

★70. Two integer 'a' and 'b' are randomly selected from the set {1, 2, ⋯} (with replacement) then if the probability of $\frac{1}{5}\left(a^2+b^2\right)$ being positive integer is $\frac{p}{q}$ (where $H.C.F\ (p, q) = 1$) then $q-2p=$ …………

71. If a and b are chosen randomly from the set consisting of numbers 1, 2, 3, 4, 5, 6 with replacement. If the probability that $\underset{x\to 0}{Lt}\left(\frac{a^x+b^x}{2}\right)^{2/x}=6$ is $\frac{p}{q}$ (where H.C.F $(p, q) = 1$) then $q-p=$ ……………..

72. A bag contained 3 maths book and 2 physics books. A book is drawn at random if it is of math, 2 more books of maths together with this book put back in the bag and if it is of physics it is not replaced in the bag. This experiment is repeated 3 time. If third draw gives math book, The probability that first two drawn books were of physics is $\frac{p}{q}$ (where H.C.F $(p, q) = 1$) then $q-8(p+1)=$ __________.

73. Two friends decide to meet at a spot between 2 p.m. and 3 p.m. whosoever arrives first agrees to wait for 15 minutes for the other. The probability that they meet is $\frac{p}{q}$ (where H.C.F $(p, q) = 1$) then $q-p=$ _______.

74. There are two bags, bag I contains 4 red and 5 white balls, while bag II contains 5 red and 4 white balls. Two balls are drawn from bag I. If they are of the same colour, another ball is drawn from bag I. If the first two drawn balls are of different colours, one ball is drawn from bag II. If the third drawn ball is red, then the probability that the first two drawn balls were white is $\frac{p}{q}$ (where H.C.F $(p, q) = 1$) then $q-3(p+1)=$ ________.

75. There are two purses. The first contains 9 fifty paise coins and a one-rupee coin, while the second purse has 10 fifty-paise coins. Nine coins are transferred from the first purse to the second randomly. Then nine coins are transferred from the second purse to the first randomly. The probability of finding a one rupee coin in the first purse after these transfers is $\frac{p}{q}$ (where H.C.F (p, q) = 1) then $q-p=$ _________.

★76. The probability that two queens, placed at random on a chess board, do not take on each other is $\frac{p}{q}$ (where H.C.F (p, q) = 1) then $q-p-5=$ __________.

★77. In an organization number of women are μ times that of men. If α things are to be distributed among them than the probability that the number of things received by men are odd is $\left(\frac{1}{2}-\left(\frac{1}{2}\right)^{\alpha+1}\right)$. Then $\mu=$ _________.

78. Six fair dice are thrown independently. The probability that there are exactly 2 different pairs (A pair is an ordered combination like 2, 2, 1, 3, 5, 6) is p, then $4p$ is______.

79. Raj and Sanchita are playing game in which they throw a die alternately till one of them gets a six. The probability that Sanchita win the game is p then the value of $5p$?

80. Two dice are thrown simultaneously what is the probability that sum of the two numbers will be 5 before 7?

81. Rajesh doesn't like to study. Probability that he will study is 1/3. If he studied, then probability that he will fail is 1/2 and if he didn't study then probability that he will fail is 3/4. If in result Rajesh failed, then what is the probability that he didn't study.

82. On a rod of length 6 units, lengths 1, 2 units are measured at random, the probability that no points of the measured lines will coincide is______.

83. In a board exam there are two sections each section has 5 questions. As per the given condition a candidate has to answer any 6 questions out of 10 questions. What is the probability that a student answered 6 questions such that not more than 4 questions selected from one section?

84. Assume that the birth of a boy or girl to a couple to be equally likely, and exhaustive. For a couple having 6 children, the probability that their "three oldest are boys" is p then the value of $10p$.

85. Two fair dice are thrown till outcome is 12. The probability that one has to do 20 throws for this is______.

86. The probability of randomly drawing five cards from a deck that has exactly one Ace is______.

87. An ordinary deck of 52 playing cards is randomly divided into 4 groups of 13 cards each. The probability that each group has exactly 1 jack?

88. In a poker game, the probability of getting a "pair" in five cards is___ [A pair consists of 2 cards of the same kind (eg, 2 kings) and 3 cards that are different from the kind of the pair (eg, different from kings) and that are all different from each other.]

89. Letters of the word MATHEMATICS are arranged in all the possible ways, and a word is selected randomly then the probability that letter C is exactly between S and H is______.

90. A group of students comprising 3 girls and 5 boys went for a picnic. During a game they were arranged in a circle then the probability that each boy has one girl on at least one side is______.

91. The probabilities that a student passes in Mathematics, physics and chemistry are m, p and c, respectively. Of these subjects, the student has a 75% chance of passing in at least one, a 50% chance of passing in atleast two, and 40% chance of passing in exactly two. Then $p + m + c$ is______.

92. Eight players $P_1, P_2, \ldots P_8$ play a knock out tournament. It is known that whenever the players P_i and P_j play, the player P_i will win if $i < j$. Assuming that the players are paired at random in each round, then the probability that the player P_4 reaches the final is______.

93. If A and B are two events such that $P(A) = 0.3$, $P(B) = 0.25$, $P(A \cap B) = 0.2$, then $10P\left(\left(\frac{A^C}{B^C}\right)^C\right)$ is equal to______.

94. A number is selected at random from the first twenty-five natural numbers. If it is a composite number, then it is divided by 5. But if it is not a composite number, it is divided by 2. The probability that there will be no remainder in the division is______.

95. There are three bags each containing 5 white balls and 2 black balls and 2 bags each containing 1 white ball and 4 black balls: a black ball having been drawn, the probability that it came from the first group is______.

96. A speaks truth 3 times out of 4, and B 7 times out of 10, they both assert that a white ball has been drawn from a bag containing 6 balls all of different colours; the probability of truth of the assertion is______.

97. A bag contains 10 different balls. Five balls are drawn simultaneously and then replaced and then seven balls are drawn. The probability that exactly three balls are common to the two drawn is p then the value of $10p$ is______.

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. If matrix $A=\begin{pmatrix}1 & -1\\ 1 & -2\end{pmatrix}$, satisfies $A^n=5I-8A$, then $n=$

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7

2. Let $A=[a_{rs}]_{n\times n}$ be a $n\times n$ matrix such that $a_{rs}=(r-s)2^{i(r-s)}$ where $i=\sqrt{-1}$, then $A=$ ($\overline{A}$ denotes $[\overline{a_{rs}}]$ complex conjugate)

(A) $\overline{A}]$ **(B)** $-\overline{A}$ **(C)** $(\overline{A})^T$ **(D)** $-(\overline{A})^T$

3. A_1 is a matrix formed by replacing all the elements in $A_{n\times n}$ by corresponding cofactors, A_2 is matrix formed by replacing all elements of A_1 by corresponding cofactors and A_3, A_4 … formed so on, if $|A_n|=K$, then $|A|=$

(A) K^{n^n} **(B)** $K^{(n-1)^n}$ **(C)** $K^{\frac{1}{(n-1)^n}}$ **(D)** $\dfrac{1}{K^{(n-1)^n}}$

4. If A is an idempotent matrix i.e. $A^2=A$, and $B=I-A$, then which of the following is incorrect:

(A) $B^2=B$ **(B)** $B^2=I$ **(C)** $AB=0$ **(D)** $BA=0$

5. $A=[a_{ij}]_{n\times n}$ be a square matrix, n is odd such that $a_{ij}=(-1)^i\ {}^nC_i\times{}^nC_j$, then trace $(A)=$

(Trace (A) denotes sum of diagonal elements of A)

(A) 0 **(B)** 1 **(C)** –1 **(D)** 2

6. Let $A=[a_{ij}]_{3\times3}, B=[b_{ij}]_{3\times3}$ where $b_{ij}=3^{i-j}a_{ij}$, $C=[c_{ij}]_{3\times3}$, where $c_{ij}=4^{i-j}b_{ij}$ be any three matrices. If $|A|=4$, then $|B|+|C|=$ ($|X|$ denotes determinant of matrix X)

(A) 8 **(B)** 48 **(C)** 192 **(D)** 4

7. If $f(x)=\begin{vmatrix}x & 1+x^2 & x^3\\ \ln(1+x^2) & e^x & \sin x\\ \cos x & \tan x & \sin^2 x\end{vmatrix}$, then :

(A) $f(x)$ is divisible by x **(B)** $f(x)=0$

(C) $f'(x)=0$ **(D)** $\lim\limits_{x\to0}\dfrac{f(x)}{x}=1$

8. System of equations $ax+4y+z=0,\ 2y+3z=1, 3x-bz=-2$ then which of the following is not true

(A) Unique solution if $ab\neq15$ **(B)** Infinitely many solutions if $a=3, b=5$

(C) No solution if $ab=15, a\neq3$ **(D)** No solution if $ab=15, a\neq5$

9. If $1, \omega, \omega^2$ are cube roots of unity then system of equations : $x + 2\omega y + 3\omega^2 z = 1 - \omega^2$, $2x + 3\omega y + \omega^2 z = \omega^2 - \omega$

$3x + \omega y + 2\omega^2 z = \omega - 1$ has

(A) unique solution **(B)** infinitely many solutions

(C) no solution **(D)** exactly 3 different solutions

10. Let *S* be the set of all 3×3 symmetric matrices whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0. A matrix is selected from set *S*, what is the probability that the selected matrix is non singular

(A) 1/4 **(B)** 1/3 **(C)** 1/2 **(D)** 2/3

11. Let $f(x) = \begin{vmatrix} x\cos x & 2x\sin x & x\tan x \\ 1 & x & 1 \\ 1 & 2x & 1 \end{vmatrix}$, then $\lim_{x \to 0} \frac{f(x)}{x^2} =$

(A) 0 **(B)** 1 **(C)** –1 **(D)** Does not exist

12. If a, b, c are the roots of the equation $x^3 + 2x^2 + 1 = 0$, then $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} =$

(A) 8 **(B)** –8 **(C)** 0 **(D)** 2

13. Matrix A is such that $A^2 = 2A - I$, where I is identity matrix, then for $n \geq 2, A^n =$

(A) $nA - (n-1)I$ **(B)** $nA - I$ **(C)** $2^{n-1}A - (n-1)I$ **(D)** $2^{n-1}A - I$

14. The system of homogeneous equations $\lambda x + (\lambda+1)y + (\lambda-1)z = 0$, $(\lambda+1)x + \lambda y + (\lambda+2)z = 0$,

$(\lambda-1)x + (\lambda+2)y + \lambda z = 0$ Has non trivial solution for :

(A) Exactly 3 real values of λ **(B)** Exactly 2 real values of λ

(C) Exactly 1 real value of λ **(D)** Infinitely many real values of λ

15. For any real values of X, Y, Z, L, M, N value of $\begin{vmatrix} \cos(X-L) & \cos(X-M) & \cos(X-N) \\ \cos(Y-L) & \cos(Y-M) & \cos(Y-N) \\ \cos(Z-L) & \cos(Z-M) & \cos(Z-N) \end{vmatrix} =$

(A) 0 **(B)** 1 **(C)** $\cos X \cos Y \cos Z + \cos L \cos M \cos N$

(D) $(\cos X - \cos Y)(\cos Y - \cos Z)(\cos Z - \cos X)(\cos L - \cos M)(\cos M - \cos N)(\cos N - \cos M)$

16. If $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, P = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, $Q = P^T AP$ then $PQ^{2014}P^T =$

(A) $\begin{pmatrix} 1 & 2^{2014} \\ 0 & 1 \end{pmatrix}$ **(B)** $\begin{pmatrix} 1 & 4028 \\ 0 & 1 \end{pmatrix}$ **(C)** $(P^T)^{2013} A^{2014} P^{2013}$ **(D)** $P^T A^{2014} P$

17. Let A be a 3×3 non-singular matrix then which of the following is not true

(A) $|-A| = -|A|$ **(B)** $|Adj(A)| = |A|^2$

(C) $Adj(Adj\ A) = |A|^2\ A$ **(D)** $A \cdot Adj\,(A) = |A| I_3$

18. If *A* be 3×3 non-singular matrix, $|A| = K$, then $|(xA)^{-1}| =$ $(where\ \ x \neq 0)$

(A) xK **(B)** $\frac{1}{xK}$ **(C)** $\frac{1}{x^3K^3}$ **(D)** $\frac{1}{x^3K}$

19. Number of distinct real values of K, such that the system of equations $x+2y+z=1$, $x+3y+4z=K$, $x+5y+10z=K^2$ has infinitely many solutions is:

(A) zero **(B)** one **(C)** two **(D)** three

Paragraph for Questions 20 - 22

Let $A=\begin{bmatrix} 4 & 1 \\ -9 & -2 \end{bmatrix}$, and $A^{100}=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then:

20. $a+d=$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

21. $\frac{c}{b}=$

(A) 9 **(B)** –9 **(C)** $\frac{-1}{9}$ **(D)** Not defined

22. Which of the following is true:

(A) $A^{200}+2A^{100}-I=0$ **(B)** $A^{200}-2A^{100}-I=0$

(C) $A^{200}-2A^{100}+I=0$ **(D)** $A^{200}+2A^{100}+I=0$

Paragraph for Questions 23 - 25

Let A be a square matrix and I be identity matrix of same order then $A-\lambda I$ is called characteristic matrix of A, λ is some complex number. $|A-\lambda I|=0$ is known as characteristic equation of matrix A; and its roots are called Characteristic roots or Eigen values of A. Every matrix satisfies its characteristic equation.

23. Eigen values of matrix $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix}$ are :

(A) $1,-1,3$ **(B)** $2,\frac{1\pm\sqrt{7}}{2}$ **(C)** $-3,3\pm2\sqrt{2}$ **(D)** $2,-2,3$

24. Which of the following matrices do not have eigen values 1 *and* -1

(A) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ **(B)** $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ **(C)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **(D)** $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

25. If $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & -2 & 3 \end{bmatrix}$ & $I=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then if $A^7-4A^6+6A^5=\alpha A^2+\beta A+\gamma I$ then (α,β,γ) is :

(A) $(5,-11,7)$ **(B)** $(-21,115,-91)$ **(C)** $(-13,44,-28)$ **(D)** None of these

MULTIPLE CORRECT ANSWERS TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONE or MORE Choices may be Correct:

26. A and B be two 3×3 matrices such that $A^5 = B^5$ and $A^4B = B^4A,\ A \neq B$ then :

(A) $A^4 = B^4$ **(B)** $|A^4 + B^4| = 0$

(C) $(A^4 - B^4)\cdot(A+B) = 0$ **(D)** $(A^4 + B^4)\cdot(A-B) = 0$

27. Let X and Y be two matrices different from I, such that $XY = YX$ and $X^n - Y^n$ is invertible for some natural number n. If $X^n - Y^n = X^{n+1} - Y^{n+1} = X^{n+2} - Y^{n+2}$, then:

(A) $I - X$ is singular **(B)** $I - Y$ is singular

(C) $X + Y = XY + I$ **(D)** $(I-X)(I-Y)$ is non singular

28. Let matrices be $X = \begin{bmatrix} 2 & 2 \\ 5 & 1 \end{bmatrix}$, $Y = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$ and $Z = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$, then (tr($A$) denotes trace of A)

(A) $\displaystyle\sum_{k=0}^{\infty} \frac{tr(X(YZ)^k)}{2^k} = 6$ **(B)** $\displaystyle\sum_{k=1}^{\infty} \frac{tr(X(YZ)^k)}{2^k} = 3$

(C) $\displaystyle\sum_{k=1}^{\infty} \frac{tr(X(YZ)^k)}{2^k} = 6$ **(D)** $\displaystyle\sum_{k=0}^{\infty} \frac{tr(X(YZ)^k)}{2^k} = 3$

29. $Adj(A) = \begin{pmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{pmatrix}$, then $|A| =$

(A) 1 **(B)** –1 **(C)** 2 **(D)** –2

30. Let $f(x) = \begin{vmatrix} (2^x - 2^{-x})^2 & (2^x + 2^{-x})^2 & 1 \\ (3^x - 3^{-x})^2 & (3^x + 3^{-x})^2 & 1 \\ (4^x - 4^{-x})^2 & (4^x + 4^{-x})^2 & 1 \end{vmatrix}$ & $g(x) = \begin{vmatrix} 2x-2 & x-1 & x-1 \\ 3x-4 & 2x-3 & x-1 \\ 3x-5 & 2x-4 & 2x-4 \end{vmatrix}$ then :

(A) $f(4) = 0$ **(B)** $f(4) = 1020$

(C) $f(x) = g(x)$ has one solution **(D)** $f(x) = g(x)$ has three solutions

31. Let $f(x) = \begin{vmatrix} 7 & 2 & x^2-12 \\ 6 & x^2-12 & 3 \\ x^2-12 & 2 & 7 \end{vmatrix}$ then :

(A) $f(x) = 0$ has 6 real roots **(B)** $f(x) = 0$ has 4 real roots

(C) Sum of real roots of $f(x) = 0$ is 0 **(D)** Sum of real roots of $f(x) = 0$ is 9

32. Let X, Y, Z be 2×2 matrices with real entries. Define $\odot$ as follows $X \odot Y = \frac{1}{2}(XY + YX)$ then :

(A) $X \odot Y = Y \odot X$ **(B)** $X \odot I = X$

(C) $X \odot X = X^2$ **(D)** $X \odot (Y \odot Z) = X \odot Y + X \odot Z$

33. If $A \& B$ are two invertible matrices of same order, then $Adj(AB) =$

(A) $|A||B|A^{-1}B^{-1}$ **(B)** $adj(A)\cdot adj(B)$ **(C)** $|A||B|(AB)^{-1}$ **(D)** $adj(B)\cdot adj(A)$

34. If $f(x) = \begin{vmatrix} (1+x)^a & (1+2x)^b & 1 \\ 1 & (1+x)^a & (1+2x)^b \\ (1+2x)^b & 1 & (1+x)^a \end{vmatrix}$, a, b being natural numbers, then:

(A) constant term of $f(x)$ is 0 **(B)** coefficient of x in $f(x)$ is 0

(C) constant term of $f(x)$ is $a-b$ **(D)** constant term of $f(x)$ is $a+b$

35. Atleast one root of the equation $\begin{vmatrix} x^2 + \sin x \cos x & x(1+\sin x) \\ x + \cos x & x+1 \end{vmatrix} = 0$ lies in:

(A) $\left(0, \frac{\pi}{6}\right)$ **(B)** $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ **(C)** $\left(\frac{-\pi}{3}, \frac{-\pi}{4}\right)$ **(D)** $\left(\frac{-\pi}{4}, \frac{\pi}{6}\right)$

36. Let in a skew symmetric matrix of order n the maximum number of non-zero elements is M_1 & let in an upper triangular matrix of order n the minimum number of zero elements is M_2 then :

(A) $\frac{M_1}{M_2} = 2$ **(B)** $M_1 = 2\sum_{k=1}^{n-1} k$ **(C)** $M_2 = \sum_{k=1}^{n} k$ **(D)** $\frac{M_1}{M_2} = \frac{2(n-1)}{n+1}$

37. Let A be a square matrix of order n, $A = [a_{ij}]_{n\times n}, a_{ij} = i^n - j^n$, then :

(A) $|A| = 0, n$ is odd

(B) $|A|$ is perfect square if n is even

(C) $|A| = 0 \,\forall\, n$

(D) A is skew symmetric when n is odd & symmetric when n is even

38. If $f(x) = \begin{vmatrix} a^{-x} & e^{x \ln a} & x^2 \\ a^{-3x} & e^{3x \ln a} & x^4 \\ a^{-5x} & e^{5x \ln a} & 1 \end{vmatrix}$ then :

(A) Graph of $f(x)$ is symmetric about origin **(B)** Graph of $f(x)$ is symmetric about y axis

(C) $f^{iv}(0) = 0$ **(D)** $f(x)\ln\left(\frac{a-x}{a+x}\right)$ is an even function

39. If A and B are square matrices of same order such that they commute then :

(A) $A^m \& B^n$ commute $m, n \in N$ **(B)** $(A+B)^n = {}^nC_0A^n + {}^nC_1A^{n-1}B + ... + {}^nC_nB^n, n \in N$

(C) $A - \lambda I, B + \mu I$ commute $\forall \lambda, \mu \in R$ **(D)** $A + \lambda I,\ \mu I - B$ commute $\forall \lambda, \mu \in R$

40. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 8 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 & 5 \\ 2 & 3 & 8 \\ 7 & 2 & 9 \end{bmatrix}$, then :

(A) $adj(adjA) = -A$ **(B)** $|adj(AB)| = 576$

(C) $|adj(adj(adj(adjA)))| = 1$ **(D)** $|adj(B^{-1})| = \frac{1}{24}$

41. If $ax+by+cz=0,\ bx+cy+az=0,\ cx+ay+bz=0$, $a,b,c\in R^{+}$ then :

(A) System have only trivial solution if $a^3+b^3+c^3>3abc$

(B) System will have non trivial solution only if $a=b=c$

(C) System have no solution if $a^3+b^3+c^3=3abc$

(D) If system have non trivial solution then minimum value of $(x-1)^2+(y-2)^2+(z-3)^2$ is 12

42. A be the set of all square matrices of order 3 with elements either 0, 1, or –1, then:

(A) $O(A)=3^9$

(B) Number of symmetric matrices in A whose trace is 0 = 7 × Number of skew symmetric matrices in A

(C) Number of matrices in A such that each of 0, 1, and –1 occurs atleast once at any position is 18150

(D) All skew symmetric matrices in A are singular

43. Let A be set of all 2×2 matrices of the form $\begin{pmatrix} a & b \\ c & a \end{pmatrix}$, such that $a,b,c\in\{0,1,2,3,4\}$ then **:**

(A) Number of matrices in A which are symmetric or skew symmetric is 25

(B) Number of matrices in A which are symmetric or skew symmetric & determinant divisible by 5 is 9

(C) Number of matrices in A for which trace is not divisible by 5 & determinant divisible by 5 is 16

(D) Number of matrices in A for which determinant is not divisible by 5 is 109

44. If $x^a y^b = e^m, x^c y^d = e^n$, $P=\begin{vmatrix} m & b \\ n & d \end{vmatrix}, Q=\begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $R=\begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then :

(A) $P\log_x e = Q\log_y e$

(B) $x=e^{P/R}, y=e^{Q/R}$

(C) $P\log_e x = Q\log_e y$

(D) $x=e^{R/P}, y=e^{R/Q}$

45. $A=\begin{pmatrix} -3 & -1 & 2 \\ 3 & 1 & -1 \\ 4 & 2 & 5 \end{pmatrix}, A\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}=\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, A\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $A\begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, B=\begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{pmatrix}$, then :

(A) Trace $(B)=-8$

(B) $|AdjB|=4$

(C) $|Adj(B)|=\frac{1}{4}$

(D) Sum of all elements of B is –10

46. Let $f(n)=\begin{vmatrix} n & n+1 & n+2 \\ {}^{n}P_{n} & {}^{n+1}P_{n+1} & {}^{n+2}P_{n+2} \\ {}^{n}C_{n} & {}^{n+1}C_{n+1} & {}^{n+2}C_{n+2} \end{vmatrix}$, where the symbols have their usual meanings. Then $f(n)$ is divisible by

(A) n^2+n+1 **(B)** $(n+1)!$ **(C)** $n!$ **(D)** None of these

47. If $A(\theta)=\begin{bmatrix} \sin\theta & i\cos\theta \\ i\cos\theta & \sin\theta \end{bmatrix}$, then which of the following is not true

(A) $A(\theta)^{-1}=A(\pi-\theta)$

(B) $A(\theta)+A(\pi+\theta)$ is a null matrix

(C) $A(\theta)$ is invertible for all $\theta\in R$

(D) $A(\theta)^{-1}=A(-\theta)$

48. If $\Delta = \begin{vmatrix} \sin\theta\cos\theta & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\sin\phi & 0 \end{vmatrix}$ then

(A) Δ is independent of θ **(B)** Δ is independent of ϕ

(C) Δ is a constant **(D)** $\left.\frac{d\Delta}{d\theta}\right]_{\theta=\frac{\pi}{2}} = 0$

49. If $\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix} = \begin{vmatrix} \alpha^2 & \beta^2 & \beta^2 \\ \beta^2 & \alpha^2 & \beta^2 \\ \beta^2 & \beta^2 & \alpha^2 \end{vmatrix}$, then

(A) $\alpha^2 = a^2+b^2+c^2$ **(B)** $\beta^2 = ab+bc+ca$ **(C)** $\alpha^2 = ab+bc+ca$ **(D)** $\beta^2 = a^2+b^2+c^2$

50. Let $a, \lambda, \mu \in R$ consider the system of linear equations $ax+2y=\lambda$, $3x-2y=\mu$

Which of the following statement(s) is (are) correct?

(A) If $a=-3$ then the system has infinitely many solutions for all values of λ and μ

(B) If $a \neq -3$, then the system has a unique solution for all values of λ and μ

(C) If $\lambda+\mu=0$ the system has infinitely many solutions for $a=-3$

(D) If $\lambda+\mu \neq 0$, then the system has no solutions for $a=-3$

51. $\Delta = \begin{vmatrix} a & a^2 & 0 \\ 1 & 2a+b & a+b \\ 0 & 1 & 2a+3b \end{vmatrix}$ is divisible by

(A) $a+b$ **(B)** $a+2b$ **(C)** $2a+3b-1$ **(D)** a^2

52. Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha$$

(A) –4 **(B)** 9 **(C)** –9 **(D)** 4

53. If $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is an orthogonal matrix of order 3, then:

(A) $a=-2$ **(B)** $a=2, b=1$ **(C)** $b=-1$ **(D)** $b=1$

54. How many 3×3 matrices M with entries from {0, 1, 2} are there for which the sum of the diagonal entries of M^TM is 5.

(A) 198 **(B)** 162 **(C)** 126 **(D)** 135

55. Let ω be a complex cube root of unity with $\omega \neq 1$ and $P = [P_{ij}]$ be a $n \times n$ matrix with $P_{ij} = \omega^{i+j}$. Then $P^2 \neq 0$, when n =

(A) 57 **(B)** 55 **(C)** 58 **(D)** 56

56. The value of θ lying between $\theta=0$ and $\theta=\frac{\pi}{2}$ and satisfying the equation $\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix}=0$ is:

(A) $\frac{7\pi}{24}$ **(B)** $\frac{5\pi}{24}$ **(C)** $\frac{11\pi}{24}$ **(D)** $\frac{\pi}{24}$

57. If maximum and minimum values of the determinant $\begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$ are α and β, then:

(A) $\alpha+\beta^{99}=4$

(B) $\alpha^3-\beta^{17}=26$

(C) $\left(\alpha^{2n}-\beta^{2n}\right)$ is always an even integer for $n\in N$

(D) A triangle can be constructed having it sides as $\alpha-\beta$, $\alpha+\beta$ and $\alpha+3\beta$

58. Let *X* and *Y* be two arbitrary, 3×3, non-zero skew-symmetric matrices and *Z* be an arbitrary 3×3, non-zero symmetric matrix. Then which of the following is (are) skew symmetric:

(A) $Y^3Z^4-Z^4Y^3$ **(B)** $X^{44}+Y^{44}$ **(C)** $X^4Z^3-Z^3X^4$ **(D)** $X^{23}+Y^{23}$

59. Which of the following is (are) not the square of a 3×3 matrix with real entries :

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ **(C)** $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ **(D)** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

60. Let *M* be a 2×2 symmetrix matrix with integer entries. Then *M* is invertible, if:

(A) The first column of *M* is the transpose of the second row of *M*.

(B) The second row of *M* is the transpose of first column of *M*

(C) *M* is a diagonal matrix with non-zero entries in the main diagonal

(D) The product of entries in the main diagonal of *M* is not the square of an integer

61. The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$ is equal to zero, if:

(A) *a*, *b*, *c* are in A.P **(B)** *a*, *b*, *c* are in G.P

(C) *a*, *b*, *c* are in H.P **(D)** $(x-\alpha)$ is a factor of $ax^2+2bx+c$

62. If *a*, *b*, *c* are non-zero real numbers such that $\begin{vmatrix} bc & ca & ab \\ ca & ab & bc \\ ab & bc & ca \end{vmatrix}=0$, then:

(A) $\frac{1}{a}+\frac{1}{b\omega}+\frac{1}{c\omega^2}=0$ **(B)** $\frac{1}{a}+\frac{1}{b\omega^2}+\frac{1}{c\omega}=0$ **(C)** $\frac{1}{a\omega}+\frac{1}{b\omega^2}+\frac{1}{c}=0$ **(D)** None of these

63. Let $\{\Delta_1, \Delta_2, \Delta_3, .. \Delta_k\}$ be the set of third order determinants that can be made with the distinct non-zero real numbers $a_1, a_2, ...a_9$, Then:

(A) $k = 9!$

(B) $\sum_{i=1}^{k} \Delta_i = 0$

(C) At least one $\Delta_i = 0$

(D) None of these

64. For 3×3 Matrices *M* and *N*, which of the following statement(s) is (are) not correct

(A) $N^T MN$ is symmetric or skew symmetric, according as *M* is symmetric or skew symmetric

(B) $MN - NM$ is skew symmetric for all symmetric matrices *M* and *N*

(C) *MN* is symmetric for all symmetric matrices *M* and *N*

(D) (adj *M*) (adj *N*) = adj(*MN*) for all invertible matrices *M* and *N*.

65. The system of equations $6x + 5y + \lambda z = 0$, $3x - y + 4z = 0$, $x + 2y - 3z = 0$, has:

(A) Only a trivial solution for $\lambda \in R$

(B) Exactly one non-trivial solution for some real λ

(C) Infinite number of non-trivial solutions for one value of λ

(D) Only one solution for $\lambda \neq -5$

MATRIX MATCH TYPE

Each of the following question contains statements given in two columns, which have to be matched. Statements in Column 1 are labelled as (A), (B), (C) & (D) whereas statements in Column 2 are labeled as p, q, r, s & t. More than one choice from Column 2 can be matched with Column 1.

66. Consider the 2×2 matrix $A = \begin{bmatrix} x & y \\ 0 & 1 \end{bmatrix}$

	Column 1		Column 2
(A)	A is idempotent	**(p)**	Either $x = 1, y = 0$ or $x = -1, y \in R$
(B)	A is involutory	**(q)**	Either $x = 0, y \in R$ or $x = 1, y = 0$
(C)	A is orthogonal	**(r)**	$x = 0, y \in R$
(D)	A is singular	**(s)**	$x = \pm 1, y = 0$

67. If $f(x)=\begin{vmatrix}(\alpha x+1)\cos^2 x & x & 1-x\\ \beta\sin x & x^2 & 2x\\ (\gamma x^2+1)\tan x & x & 1-x^2\end{vmatrix}$

	Column 1		Column 2
(A)	$\lim\limits_{x\to 0}\frac{f(x)}{x}+\lim\limits_{x\to 0}\frac{f(x)}{x^2}$	**(p)**	0
(B)	$\lim\limits_{x\to 0}\frac{f'(x)}{x}+f'(0)$	**(q)**	–1
(C)	$\lim\limits_{x\to 0}\frac{1}{x^6}\int_{x^2}^{x^3} f(x)dx=A,[A]$ is ([.] denotes greatest integer function)	**(r)**	–2
(D)	If $\alpha=\beta=\gamma=0, g(x)=\frac{f(x)}{x^2}$ then $\left[g\left(\frac{\pi}{4}\right)\right]$ is ([.] denotes greatest integer function)	**(s)**	$f''(0)$

68. The elements of 3×3 matrix A are either 1 or –1, then

	Column 1		Column 2
(A)	Total number of such A which are symmetric	**(p)**	3
(B)	Maximum value of determinant $\lvert A\rvert$	**(q)**	4
(C)	Minimum value of determinant $\lvert A\rvert$	**(r)**	–4
(D)	Maximum value of trace of A	**(s)**	64

69. A is non singular matrix of order *n*.

	Column 1		Column 2
(A)	$(adj(A))^{-1}$	**(p)**	$\frac{A}{\lvert A\rvert}$
(B)	$adj(kA)$	**(q)**	$\frac{adj(adj\ A)}{\lvert A\rvert^{n-1}}$
(C)	$adj(adj(kA))$	**(r)**	$k^{n-1}(adj\ A)$
(D)	$adj(A^{-1})$	**(s)**	$k^{(n-1)^2}\lvert A\rvert^{n-2}A$

70. $x+y+z=6,\ x+2y+3z=10,\ x+2y+\alpha z=\beta$

	Column 1		Column 2
(A)	Unique solution	**(p)**	$\alpha=3$
(B)	No solution	**(q)**	$\alpha\neq 3$
(C)	Infinitely many solutions	**(r)**	$\beta=10$
(D)	Atleast 2 solutions	**(s)**	$\beta\neq 10$

NUMERICAL VALUE TYPE

This section has Numerical Value Type Questions. The answer to each question is a NUMERICAL/INTEGER VALUE. For each question, enter the correct numerical value of the answer. If the answer is a decimal numerical value, then round-off the value to TWO decimal places.

71. Let $A=[a_{ij}]_{n\times n}$, n is odd natural number. Then determinant of matrix $(A-A^T)^{2015}$ is _______.

72. If x, y, z distinct common roots of $z^6-1=0$ and $z^{21}-1=0$ then $\begin{vmatrix} x-y-z & 2x & 2x \\ 2y & y-x-z & 2y \\ 2z & 2z & z-x-y \end{vmatrix}$ is equal to_____.

73. Let $A=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ & $P=\begin{bmatrix} \cos\frac{\pi}{12} & \sin\frac{\pi}{12} \\ -\sin\frac{\pi}{12} & \cos\frac{\pi}{12} \end{bmatrix}$ and $Q=P^TAP$, then if $PQ^{2014}P^T=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then sum of digits of b is____.

74. Let ω be complex cube root of unity. Let $S=\begin{bmatrix} 1 & a & b \\ \omega^4 & 1 & c \\ \omega^2 & \omega^7 & 1 \end{bmatrix}$, where each of a, b, c are either ω or ω^2. Then number of distinct non singular possible such matrices S is________.

75. Let $\begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix} = ax^4+bx^3+cx^2+dx+e$ be an identity in x, then $-\left[\left(\dfrac{a+b+c+d+e}{a+e}\right)\right]$ is ____. ([.]) denotes greatest integer function).

76. $\begin{vmatrix} {}^5C_1 & {}^5C_2 & {}^5C_3 \\ {}^4C_1 & {}^4C_2 & {}^4C_3 \\ {}^3C_1 & {}^3C_2 & {}^3C_3 \end{vmatrix} - \begin{vmatrix} {}^5C_1 & {}^6C_2 & {}^7C_3 \\ {}^4C_1 & {}^5C_2 & {}^6C_3 \\ {}^3C_1 & {}^4C_2 & {}^5C_3 \end{vmatrix} =$ ______.

77. $A=\begin{bmatrix} \frac{1}{2}|[x]| & |\sin y| \\ \cos z & 1 \end{bmatrix}, B=\begin{bmatrix} \{x\} & \{y\} \\ \{z\} & 1 \end{bmatrix}$, if $x\in[-2,2]$, $y, z\in(-\pi,\pi)$ if number of triplets (x, y, z) such that $A=B$ is k, then value of $k/7$ is _______.

78. $A=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, then $A^{2014}=\lambda A^{2013}+\mu A^{2012}, \lambda+\mu=$ ___________.

79. Let A be a 3×3 matrix which contains five 'a' & four 'b' then number of symmetric matrices possible is k, number of zeros at the end of $k!$ is _________.

80. Matrix A satisfies $A^2=3A-2I$, and $A^{-1}=\dfrac{\lambda I+kA^3}{\mu}$, then $\lambda+k\mu$ is ________.

81. In a ΔABC, if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix}=0$, then $\sin^2 A+\sin^2 B+\sin^2 C=$_____.

82. For all values of $\theta \in \left[0, \frac{\pi}{2}\right]$, the determinant of the matrix $\Delta = \begin{bmatrix} -2 & \tan\theta + \sec^2\theta & 3 \\ -\sin\theta & \cos\theta & \sin\theta \\ -3 & -4 & 3 \end{bmatrix}$ is always greater than or equal to _____.

83. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If $Q = [q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\frac{q_{31} + q_{32}}{q_{21}}$ equals _____.

84. Number of positive integral solutions of the equation $\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix} = 30$ are _____.

85. Let $A = \begin{bmatrix} 1 & \frac{-1-i\sqrt{3}}{2} \\ \frac{-1+i\sqrt{3}}{2} & 1 \end{bmatrix}$, Then $A^{100} = 2^k . A$ where k is ____.

86. If the system of linear equations $x + ky + 3z = 0$, $3x + ky - 2z = 0$, $2x + 4y - 3z = 0$ has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to ____.

87. If $S_r = \alpha^r + \beta^r + \gamma^r$ the value of $\begin{vmatrix} S_0 & S_1 & S_2 \\ S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \end{vmatrix}$ is equal to $(\alpha - \beta)^k (\beta - \gamma)^{2k-2} (\gamma - \alpha)^{k^2-2}$. Then k is ____.

88. Consider three matrices $A = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$, then the value of the sum

$tr(A) + tr\left(\frac{ABC}{2}\right) + tr\left(\frac{A(BC)^2}{4}\right) + tr\left(\frac{A(BC)^3}{8}\right) + ... + \infty$ is ____

89. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$ then $A + 2B$ equals ____

90. Let $\{\Delta_1, \Delta_2, \Delta_3, ... \Delta_k\}$ be the set of third order determinants that can be made with the distinct nonzero real numbers $a_1, a_2, a_3, ... a_9$ then $k = (a + b)!$ where $a + b$ equals ____

91. Let k be a positive real number and let $A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$

If det (adj A) + det (adj B) = 10^6, then greatest integer of k is equal to ____

92. For a real number α, if the system $\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ of linear equations, has infinitely many solutions, then $1+\alpha+\alpha^2 =$

93. If $\begin{vmatrix} x & x+y & x+y+z \\ 2x & 3x+2y & 4x+3y+2z \\ 3x & 6x+3y & 10x+6y+3z \end{vmatrix}=64$, then the real value of x is____

94. Let $\Delta_1=\begin{vmatrix} a & b & a-b \\ c & d & c+d \\ a & b & a+b \end{vmatrix}$ and $\Delta_2=\begin{vmatrix} a & c & a+c \\ b & d & b+d \\ a & c & a+b+c \end{vmatrix}$ then the value of $\left|\dfrac{\Delta_1}{\Delta_2}\right|$, where $b \neq 0$ and $ad \neq bc$ is

95. Let M be a 3×3 matrix satisfying $M\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}=\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$, $M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ and $M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$. Then the sum of diagonal entries of M is ____.

96. Let ω be the complex number $\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}$. Then, the number of distinct complex numbers z satisfying $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix}=0$ is equal to____.

97. The characteristic equation of a matrix A is $\lambda^3-5\lambda^2-3\lambda+2=0$ then $|\text{adj } A| =$

98. If $a_i^2+b_i^2+c_i^2=1, \ (i=1, 2, 3)$ and $a_ia_j+b_ib_j+c_ic_j=0 \ (i\neq j;\ i, j=1, 2, 3)$ then the value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ is

99. If $x_1, x_2, x_3, ..., x_{13}$ are in A.P then the value of $\begin{vmatrix} e^{x_1} & e^{x_4} & e^{x_7} \\ e^{x_4} & e^{x_7} & e^{x_{10}} \\ e^{x_7} & e^{x_{10}} & e^{x_{13}} \end{vmatrix}$ is

100. For $a, b, c, x, y, z \in R$, if $\Delta_1=\begin{vmatrix} (a-x)^2 & (b-x)^2 & (c-x)^2 \\ (a-y)^2 & (b-y)^2 & (c-y)^2 \\ (a-z)^2 & (b-z)^2 & (c-z)^2 \end{vmatrix}$ and $\Delta_2=\begin{vmatrix} (1+ax)^2 & (1+bx)^2 & (1+cx)^2 \\ (1+ay)^2 & (1+by)^2 & (1+cy)^2 \\ (1+az)^2 & (1+bz)^2 & (1+cz)^2 \end{vmatrix}$ then $|\Delta_1/\Delta_2|=$_____.

JEE Advanced Revision Booklet	Statistics

SINGLE CORRECT ANSWER TYPE

Each of the following Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct.

1. If standard deviations for two variables X and Y are 3 and 4 respectively and their covariance is 8, then correlation coefficient between them is:

(A) $\frac{2}{3}$ **(B)** $\frac{8}{3\sqrt{2}}$ **(C)** $\frac{9}{8\sqrt{2}}$ **(D)** $\frac{2}{9}$

2. The arithmetic mean of a set of observation is $\overline{X}$. If each observation is divided by α and then is increased by 10, then the mean of the new series is:

(A) $\frac{\overline{X}}{\alpha}$ **(B)** $\frac{\overline{X}+10}{\alpha}$ **(C)** $\frac{\overline{X}+10\alpha}{\alpha}$ **(D)** $\alpha\overline{X}+10$

3. Median of ${}^{2n}C_0, {}^{2n}C_1, {}^{2n}C_2, {}^{2n}C_3, \ldots\ldots, {}^{2n}C_n$ (where n is even) is:

(A) ${}^{2n}C_{n/2}$ **(B)** ${}^{2n}C_{\frac{n+1}{2}}$ **(C)** ${}^{2n}C_{\frac{n-1}{2}}$ **(D)** 0

4. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set:

(A) is increased by 2 **(B)** is decreased by 2
(C) is two times the original median **(D)** remains the same as that of the original set

5. The means and variance of n observations $x_1, x_2, x_3, \ldots x_n$ are 5 and 0 respectively. If $\sum_{i=1}^{n} x_i^2 = 400,$ then the value of n is equal to:

(A) 80 **(B)** 25 **(C)** 20 **(D)** 16

6. If $x_1, x_2, \ldots\ldots, x_{18}$ are observations such that $\sum_{j=1}^{18}(x_j - 8) = 9$ *and* $\sum_{j=1}^{18}(x_j - 8)^2 = 45$, then the standard deviation of these observations is:

(A) 80 **(B)** 25 **(C)** 20 **(D)** 16

7. If the mean of n observations $1^2, 2^2, 3^2, \ldots n^2$ is $\frac{46n}{11}$ then n is equal to:

(A) 11 **(B)** 12 **(C)** 23 **(D)** 22

8. The standard deviation of n observations $x_1, x_2, \ldots, x_n$ is 2. If $\sum_{i=1}^{n} x_i = 20$ and $\sum_{i=1}^{n} x_i^2 = 100$, then n is:

(A) 10 or 20 **(B)** 5 or 10 **(C)** 5 or 20 **(D)** 5 or 15

9. If σ is the standard deviation of a random variable x, then the standard deviation of the random variable $ax+b$, where $a, b \in R$ is:

(A) $a\sigma + b$ **(B)** $|a|\sigma$ **(C)** $|a|\alpha + b$ **(D)** $a^2\sigma$

10. Let $x_1, x_2, ...x_n$ be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$. Then a possible value of n among the following is:

(A) 15 **(B)** 18 **(C)** 9 **(D)** 12

11. For a frequency distribution mean deviation from mean is computed by:

(A) $M.D. = \dfrac{\sum f}{\sum f|d|}$ **(B)** $M.D. = \dfrac{\sum d}{\sum f}$ **(C)** $M.D. = \dfrac{\sum fd}{\sum f}$ **(D)** $M.D. = \dfrac{\sum f|d|}{\sum f}$

12. For a frequency distribution standard deviation is computed by applying the formula.

(A) $\sigma = \sqrt{\dfrac{\sum fd^2}{\sum f} - \left(\dfrac{\sum fd}{\sum f}\right)^2}$ **(B)** $\sigma = \sqrt{\left(\dfrac{\sum fd}{\sum f}\right)^2 - \dfrac{\sum fd^2}{\sum f}}$

(C) $\sigma = \sqrt{\dfrac{\sum fd^2}{\sum f} - \dfrac{\sum fd}{\sum f}}$ **(D)** $\sigma = \sqrt{\left(\dfrac{\sum fd}{\sum f}\right)^2 - \dfrac{\sum fd^2}{\sum f}}$

13. If r is the variance and 6 is the standard deviation, then:

(A) $r = 1/\sigma^2$ **(B)** $r = 1/\sigma$ **(C)** $r = \sigma^2$ **(D)** $r^2 = \sigma$

14. The mean deviation from the median is:

(A) Equal to that measured from another value
(B) Minimum if all observations are positive
(C) Greater than that measured from any other value
(D) Less than that measured from any other value

15. The standard deviation of the data:

$x:$ 1 a a^2...............................a^n

$f:$ ${}^nC_\circ$ ${}^nC_1$ ${}^nC_2$...............................${}^nC_n$ is:

(A) $\left(\dfrac{1+a^2}{2}\right)^n - \left(\dfrac{1+a}{2}\right)^n$ **(B)** $\left(\dfrac{1+a^2}{2}\right)^{2n} - \left(\dfrac{1+a}{2}\right)^n$

(C) $\left(\dfrac{1+a}{2}\right)^{2n} - \left(\dfrac{1+a^2}{2}\right)^n$ **(D)** 0

16. The mean deviation of the series a, $a + d$, $a + 2d$, ..., a + 2nd from its mean is:

(A) $\dfrac{(n+1)d}{2n+1}$ **(B)** $\dfrac{nd}{2n+1}$ **(C)** $\dfrac{n(n+1)d}{(2n+1)}$ **(D)** $\dfrac{(2n+1)d}{n(n+1)}$

17. A batsman scores runs in 10 innings as 38, 70, 48, 34, 42, 55, 63, 46, 54 and 44. The mean deviation about mean is:

(A) 8.6 **(B)** 6.4 **(C)** 10.6 **(D)** 7.6

18. The mean deviation of the members 3, 4, 5, 6, 7 from the mean is:

(A) 25 **(B)** 5 **(C)** 1.2 **(D)** 0

19. Let $x_1, x_2, x_3, ..., x_n$ be the values taken by a variable X and $y_1, y_2, ..., y_n$ be the values taken by a variable Y such that $y_i = ax_i + b, i = 1, 2, ..., n$. Then:

(A) $Var(Y) = a^2 Var(X)$ **(B)** $Var(X) = a^2 Var(Y)$
(C) $Var(Y) = Var(X) + b$ **(D)** None of these

20. If the standard deviation of a variable X is σ, then the standard deviation of variable $\frac{ax+b}{c}$ is:

(A) $a\sigma$ **(B)** $\frac{a\sigma}{c}$ **(C)** $\left|\frac{a}{c}\right|\sigma$ **(D)** $\frac{a\sigma + b}{c}$

21. If the standard deviation of a set of observations is 8 and if each observation is divided by –2, the standard deviation of the new set of observations will be:
(A) –4 **(B)** –8 **(C)** 8 **(D)** 4

22. If two variants X and Y are connected by the relation $Y = \frac{aX + b}{c}$, where a, b, c are constants such that $ac < 0$, then:

(A) $\sigma_y = \frac{a}{c}\sigma_x$ **(B)** $\sigma_y = \frac{-a}{c}\sigma_x$ **(C)** $\sigma_y = \frac{a}{c}\sigma_x + b$ **(D)** None of these

23. Let $x_1, x_2, ..., x_n$ be n observations such that $\sum x_i^2 = 400$ and $\sum x_i = 80$, then a possible value of n is:
(A) 9 **(B)** 12 **(C)** 15 **(D)** 18

24. In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class are 72, then average marks of girls is:
(A) 73 **(B)** 65 **(C)** 68 **(D)** 74

25. The median of a set of 9 distinct observations is 20.5. If each of the largest four observations of the set in increased by 2, then the median of the new set.
(A) is increased by 2 **(B)** is decreased by 2
(C) is 2 times of the original median **(D)** remains the same as the original set

26. Suppose a population A has 100 observations 101, 102, ..., 200 and another population B has 100 observations 151, 152, ... 250. If V_A *and* V_B represent the variance of the two populations respectively, then $\frac{V_A}{V_B}$ is:

(A) 1 **(B)** $\frac{9}{4}$ **(C)** $\frac{4}{9}$ **(D)** $\frac{2}{3}$

27. Quartile deviation for a frequency distribution is:

(A) $Q = Q_3 - Q_1$ **(B)** $Q = \frac{1}{2}(Q_3 - Q_1)$ **(C)** $Q = \frac{1}{3}(Q_3 - Q_1)$ **(D)** $Q = \frac{1}{4}(Q_3 - Q_1)$

28. If the coefficient of variation of a distribution is 45% and the mean is 12, then its standard deviation is:
(A) 5.2 **(B)** 5.3 **(C)** 5.4 **(D)** None of these

29. In an experiment with 15 observations on x, the following results were available $\sum x^2 = 2830, \sum x = 170$. One observation that was 20 was found to be wrong and was replaced by the correct value 30, then the correct variance is:
(A) 78.00 **(B)** 188.66 **(C)** 177.33 **(D)** 8.33

30. Mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80, then the possible values of a and b respectively are:
(A) 5, 2 **(B)** 1, 6 **(C)** 3, 4 **(D)** 0, 7

31. The mean and standard deviation of the marks of 200 candidates were found to be 40 and 15 respectively. Later it was discovered that a score of 40 was wrongly used as 50. The correct mean and standard deviation respectively are
(A) 14.98, 39.95 **(B)** 39.95, 14.98 **(C)** 39.95, 224.5 **(D)** None of these

Assertion & Reason

Each of the following question contains two statements:
Statement -1 (**Assertion**) and Statement -2 (**Reason**)
Each of these questions also has four alternative choices, only one of which is correct. Select the correct choice.

32. Let $x_1, x_2 x_3 -----x_n$ be n given numbers and a is a variable number
$A^2 = (x_1 - a)^2 + (x_2 - a)^2 + (x_3 - a)^2 -----+(x_n - a)^2$ consider the following statements:
Statement - 1: A^2 is minimum when $a = \frac{x_1 + x_2 + x_3 + -----+x_n}{n}$
Statement-2: Minimum value of $A^2 = |x_1 - \bar{x}| + |x_2 - \bar{x}| + ------+|x_n - \bar{x}|$, where $\bar{x} = \frac{x_1 + x_2 + x_3 + -----+x_n}{n}$
Which of the following is true?
(A) **Statement – 1:** is true, statement – 2 is true;
Statement – 1: is a correct explanation for statement – 2
(B) **Statement – 1:** is true, statement – 2 is true;
Statement – 1: is not a correct explanation for statement – 2
(C) **Statement – 1:** is true, statement – 2 is false.
(D) **Statement – 1:** is false, Statement – 2 is true.

Paragraph for Questions 33 – 34

Consider the observation $x_1 = 1,\ x_2 = 2,\ x_3 = 3, ------x_{100} = 100,\ x_{101} = 101,\ x_{102} = 102,\ x_{103} = 103,\ x_{104} = 104$

33. Median of the given data is:
(A) 51 **(B)** 52.5 **(C)** 51.5 **(D)** 53

34. Mean deviation from the median of the given data is
(A) $\frac{51}{2}$ **(B)** 26 **(C)** $\frac{51 \times 52}{103}$ **(D)** None of these

SUBJECTIVE

35. Find the mean of the binomial coefficients in the expansion of $(1+x)^n$

36. **(a)** Show that the sum of the squares of the derivations of a set of values is minimum when taken about mean.
(b) If a variate X is expressed as a linear function of two variates U and V in the form $X = aU + bV$, then find the mean $\overline{X}$ of X.

37. Find the variance of first n even numbers.

38. **(a)** If each observation of a raw data whose variance is σ^2 is multiplied by k then find the variance of new set.
(b) The median and standard deviation of a distribution are 20 & 4 respectively. If each item is increased by 2 then find the new median & standard deviation.

39. **(a)** If coefficient of variation of a series is 50. Its S.D. is 21.2. Then find its arithmetic mean?

(b) The mean of two samples of sizes 200 and 300 were found to be 25, 10 respectively. Their standard deviations were 3 and 4 respectively. Find the variance of combined sample of size 500.

40. The mean of *n* observations $x_1, x_2, x_3, ... x_n$ is $\overline{X}$. If $(a - b)$ is added to each of the observations, show that the mean of the new set of observations is $\overline{X} + (a - b)$.

41. The mean monthly salary of 10 members of a group is 1445, one more member whose monthly salary is Rs. 1500 has joined in group. Find the mean monthly salary of 11 members of the group.

42. The sum of the deviations of a set of *n* values $x_1, x_2, x_3, ..., x_n$ measured from 50 is – 10 and the sum of deviations of the values from 46 is 70. Find the value of *n* and the mean.

43. If $\overline{X}$ is the mean of 10 natural numbers $x_1, x_2, x_3, ..., x_{10}$. Show that $(x_1 - \overline{x}) + (x_2 - \overline{x}) + (x_3 - \overline{x}) + ... (x_{10} - \overline{x}) = 0$.

44. The mean of 200 items was 50. Later on, it was discovered that the two items were misread 92 and 8 instead of 192 and 88. Find the correct mean.

45. Thirty children were asked about the number of hours they watched TV programs in the previous week. The results were as follows:

1	6	2	3	5	12	5	8	4	8
10	3	4	12	2	8	15	1	17	6
3	2	8	5	9	6	8	7	14	12

(i) Make a grouped frequency distribution table for this data, taking class width 5 and one of the class intervals as 5-10.

(ii) How many children watched television for 15 or more hours a week?

46. The mean of marks scored by 100 students was found to be 40. Later on it was discovered that a score of 53 was misread as 83. Find correct mean.

47. Find values of *n* and $\overline{x}$ in following case: $\sum_{i=1}^{n}(x_i - 12) = -10$ and $\sum_{i=1}^{n}(x_i - 3) = 62$

48. Find the median of following data: 41, 43, 127, 99, 61, 92, 71, 58, 51. If 58 is replaced by 85, what will be the new median.

49. The following observations have been arranged in ascending order. If the median of data is 63. Find the value of *x*. 29, 32, 48, 50, *x*, *x* + 2, 72, 78, 84, 95.

50. The mean height of 29 male workers is 71 cms and 31 female workers is 48 cms. Find the combined mean height of all 60 workers in the factory.

51. The price of a commodity is increased by 5% from 1997 to 1998, 8% from 1998 to 1999 and 53% from 1999 to 2000. Find the average increase percent from the period 1997 to 2000.

52. The arithmetic mean of 4 observations was calculated as 22. It was later observed that one of the observations was recorded a 14 instead of 40. Find the correct arithmetic mean.

53. The weighted arithmetic mean of 10 observations was 36. However, a particular observation was recorded as 60 instead of 40. In what ratio should be the weights of correct and incorrect reading be so as to have no change in AM.

54. **(a)** The geometric mean of *n* items is G. If first term is kept same, second made twice, third made thrice….. and so on, find the new mean.

(b) If each item is made *n* times, then prove that mean also becomes *n* times.

55. Show that the mean deviation from the mean of the A.P $a, a+d, a+2d, ..., a+2nd$ is independent of the common difference of A.P.

56. If the observations $x_1, x_2, x_3 .... x_n$ are changed to $x_1 + y, x_2 + y, ... x_n + y$ where y is a positive or a negative number, show that the variance remains unchanged.

57. The mean and standard deviation of one sample are respectively 54.8 and 8, the mean and standard deviation of another sample are 50.3 and 7 respectively. The size of the first sample is 50 and that of the second is 100. Find the mean and standard deviation of the composite sample (size 150) combining the above two samples.

58. The geometric mean of 6 observations was calculated as 11. If was later observed that one of the observation was recorded as 1 instead of 64. Find the correct geometric mean.

59. The mean annual salaries paid to 1000 employees of a company was Rs. 5000. The mean annual salaries paid to male and female employees were Rs. 200 and Rs. 4200 respectively. Determine the percentage of males and females employed by the company.

60. If a vehicle covers the distance along four sides of a square with four speeds $x, 2x, 3x$ and $4x\ m/\sec$ respectively, then show that harmonic mean of speeds is better average than arithmetic mean and hence find the average speed.

61. The mean and standard deviation of a set of 100 observations were worked out as 40 and 5 respectively. But by mistake a value 50 was taken in place of 40 for the observation. Recalculate the correct mean and standard deviation.

62. Prove that sum of squares of the deviations of a set of values is minimum when taken about mean.

63. Calculate the mean and standard deviation of the following distribution:

x :	$2.5-7.5$	$7.5-12.5$	$12.5-17.5$	$17.5-22.5$	
f :	12	28	65	121	
x :	$22.5-27.5$	$27.5-32.5$	$32.5-37.5$	$37.5-42.5$	$42.5-47.5$
f :	175	198	176	120	66
x :	$47.5-52.5$	$52.5-57.5$	$57.5-62.5$		
f :	27	9	3		

64. **(a)** The arithmetic mean and variance of a set of 10 figures are known to be 17 and 33 respectively. Of the 10 figures, one figure (i.e., 26) was subsequently found inaccurate, and was weeded out. What is the resulting (a) arithmetic mean and (b) standard deviation.

(b) The mean and standard deviation of 20 items is found to be 10 and 2 respectively. At the time of checking, it was found one item 8 was incorrect. Calculate the mean and standard deviation if (i) the wrong item is omitted, and (ii) it is replaced by 12.

(c) For a frequency distribution of marks in statistics of 200 candidates (grouped in intervals 0 –5, 5 – 10, …., etc.), the mean and standard deviation were found to be 40 and 15 respectively. Later it was discovered that the score 43 was misread as 53 in obtaining the frequency distribution. Find the corrected mean and standard deviation corresponding to the corrected frequency distribution.

65. The mean of 5 observations is 4.4 and variance is 8.24. It three of the five observations are 1, 2 and 6, find the other two.

66. **(a)** Scores of two golfers for 24 rounds were as follows:

Golfer A: 74, 75, 78, 72, 77, 79, 78, 81, 76, 72, 72, 77, 74, 70, 78, 79, 80, 81, 74, 80, 75, 70,71, 73

Golfer B: 86, 84, 80, 88, 89, 85, 86, 82, 82, 79, 86, 80, 82, 76, 86, 89, 87, 83, 80, 88, 86, 81, 81, 87

For which golfer may be considered to be a more consistent player?

(b) The sum and sum of squares corresponding to length X (in cms.) and weight Y (in gms.) of 50 tapioca tubers are given below:

$\Sigma X = 212, \quad \Sigma X^2 = 902.8$

$\Sigma Y = 261, \quad \Sigma Y^2 = 1457.6$

Which is more varying, the length or weight?

67. **(a)** A frequency is distribution is divided into two parts. The mean and standard deviation of the first part are m_1 and s_1 and those of second part are m_2 and s_2 respectively. Obtain the mean and standard deviation for the combined distribution.

(b) The means of two samples of size 50 and 100 respectively are 54.1 and 50.3 and the standard deviations are 8 and 7. Obtain the mean and standard deviation of the sample of size 150 obtained by combining the two samples.

(c) A distribution consists of three components with frequencies 200, 250 and 300 having means 25, 10 and 15 and standard deviations 3, 4 and 5 respectively. Show that the mean of the combined group is 16 and its standard deviation is 7.2 approximately.

68. In a certain test for which the pass marks is 30, the distribution of marks of passing candidates classified by sex (boys and girls) were as given below:

Marks	**Frequency**	
	Boys	**Girls**
30-34	5	15
35-39	10	20
40-44	15	30
45-49	30	20
50-54	5	5
55-59	5	–
Total	70	90

The overall means and standard deviation of marks for boys including the 30 failed were 38 and 10. The corresponding figures for girls including the 10 failed were 35 and 9.

(i) Find the mean and standard deviation of marks obtained by the 30 boys who failed in the test.

(ii) The moderation committee argued that percentage of passed among girls is higher because the girls are very studious and if the intention is to pass those who are really intelligent, a higher pass marks should be used for girls. Without questioning the propriety of this argument, suggest what the pass mark should be which would allow only 70% of the girls to pass.

(iii) The prize committee decided to award prizes to the best 40 candidates (irrespective of sex) judged on the basis of marks obtained in the test. Estimate the number of girls who would receive prizes.

69. Find the mean and variance of first n-natural numbers.

70. In a frequency distribution, the n intervals are 0 to 1, 1 to 2, ….., $(n - 1)$ to n with equal frequencies. Find the mean deviation and variance.

71. If the mean and standard deviation of a variable x and m and σ respectively, obtain the mean and standard deviation of $(ax - b)/c$, where a, b and c are constants.

72. In a series of measurements we obtain m_1 values of magnitude x_1, m_2 values of magnitude x_2, and so on. If $\bar{x}$ is the mean value of all the measurements, prove that the standard deviation is $\sqrt{\dfrac{\Sigma m_r (k - x_r)^2}{\Sigma m_r} - \delta^2}$ where $\bar{x} = k + \delta$ and k is any constant.

73. **(a)** Show that in a discrete series if deviations are small compared with mean M so that $(x/M)^2$ and higher powers of (x/M) are neglected, prove that (i) $MH = G^2$ (ii) $M - 2G + H = 0,$ where G is geometric mean and H is harmonic mean.

(b) The mean and standard deviation of a variable x are m and σ respectively. If the deviations are small compared with the value of the mean, show that

(i) $\text{Mean}(\sqrt{x}) = \sqrt{m}\left(1 - \dfrac{\sigma^2}{8m^2}\right)$ **(ii)** $\text{Mean}\left(\dfrac{1}{\sqrt{x}}\right) = \dfrac{1}{\sqrt{m}}\left(1 + \dfrac{3\sigma^2}{8m^2}\right)$ approximately.

(c) If the deviation $X_i = x_i - M$ is very small in comparison with mean M and $(X_i / M)^2$ and higher powers of (X_i / M) are neglected prove that $V\sqrt{\dfrac{2(M - G)}{M}}$ where G is the geometric mean of the values $x_1, x_2, \ldots, x_n$ and V is the coefficient of dispersion (σ / M).

74. From a sample of observations the arithmetic mean and variance are calculated. It is then found that one of the values, x_1, is in error and should be replaced by x_1'. Show that the adjustment to the variance to correct this error is $\dfrac{1}{n}(x_1' - x_1)\left(x_1' + x_1 - \dfrac{x_1' - x_1 + 2T}{n}\right)$ where T is the total of the original results.

75. Show that, if the variable takes the values 0, 1, 2, …., n with frequencies proportional to the binomial coefficients ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$ respectively then the mean of the distribution is $(n/2)$, the mean square deviation about $x = 0$ is $n(n+1)/4$ and the variance is $n/4$.

76. **(a)** Let r be the range and s be the standard deviation of a set of observations $x_1, x_2, \ldots, x_n$, then prove by general reasoning or otherwise that $s \le r$.

(b) Let r be the range and $S\left(\dfrac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2\right)^{1/2}$ be the standard deviation of a set of observations $x_1, x_2, \ldots, x_n$, then prove that $S \le r\left(\dfrac{n}{n-1}\right)^{1/2}$.

VMC Vidyamandir Classes SINCE 1986
IIT JEE | MEDICAL | FOUNDATION

Answers to JEE Advanced Revision Booklet | Mathematics

QUADRATIC EQUATIONS

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	C	A	B	C	B	D	A	C	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	A	C	D	A	B	A	D	ABCD	ABC
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
AD	AC	AD	ABD	ABCD	ACD	CD	CD	BD	BCD
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
[A-T] [B-R, S] [C-P, S] [D-T]	[A-Q] [B-R] [C-P] [D-T]	2	0	5	1	0	4	5	7

TRIGONOMETRY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
B	B	D	C	A	D	B	A	C	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	A	B	B	B	B	A	A	B	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	C	B	B	D	BC	BD	A	ABD	BD
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
AD	CD	BD	ABC	AB	BD	BD	AB	AD	ABD
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
BD	BD	ABD	ABCD	ABCD	ACD	ABCD	ABC	BCD	CD
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
BC	ABC	BC	AC	ABC	AC	AB	ABD	ABD	2
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
3	6	6	4	1	2	9	8	6	3
71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
2	3	1	3	9	6	7	8	8	3
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	
1	2	2	7	9	16	65	1	8	

SEQUENCE AND SERIES

1	2	3	4	5	6	7	8	9	10
D	D	B	D	B	B	C	B	D	A
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	B	C	B	C	D	AD	BCD	CD	AB
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
ABD	ABC	AD	AB	CD	AC	BC	ABCD	BC	ACD
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
ABC	BCD	BC	BC	BD	ABCD	**ABCD**	**BD**	[A-Q] [B-T] [C-P] [D-R]	[A-T] [B-Q] [C-S] [D-P]
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
2	6	661750	289	69	352	0	2017	2018	4

COMPLEX NUMBERS

1	2	3	4	5	6	7	8	9	10
D	A	D	A	B	B	A	B	C	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	A	A	D	C	C	A	C	C	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
A	D	C	ABC	BD	ABCD	D	BD	BC	AD
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
ABCD	ABCD	AB	BD	BC	AB	CD	ABC	AD	BD
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
BD	AB	AC	BC	ABC	CD	ABCD	AB	ABD	AB
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
AB	AD	CD	BC	ABC	AD	BD	AC	AB	ABCD
61	**62**	**63**	**64**		**65**		66		
AC	AB	CD	[A-r] [B-s] [C-q] [D-p]		[A-q] [B-s] [C-r] [D-p]		[A – q] [B-p] [C-r] [D-s]		
67	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**
1	5	7	6	5	5	0	0	3	9
77	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**
1	0.414	0	6.25	1	0.20	10	1	1	10
87	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**
0.50	0	4	1	5	4.88	0.875	18.75	5	6.25

PERMUTATIONS & COMBINATIONS

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
D	D	B	C	B	B	A	D	C	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	C	A	D	D	B	C	B	D	D
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
A	B	C	C	A	B	C	ABD	BC	BC
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
BC	BCD	ABCD	CD	BD	BD	ABCD	BD	ACD	AC
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
BD	CD	ABC	ABC	AB	AD	ABCD	ABCD	ABD	ABCD
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
ABC	AB	ABC	AC	ABCD	BC	ABC	BD	ABC	AD
61	**62**	**63**	**64**	65			66		
BC	ABC	AD	ABD	[A-r [B-s] [C-p] [D-q]]			[A-r] [B-s] [C-p] [D-q]		
67		68		69				**70**	**71**
[A-q] [B-s] [C-p] [D-r]		[A-s] [B-r] [C-p] [D-q]		[A-p, r, s] [B-p, r, s] [C-q, t] [D-q, t]				1	3
72	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**	**81**
7	7	9	5	9	2	6	9	47	23
82	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**
14	4	10	3.150	243	6	240	15.68	10	729
92	**93**	**94**	**95**	**96**	**97**	**98**	**99**		
315	2	7	7	2500	10	4	5		

BINOMIAL THEOREM

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
D	B	A	B	B	C	D	B	D	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	D	A	AD	ABCD	BCD	ABC	BC	CD	ABCD
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	
BCD	CD	ABC	ACD	AC	CD	ABC	BD	[A-P, Q] [B-R, T] [C-P, Q] [D-P, Q, S]	
30		**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**
[A-T] [B-Q][C-S] [D-R]		2250000	21	1024	2252	13	1	1	2
39	**40**								
1	3								

STRAIGHT LINE

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
C	A	D	D	C	B	B	D	C	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	B	A	B	B	B	B	D	B	C
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
B	D	A	BD	ABCD	AB	ABCD	AC	ABD	BD
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
BC	BC	AD	ABC	ABC	ABCD	BD	AB	BCD	AD
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
ABC	ABD	AB	ABC	ABD	BC	ABC	ABCD	ABC	AC
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
ABCD	ABC	AB	ACD	BC	ABCD	ABC	ABCD	AB	CD
61	**62**	**63**	**64**		**65**		**66**		
ABC	ABCD	ACD	[A-p] [B-s] [C-q] [D-s]		[A-q] [B-p] [C-s] [D-r]		[A-s] [B-p] [C-q] [D-r]		
67		**68**		**69**			**70**	**71**	**72**
[A-s] [B-p] [C-q] [D-r]		[A-r] [B-p] [C-s] [D-q]		[A-r] [B-s] [C-q] [D-p]			2	6	2
73	**74**	**75**	**76**	**77**	**78**	**79**	**80**	**81**	**82**
7	6	8	2	2	3	13	3	9	16
83	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**
3	1.80	4	3	16	24	6	4	5	7
93	**94**	**95**	**96**	**97**	**98**				
4	2	5	12	27	9				

CIRCLE

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	B	A	B	B	A	A	B	D	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	A	B	A	A	C	D	C	B	D
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
B	D	d	d	d	c	c	d	BC	AD
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
AB	ABD	AB	ABCD	AB	AC	AC	ACD	ACD	AB
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
ABC	ABC	ABCD	ABC	BC	ABD	ABD	AD	ABC	BCD
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
CD	CD	AD	BC	BCD	AB	ABCD	ABC	BC	BD
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	
BC	AB	ABD	ACD	AC	AC	BC	ABC	AC	
70		**71**		**72**		**73**		**74**	
[A-q] [B-p][C-t] [D-r]		[A-r] [B-p][C-q] [D-q]		[A-s] [B-q][C-t] [D-r]		[A-r, B-s, C-p, D-q]		[A-r, B-p, C-s, D-p]	
75	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**
8	4	6	4	1	0	2	25	0.5	1.66
85	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**
1	8	4	40	5	1	3	3	0	8
95	**96**	**97**	**98**	**99**	**100**	**101**			
5	2	5	6	7	4	3			

CONIC SECTION

1	2	3	4	5	6	7	8	9	10
A	A	B	B	B	D	B	B	B	A
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	D	A	A	A	B	A	B	C	B
21	**22**	**23**	**24**	**25**	26	27	28	**29**	**30**
A	C	C	D	B	A	D	B	B	A
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
B	A	C	B	AC	BCD	BD	AD	AC	AC
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
CD	ABCD	AC	AC	AC	ABC	AC	AC	ABC	AD
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
A	ABC	B	ABCD	AD	BCD	AB	CD	ABC	BC
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
CD	ABCD	AB	BD	BCD	AC	ABD	**BC**	**AD**	**AC**
71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
ABD	ABCD	ABC	BC	BD	BC	AB	AB	AB	D
81		**82**		**83**		**84**		**85**	
[A-r] [B-s] [C-p] [D-q]		[A-p, r] [B-p, q, r] [C-q] [D-q, s]		[A-p,q,r,s] [B-p,q,r,s] [C-q,r,s] [D-p]		[A-q] [B-q] [C-s] [D-p]		[A-p, s] [B-q] [C-r] [D-p, q, s]	
86	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**
A	1	4	1	2	1	6	3	4	8
96	**97**	**98**	**99**	**100**	**101**	**102**	**103**	**104**	**105**
4	2	0	0	3	6.33	2	4	80	2
106	**107**	**108**	**109**	**110**	**111**	**112**	**113**	**114**	**115**
4	2	8	3	3	1	3	3	1	4
116									
1.32									

FUNCTIONS

1	2	3	4	5	6	7	8	9	10
B	D	C	C	A	A	B	A	A	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
A	A	B	B	C	ACD	AC	ABC	ABC	ABCD
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	
ABCD	AD	AC	BD	AB	AD	ABCD	BCD	CD	
30		**31**		**32**		**33**	**34**	**35**	**36**
[A-s] [B-r] [C-q] [D-p]		[A-r] [B-s] [C-p] [D-p]		[i-c] [ii-d][iii-b] [iv-c]		2	3	22	5
37	**38**	**39**	**40**	**41**	**42**	**43**	**44**		
8	4	2049	2	37	7	12	1		

DIFFERENTIAL CALCULUS-1

1	2	3	4	*5*	*6*	*7*	8	9	10
D	D	C	A	B	D	C	C	B	B
11	*12*	13	14	*15*	16	17	18	19	20
A	A	B	D	A	B	C	B	B	AB
21	22	23	24	25	26	27	28	29	*30*
AC	AC	AB	AB	AC	AB	AC	ABD	AD	AC
31	32	*33*	34	35	*36*	37	38	39	40
ABCD	ABD	AC	ABCD	AC	BC	ABC	AC	AD	ABD
41	42	43	44	45	46	47	48	49	50
B, C	ABC	AC	BCD	AD	AC	ACD	AB	BCD	BCD
51	52	53	54	55	56	57	58		
BC	BC	AC	BC	BD	ABC	ABCD	[A-p, q, r, s] [B-p, q, r, s] [C-p, q, r, s] [D-r]		
59		60		61			62	63	64
[A-s] [B-p] [C-p] [D-p]		[A-s] [B-r] [C-q, r] [D-p]		[A-p, q] [B-p, s] [C-q] [D-r, t]			1	1	6
65	66	67	68	69	70	71	72	73	74
1	8	2	2	4	8	5	3	4	1
75	76	77	78	79	80	81	82	83	84
9	1.41	0	3	2	3	1	1.5	4	2
85	86	87	88	89	90				
3.14	8	3	12.07	6	2				

DIFFERENTIAL CALCULUS-2

1	2	3	4	5	6	7	8	9	10
D	B	A	B	A	D	C	A	C	D
11	**12**	**13**	**14**	**15**	**16**	***17***	**18**	**19**	**20**
D	A	D	A	D	B	B	A	A	D
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
C	C	B	D	AB	ABCD	AC	ABCD	AD	ABCD
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
AB	BD	BCD	ABCD	ABC	ACD	ABC	BCD	AC	CD
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
BCD	BCD	BCD	ABC	AC	BCD	BC	ABC	ABC	ACD
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
ABC	AC	AC	BD	ABCD	AD	AD	**B**	D	D
61	**62**	**63**	**64**			**65**		**66**	
ABD	BD	BC	[A-p, q, r, s] [B-p, r] [C-p, q, r, s] [D-p, q, r, s]			[A-p] [B-q] [C-r] [D-s]		[A-q] [B-r] [C-p] [D-t]	
67	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**
9	0	2	0	2	1	3	2	9	5
77	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**
4	2	3	5	1200	1	0	1	2	2
87	**88**	**89**	**90**	**91**	**92**	**93**	**94**		
2	0	5	9	9	3	48	0		

INTEGRAL CALCULUS-1

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
C	D	B	A	A	B	D	A	D	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	C	C	A	C	B	C	D	A	B
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
C	AB	AB	BD	AC	ABD	AC	AB	ABC	BCD
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
ABC	BC	CD	ABD	ABC	BC	BC	CD	AD	AD
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
B,C	AC	ABC	ACD	AC	ABCD	BC	AC	AB	ABC
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	
CD	CB	BC	BD	AC	ABC	ABD	BC	AD	
60		**61**		**62**		**63**	**64**	**65**	**66**
[A-p, q] [B-r, s] [C-p] [D-p, q]		[A-r] [B-s] [C-q] [D-p]		[A-s] [B-t] [C-r] [D-q]		1	4	2	3
67	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**
0	2	1	10	6	4	3015	521	8	3
77	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**	**86**
2.5	4	0.5	3.6	2.05	1	403	1	0	12
87	**88**	**89**	**90**	**91**	**92**				
1.59	2019	1.8	2.14	2	11				

INTEGRAL CALCULUS-2

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
A	C	C	B	C	A	C	D	C	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	D	C	D	C	D	D	A	D	A
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
B	D	B	D	D	D	C	B	D	B
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A	A	AB	A	ABC	ABD	ABC	AB	ACD	AD
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
ABC	BC	ABD	ABC	AB	BC	ABD	AB	ACD	AC
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
AD	AB	BD	ACD	AC	ABCD	ABC	BD	A,B,C,D	A,B
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
A,B,C	A,B,D	A,B	A,B	A,B,C	A,B,C	A,B,C,D	B,C	BD	A,B,C
71	**72**	**73**	**74**	**75**	**76**	**77**		**78**	
ABC	A,B,C	C,D	A,B,C	CD	A,B,D	[A-s] [B-s] [C-r] [D-q]		[A-r] [B-p] [C-s] [D-q]	
79		**80**		**81**		**82**	**83**	**84**	**85**
[A-q] [B-r, s] [C-p] [D-p]		[A-p, q] [B-p, q, r] [C-q, s] [D- s]		[A-r] [B-p] [C-q] [D-s]		3	4	6	8
86	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**
8	4	0	1	1	2	0.72	3.14	101	4
96	**97**	**98**	**99**	**100**	**101**	**102**	**103**	**104**	**105**
8	10.50	2	0	8.15	2	1	8	85	101
106	**107**	**108**	**109**						
153	61	10	4.14						

DIFFERENTIAL EQUATIONS

1	2	3	4	5	6	7	8	9	10
C	C	B	A	A	C	C	A	A	D
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	C	C	D	B	A	C	D	B	C
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
A	C	ABCD	AB	CD	AB	BC	AD	AB	AB
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
AC	ABCD	BCD	CD	ABD	ABD	CD	BC	AC	AB
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
BC	ABC	BCD	ACD	AC	ABCD	A,B,C	AD	A,B,C	A,B,D
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	
C,D	A,B,C,D	B,C	A,B	A,B, C	B,C	A,D	C	C,D	
60		**61**		**62**		**63**		**64**	
[A-u] [B-s] [C-q] [D-p]		[A-r] [B-s] [C-p] [D-q]		[A-r] [B-s] [C-q] [D-p]		[A-q, s] [B-p] [C-p] [D-q, r, s]		[A-q] [B-r] [C-p] [D-s]	
65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**
4	2	8	2	1	1	8	2	4	1
75	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**
8	1	1	1	3	1	3	7	8	8
85	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	
2	3	2	5	62	8	1	1	0	

VECTORS

1	2	3	4	5	6	7	8	9	10
D	A	B	A	D	C	C	D	A	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	C	C	D	C	A	A	B	A	D
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
B	B	C	D	A	C	D	B	B	A
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A	B	C	C	A	B	A	B	D	C
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
CD	AB	AC	AC	ABC	ABCD	AB	BC	ABC	AC
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
BD	BD	AB	AD	ABCD	AC	BC	AB	CD	AC
61	**62**	63		64		**65**		**66**	
BC	AC	[A-q, r, s] [B-p, s] [C-p, r] [D-p, q]		[A-r] [B-p] [C-s] [D-q]		[A-p, q, r, s] [B-p, q] [C-p, r] [D-r]		[A-r, s] [B-q, r] [C-q, r] [D-p, q, r]	
67		**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
[A-r] [B-p][C-p, q] [D-s]		5	9	3	1	6	7	5	**0**
76	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**	
1	4	0	2	$2\sqrt{5/7}$	7	1	3	2	

THREE DIMENSIONAL GEOMETRY

1	2	3	4	5	6	7	8	9	10
D	A	D	C	A	D	A	A	A	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	A	A	C	A	ABC	A	B	D	A
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
C	B	A	D	C	B	A	B	A	D
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
B	D	D	D	B	C	B	A	BC	AB
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
A	A	ABCD	AB	CD	C	AD	BD	AB	AB
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	
AC	ABCD	AC	ABC	BC	BC	AC	ABC	AB	
60		61		62		**63**		**64**	
[A-p,q][B-p,q] [C-r,s][D-q]		[A-q] [B-p][C-s] [D-r]		[A-t][B-p][C-s][D-r]		[A-q,s][B-s][C-r][D-p]		[A-q] [B-p][C-p] [D-r]	
65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**
2	1	4	5	6	3	4	1	4	7
75	**76**	**77**	**78**	**79**	**80**				
4	5	2	4	6	2				

PROBABILITY

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
C	D	D	D	D	A	B	D	C	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
B	A	B	C	D	C	B	D	D	C
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
C	A	D	A	C	B	A	C	D	C
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
A	C	A	B	ABCD	ABC	BD	BC	AD	AD
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
ABCD	AC	AC	AC	ABCD	ABCD	BC	ABCD	AC	BD
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
ABCD	BCD	ACD	AC	AB	AD	ACD	ABCD	ABC	AD
61	**62**	**63**	**64**	**65**		**66**		**67**	
BC	AD	AC	ABC	[A-q,s][B-p,t][C-r,t]		[A-t][B-q] [C-s][D-r]		[A-r][B-s][C-p][D-q]	
68		**69**		**70**	**71**	**72**	**73**	**74**	**75**
[A-r] [B-p][C-q][D-s]		[A-q][B-p][C-r][D-q]		7	8	3	9	8	9
76	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**	**85**
8	3	1.44	2.27	0.40	0.75	0.45	0.9523	1.25	0.0130
86	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**
0.299	0.1054	0.422	0.018181	0.142857	1.35	0.1142	1.33	0.20	0.3488
96	**97**								
0.9722	4.17								

MATRICES & DETERMINANTS

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
C	D	C	B	C	A	D	D	A	C
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
C	A	A	C	A	B	C	D	C	C
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
B	C	A	C	B	BCD	ABC	AB	CD	AD
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
AC	ABC	CD	AB	BCD	AB	AB	ACD	ABCD	ABC
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
ABD	ABCD	ABC	AB	ACD	AC	ABC	BD	AB	BCD
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
AC	BC	AC	A	BCD	AC	ABC	CD	AC	CD
61	**62**	**63**	**64**	**65**	**66**		**67**		
BD	ABC	AB	CD	CD	[A-q, r] [B-p, s] [C-s] [D-r]		[A-q] [B-r, s] [C-p] [D-p]		
68		**69**		**70**		**71**	**72**	**73**	**74**
[A-s] [B-q] [C-r] [D-p]		[A-p, q] [B-r] [C-s] [D-p, q]		[A-q] [B-p, s] [C-p, r] [D-p, r]		0	0	7	2
75	**76**	**77**	**78**	**79**	**80**	**81**	**82**	**83**	**84**
9	0	6	7	2	1	2.25	3	103	3
85	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**
99	10	2	6	6	9	4	1	4	2
95	**96**	**97**	**98**	**99**	**100**				
9	1	4	1	0	1				

STATISTICS

1	2	3	4	5	6	7	8	9	10
A	C	A	D	D	D	A	C	B	B
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
D	A	C	D	A	C	A	C	A	C
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
D	B	D	B	D	A	B	C	A	C
31	**32**	**33**	**34**						
B	C	B	B						

SUBJECTIVE

35	36	37	38	39
$\frac{2^n}{n+1}$	**(B)** $a\bar{U}+b\bar{V}$	$\frac{n^2-1}{3}$	**(A)** $k^2\sigma^2$ **(B)** Median will go by 2 and S.D. will remain constant.	**(A)** 42.4 **(B)** 67.2

41	42	44	45	46	47	48
1450	n = 26, mean=49.5	50.9	(ii) Two children	39.7	$n = 8, \bar{x} = 10.75$	61.71

49	50	51	52	53	54	55	57
62	59.12	24.50	19.5	6 : 1	(A) $\bar{X}+\frac{k+1}{2}$	$\frac{(n+1)n}{2n+1}$	$\bar{X}_{12} = 51.67$; $\sigma_{12} = 7.6$

58	59	60	61	63	64
22	22	80% and 20%	$\bar{X} = 39.9$; $\sigma = 4.9$	Mean = 30.005, Standard Deviation = 0.01	Mean = 39.95, S.D. = 14.974

65	66	67	68
4, 9	Golfer B is more consistent player.	Combined mean = 51.57, Combined S.D. = 7.5 approx.	**(i)** $\bar{x}$ = 22.83, σ_2 = 8.27 **(ii)** 39 **(iii)** 15

69	70	71
$\bar{x} = \frac{n+1}{2}, \sigma_2 = \frac{n^2-1}{12}$	n is odd, M.D. = $\frac{n^2-1}{4n}$; n is even, M.D. = $\frac{n}{4}$; Variance = $\frac{n^2-1}{12}$	$\bar{u} = \frac{1}{c}(a\bar{x}-b), \sigma_M = \left\|\frac{a}{c}\right\|\sigma$